Outline:

- Fine grain vs. coarse grain parallelism
- Manually splitting loops between threads
- Examples with bugs

Reading:

- class notes: OpenMP section of Bibliography
- $UWHPSC/codes/openmp$
- https://computing.llnl.gov/tutorials/openMP/
Fine vs. coarse grain parallelism

Fine grain: Parallelize at the level of individual loops, splitting work for each loop between threads.

Coarse grain: Split problem up into large pieces and have each thread deal with one piece.

May need to synchronize or share information at some points.
Fine vs. coarse grain parallelism

**Fine grain**: Parallelize at the level of individual loops, splitting work for each loop between threads.

**Coarse grain**: Split problem up into large pieces and have each thread deal with one piece.

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More similar to what must be done in MPI.
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More similar to what must be done in MPI.

**Domain Decomposition:** Splitting up a problem on a large domain (e.g. three-dimensional grid) into pieces that are handled separated (with suitable coupling).
Solution of independent ODEs by Euler’s method

Solve \( u_i'(t) = c_i u_i(t) \) for \( t \geq 0 \)
with initial condition \( u_i(0) = \eta_i \).

Decoupled system of ODEs
for \( i = 1, 2, \ldots, n \)
Solve $u'_i(t) = c_i u_i(t)$ for $t \geq 0$
with initial condition $u_i(0) = \eta_i$. Decoupled system of ODEs
for $i = 1, 2, \ldots, n$

Exact solution: $u_i(t) = e^{c_i t} \eta_i$. 

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Solution of independent ODEs by Euler’s method

Solve \( u'_i(t) = c_i u_i(t) \) for \( t \geq 0 \)
with initial condition \( u_i(0) = \eta_i \).  \( i = 1, 2, \ldots, n \)
\( \)  Decoupled system of ODEs

Exact solution: \( u_i(t) = e^{c_i t} \eta_i \).

Euler method: \( u_i(t + \Delta t) \approx u_i(t) + \Delta t c_i u_i(t) = (1 + c_i \Delta t) u_i(t) \).

Implement this for large number of time steps for large \( n \).
Solution of independent ODEs by Euler’s method

Solve \( u_i'(t) = c_i u_i(t) \) for \( t \geq 0 \) with initial condition \( u_i(0) = \eta_i \). Decoupled system of ODEs for \( i = 1, 2, \ldots, n \)

Exact solution: \( u_i(t) = e^{c_i t} \eta_i \).

Euler method: \( u_i(t + \Delta t) \approx u_i(t) + \Delta t c_i u_i(t) = (1 + c_i \Delta t) u_i(t) \).

Implement this for large number of time steps for large \( n \).

For each \( i \) time stepping can’t be easily made parallel.

But for large \( n \), this problem is embarassingly parallel:

Problem for each \( i \) is completely decoupled from problem for any other \( i \). Could solve them all simultaneously with no communication needed.
Fine grain solution with parallel do loops

```fortran
!$omp parallel do
do i=1,n
  u(i) = eta(i)
enddo

do m=1,nsteps
  !$omp parallel do
do i=1,n
    u(i) = (1.d0 + dt*c(i))*u(i)
  enddo
enddo

Note that threads are forked nsteps+1 times.

Requires shared memory:
  don’t know which thread will handle each i.
```
Fine grain solution with parallel do loops

Might try to fork threads only once via: Wrong!

```fortran
!$omp parallel private(m)
!$omp do
  do i=1,n
    u(i) = eta(i)
  enddo
  do m=1,nsteps
    !$omp do
      do i=1,n
        u(i) = (1.d0 + dt*c(i))*u(i)
      enddo
    enddo
  enddo
!$omp end parallel
```

Error: the loop on m will be done independently by each thread. (Actually works in this case but not good coding.)
Fine grain solution with parallel do loops

Might try to fork threads only once via:  Wrong!

!$omp parallel private(m)
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do i=1,n
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do m=1,nsteps
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do i=1,n
        u(i) = (1.d0 + dt*c(i))*u(i)
        enddo
    enddo
!$omp end parallel

Error: the loop on m will be done independently by each thread.
(Actually works in this case but not good coding.)
Can rearrange loops:

```fortran
!$omp parallel private(m)
!$omp do
do i=1,n
  u(i) = eta(i)
enddo
!$omp do
do i=1,n
  do m=1,nsteps
    u(i) = (1.d0 + dt*c(i))*u(i)
  enddo
enddo
!$omp end parallel
```

Only works because ODEs are decoupled — can take all time steps on \( u_1(t) \) without interacting with \( u_2(t) \), for example.
Fine grain solution with parallel do loops

Can rearrange loops:

```c
!$omp parallel private(m)
!$omp do
do i=1,n
    u(i) = eta(i)
enddo

!$omp do
do i=1,n
    do m=1,nsteps
        u(i) = (1.d0 + dt*c(i))*u(i)
    enddo
enddo
!$omp end parallel
```

Only works because ODEs are decoupled — can take all time steps on $u_1(t)$ without interacting with $u_2(t)$, for example.
Coarse grain solution of ODEs

Split up $i = 1, 2, \ldots, n$ into $n$ threads disjoint sets. A set goes from $i = \text{istart}$ to $i = \text{iend}$.

These private values are different for each thread.

Each thread handles 1 set for the entire problem.

```fortran
!$omp parallel private(istart,iend,i,m)
istart = ??
iend = ??

do i=istart,iend
    u(i) = eta(i)
enddo

do m=1,ntssteps
    do i=istart,iend
        u(i) = (1.d0 + dt*c(i))*u(i)
        enddo
enddo
!$omp end parallel
```

Threads are forked only once,
Each thread only needs subset of data.
Setting \texttt{istart} and \texttt{iend}

Example: If $n=100$ and $nthreads = 2$, we would want:

Thread 0: \texttt{istart= 1 and iend= 50},  
Thread 1: \texttt{istart=51 and iend=100}.

If \texttt{nthreads} divides \texttt{n} evenly...

\begin{verbatim}
points_per_thread = n / nthreads
!$omp parallel private(thread_num, istart, iend, i)
  thread_num = 0       ! needed in serial mode
  !$ thread_num = omp_get_thread_num()
  istart = thread_num * points_per_thread + 1
  iend = (thread_num+1) * points_per_thread
  do i=istart,iend
    ! work on thread’s part of array
    enddo
  ...
!$omp end parallel
\end{verbatim}
Setting \texttt{istart} and \texttt{iend} more generally

Example: If \(n=101\) and \(n\text{threads} = 2\), we would want:

Thread 0: \(\text{istart}= 1\) and \(\text{iend}= 51\),
Thread 1: \(\text{istart}=52\) and \(\text{iend}=101\).

If \(n\text{threads} \text{ might not divide } n\) evenly...

\texttt{points\_per\_thread} = \((n + n\text{threads} - 1) / n\text{threads}\)

\begin{verbatim}
!$omp parallel private(thread_num, istart, iend, i)
  thread_num = 0  ! needed in serial mode
  !$ thread_num = omp_get_thread_num()

  istart = thread_num * points_per_thread + 1
  iend = min((thread_num+1) * points_per_thread, n)

  do i=istart,iend
    ! work on thread’s part of array
    enddo
...

!$omp end parallel
\end{verbatim}
Example: Normalizing a vector

Given a vector (1-dimensional array) $x$,
Compute the normalized vector $x/\|x\|_1$, with $\|x\|_1 = \sum_{i=1}^n |x_i|$

Fine-grain: Using parallel do loops.

```fortran
norm = 0.d0
!$omp parallel do reduction(+ : norm)
do i=1,n
   norm = norm + abs(x(i))
endo do

!$omp parallel do
do i=1,n
do i=1,n
   x(i) = x(i) / norm
endo do
```

Note: Must finish computing norm before using for any $x(i)$, so we are using the implicit barrier after the first loop.
Example: Normalizing a vector

Another fine-grain approach, forking threads only once:

! from $UWHPSC/codes/openmp/normalize1.f90
norm = 0.d0
!$omp parallel private(i)

!$omp do reduction(+ : norm)
do i=1,n
    norm = norm + abs(x(i))
  enddo
!$omp barrier ! not needed (implicit)

!$omp do
do i=1,n
    x(i) = x(i) / norm
  enddo
!$omp end parallel
Example: Normalizing a vector

Compute the normalized vector $x/\|x\|_1$, with $\|x\|_1 = \sum_{i=1}^{n} |x_i|$

Coarse grain version:

Assign blocks of $i$ values to each thread. Threads must:

- Compute thread’s contribution to $\|x\|_1$,

  $$\text{norm\_thread} = \sum_{\text{istart}}^{\text{iend}} |x_i|,$$

- Collaborate to compute total value $\|x\|_1$:

  $$\|x\|_1 = \sum_{\text{threads}} \text{norm\_thread}$$

- Loop over $i = \text{istart, iend}$ to divide $x_i$ by $\|x\|_1$. 

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Example: Normalizing a vector

! from $UWHPSC/codes/openmp/normalize2.f90

    norm = 0.d0
    !$omp parallel private(i,norm_thread, &
    !$omp               istart,iend,thread_num)
    !$ thread_num = omp_get_thread_num()
    istart = thread_num * points_per_thread + 1
    iend = min((thread_num+1) * points_per_thread, n)

    norm_thread = 0.d0
    do i=istart,iend
       norm_thread = norm_thread + abs(x(i))
    enddo

    ! update global norm with value from each thread:
    !$omp critical
       norm = norm + norm_thread
    !$omp end critical

    !$omp barrier  !! needed here
    do i=istart,iend
       y(i) = x(i) / norm
    enddo

    !$omp end parallel
Example: Normalizing a vector — parallel block

```fortran
norm_thread = 0.d0
do i=istart,iend
    norm_thread = norm_thread + abs(x(i))
  enddo

! update global norm with value from each thread:
!$omp critical
  norm = norm + norm_thread
!$omp end critical

!$omp barrier  !! needed here

do i=istart,iend
  y(i) = x(i) / norm
  enddo
```
1. Not declaring proper variables private
1. Not declaring proper variables private

2. Setting \( \text{norm} = 0.\text{d}0 \) inside parallel block.

Ok if it’s in a \texttt{omp single} block. Otherwise second thread might set to zero after first thread has updated by \texttt{norm_thread}.
Normalizing a vector — possible bugs

1. Not declaring proper variables private

2. Setting $\text{norm} = 0.d0$ inside parallel block.
   Ok if it’s in a $\text{omp single}$ block. Otherwise second thread might set to zero after first thread has updated by $\text{norm\_thread}$.

3. Not using $\text{omp critical}$ block to update global $\text{norm}$.
   Data race.
Normalizing a vector — possible bugs

1. Not declaring proper variables private

2. Setting $\text{norm} = 0.d0$ inside parallel block.

   Ok if it's in a `omp single` block. Otherwise second thread might set to zero after first thread has updated by `norm_thread`.

3. Not using `omp critical` block to update global `norm`.

   Data race.

4. Not having a `barrier` between updating norm and using it.

   First thread may use `norm` before other threads have added their contributions.
Normalizing a vector — possible bugs

1. Not declaring proper variables private.

2. Setting $\text{norm} = 0.\text{d0}$ inside parallel block.

Ok if it's in a \texttt{omp single} block. Otherwise second thread might set to zero after first thread has updated by \texttt{norm\_thread}.

3. Not using \texttt{omp critical} block to update global \texttt{norm}.

Data race.

4. Not having a \texttt{barrier} between updating norm and using it.

First thread may use \texttt{norm} before other threads have added their contributions.

None of these bugs would give compile or run-time errors! Just wrong results (sometimes).
OpenMP example with shared exit criterion

Solve $u'_i(t) = c_i u_i(t)$ for $t \geq 0$

with initial condition $u_i(0) = \eta_i$.

Exact solution: $u_i(t) = e^{c_i t} \eta_i$.

Euler method: $u_i(t + \Delta t) \approx u_i(t) + \Delta t c_i u_i(t) = (1 + c_i \Delta t) u_i(t)$.

New wrinkle: Stop time stepping when any of the $u_i(t)$ values exceeds 100.

(Will certainly happen as long as $c_j > 0$ for some $j$.)
OpenMP example with shared exit criterion

Stop time stepping when any of the $u_i(t)$ values exceeds 100.

Idea:

Each time step, compute $u_{\text{max}}$ = maximum value of $u_i$ over all $i$ and exit the time-stepping if $u_{\text{max}} > 100$. 

Study code in `$UWHPSC/codes/openmp/umax1.f90`.

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Stop time stepping when any of the $u_i(t)$ values exceeds 100.

Idea:

Each time step, compute $\text{umax} = \text{maximum value of } u_i \text{ over all } i$ and exit the time-stepping if $\text{umax} > 100$.

Each thread has a private variable $\text{umax\_thread}$ for the maximum value of $u_i$ for its values of $i$. Updated for each $i$. 
Stop time stepping when any of the $u_i(t)$ values exceeds 100.

Idea:

Each time step, compute $\text{umax} = \text{maximum value of } u_i \text{ over all } i$ and exit the time-stepping if $\text{umax} > 100$.

Each thread has a private variable $\text{umax}_\text{thread}$ for the maximum value of $u_i$ for its values of $i$. Updated for each $i$.

Each thread updates shared $\text{umax}$ based on its $\text{umax}_\text{thread}$. This needs to be done in critical section.
Stop time stepping when any of the $u_i(t)$ values exceeds 100.

Idea:

Each time step, compute $\text{umax} = \text{maximum value of } u_i$ over all $i$ and exit the time-stepping if $\text{umax} > 100$.

Each thread has a private variable $\text{umax\_thread}$ for the maximum value of $u_i$ for its values of $i$. Updated for each $i$.

Each thread updates shared $\text{umax}$ based on its $\text{umax\_thread}$. This needs to be done in critical section.

Also need two barriers to make sure all threads are in synch at certain points.
Stop time stepping when any of the $u_i(t)$ values exceeds 100.

**Idea:**

Each time step, compute $\text{umax} = \text{maximum value of } u_i \text{ over all } i$ and exit the time-stepping if $\text{umax} > 100$.

Each thread has a private variable $\text{umax\_thread}$ for the maximum value of $u_i$ for its values of $i$. Updated for each $i$.

Each thread updates shared $\text{umax}$ based on its $\text{umax\_thread}$. This needs to be done in critical section.

Also need two barriers to make sure all threads are in synch at certain points.

**Study code in** $\$UWHPSC/codes/openmp/umax1.f90$. 

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OpenMP example with shared exit criterion

!$omp parallel private(i,m,umax_thread, &
!$omp               istart,iend,thread_num)
!$ thread_num = omp_get_thread_num()
istart = thread_num * points_per_thread + 1
iend = min((thread_num+1) * points_per_thread, n)
do m=1,nsteps
   umax_thread = 0.d0
   !$omp single
      umax = 0.d0
   !$omp end single
   do i=istart,iend
      u(i) = (1.d0 + c(i)*dt) * u(i)
      umax_thread = max(umax_thread, u(i))
   enddo

   !$omp critical
      umax = max(umax, umax_thread)
   !$omp end critical
   !$omp barrier
   if (umax > 100) exit
   !$omp barrier
   enddo
!$omp end parallel
do loop in parallel block:

\[ \text{do m=1,nsteps} \]
\[ \text{umax\_thread} = 0.d0 \]
\[ \text{(!$omp\ single)} \]
\[ \text{umax} = 0.d0 \]
\[ \text{(!$omp\ end\ single)} \]
\[ \text{do i=istart,iend} \]
\[ \text{u(i)} = (1.d0 + c(i)\times dt) \times u(i) \]
\[ \text{umax\_thread} = \text{max}(\text{umax\_thread}, u(i)) \]
\[ \text{enddo} \]
\[ \text{(!$omp\ critical)} \]
\[ \text{umax} = \text{max}(\text{umax}, \text{umax\_thread}) \]
\[ \text{(!$omp\ end\ critical)} \]
\[ \text{(!$omp\ barrier)} \]
\[ \text{if (umax > 100) exit} \]
\[ \text{(!$omp\ barrier)} \]
\[ \text{enddo} \]
OpenMP example with shared exit criterion

If there were no barriers, the following could happen:

Thread 0 executes critical section first, setting $\text{umax}$ to 0.5.
Thread 0 checks if $\text{umax} > 100$. False, starts next iteration.
Thread 1 executes critical section, updating $\text{umax}$ to 110.
Thread 1 checks if $\text{umax} > 100$. True, so it exits.
Thread 0 next sets $\text{umax}$ to 0.4.

Thread 0 might never reach $\text{umax} > 100$. Runs forever.
OpenMP example with shared exit criterion

If there were no barriers, the following could happen:

Thread 0 executes critical section first, setting $\text{umax}$ to 0.5.
Thread 0 checks if $\text{umax} > 100$. False, starts next iteration.
Thread 1 executes critical section, updating $\text{umax}$ to 110.
Thread 1 checks if $\text{umax} > 100$. True, so it exits.
Thread 0 next sets $\text{umax}$ to 0.4.

Thread 0 might never reach $\text{umax} > 100$. Runs forever.

With only first barrier, the following could happen:

$\text{umax} < 100$ in iteration $m$.
Thread 1 checks if $\text{umax} > 100$. Go to iteration $m + 1$.
Thread 1 does iteration on $i$ and sets $\text{umax} > 100$,
Stops at first barrier.
Thread 0 (iteration $m$) checks if $\text{umax} > 100$. True, Exits.

Thread 0 never reaches first barrier again, code hangs.