Graphs of Functions & Their Inverses

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Given a tube partway filled with liquid will have a height dependent on the temperature. That is, we have height \( h \) (in cm) as a function of Temperature \( T \) (in F).

   (a) What does \( h(32) = 1 \) mean in physical terms?
   (b) What does \( h(212) = 10 \) mean in physical terms?
   (c) Describe the inverse function \( h^{-1} \). What are the inputs? Outputs? Is there a device that we have that performs this in real life?

2. Let \( m \) be the function completely defined by the table:

\[
\begin{array}{c|c|c|}
\pi & \sqrt{2} & 2 \\
\frac{3}{2} & 2 & \sqrt{2} \\
\end{array}
\]

   (a) Complete the table above to define \( q^{-1} \).
   (b) Plot the graph of \( q \) on the set of axes provided.
   (c) Use a different mark (or color) to graph \( q^{-1} \) on the same set of axes.
   (d) Notice the point \( (\frac{3}{2}, 2) \) is on the graph of \( q \) and \( (2, \frac{3}{2}) \) is on the graph of \( q^{-1} \).
   (e) Find the domain of \( q \) and range of \( q^{-1} \). Are there any similarities?

The observations you made in (e) & (f) are true in general, that is:
if \( f \) is the inverse of \( g \) then: Domain of \( f \) = Range of \( g \)  Range of \( f \) = Domain of \( g \)
3. Let \( n \) be the function defined by the following graph:

(a) Will \( n \) have an inverse? Why?

(b) Use the observations from \#2d to graph \( n^{-1} \).

4. Let \( p \) be the function defined by \( p(x) = x^2 - 1 \).

(a) Draw the graph of \( p \).

(b) Will \( p \) have an inverse? Why?

(c) What might we do to try and build something "kind of like an inverse"?