Today:
- Cache considerations
- Optimizing Fortran codes
- Debugging Fortran

Wednesday:
- Software packages
- LAPACK and BLAS

Read: Class notes and references
There are several new sections!

Code optimization

We will look at some basics worth keeping in mind.

However:
- Also important to consider programmer time.
- Writing readable code is very important in getting program correct.
- Some optimizations not worth spending time on.
- Often best to first get code working properly and then determine whether optimization is necessary.
  “Premature optimization is the root of all evil” (Don Knuth)
- If so, determine which parts of code need to be improved and spend effort on these sections.
- Use optimized software such as BLAS, LAPACK.

Array ordering — which loop is faster?

```
integer, parameter :: m = 4097, n = 10000
real(kind=8), dimension(m,n) :: a

do i = 1,m
    do j=1,n
        a(i,j) = 0.d0
    enddo
enddo
```

```
do j = 1,n
    do i=1,m
        a(i,j) = 0.d0
    enddo
enddo
```

First: 0.72 seconds, Second: 0.19 seconds

Memory Hierarchy

Between registers and memory there are 2 or 3 levels of cache, each larger but slower.

Registers: access time 1 cycle
L1 cache: a few cycles
L2 cache: ~ 10 cycles
(Main) Memory: ~ 250 cycles
Hard drive: 1000s of cycles
Much worse if \( m \) is high power of 2

```fortran
integer, parameter :: m = 4096, n = 10000
real(kind=8), dimension(m,n) :: a

do i = 1, m
    do j = 1, n
        a(i,j) = 0.d0
    enddo
endo

do j = 1, n
    do i = 1, m
        a(i,j) = 0.d0
    enddo
endo
```

First: 2.4 seconds, Second: 0.19 seconds

---

More about cache

Simplified model of one level direct mapped cache.

32-bit memory address: \( 4.3 \times 10^9 \) addresses

Suppose cache holds \( 512 = 2^9 \) cache lines (9-bit address)

A given memory location cannot go anywhere in cache.

9 low order bits of memory address determine cache address.

For a memory fetch:
- Determine cache address, check if this holds desired words from memory.
- If so, use it.
- If not, check “dirty bit” to see if has been modified since load.
- If so, write to memory before loading new cache line.

---

Cache collisions

Return to example where matrix has \( 4096 = 2^{12} \) rows.

Cache line holds 64 bytes = 8 floats. \( 4096/8 = 512 \) cache lines per column of matrix.

Loading one column of matrix will fill up cache lines 0,1,2,...,511.

Second column will go back to cache line 0.
But all elements in cache have been used before this happens,

Prefetching can be done by optimizing compiler.

Worse — Going across the rows:

The first 8 elements of column 1 go to cache line 0.
The first 8 elements of column 2 also map to cache line 0.
Similarly for all columns. The rest of cache stays empty.

---

More about cache

If cache holds more lines:

1024 lines \( \implies \)

- first 8 bytes of column 1 go to cache line 0,
- first 8 bytes of column 2 go to cache line 512,
- first 8 bytes of column 3 go to cache line 0,
- first 8 bytes of column 4 go to cache line 512.

Still only using 1/512 of cache.

In practice cache is often set associative: small number of cache addresses for each memory address.
Matrix dimensions that are high powers of 2 should usually be avoided.

Even though natural for some algorithms such as FFTs

May be worth declaring larger arrays and only using part of it.

---

**Padding**

do j=1,n
  do i=1,n
    b(j,i) = a(i,j)
  enddo
enddo

Accessing *a* by column but *b* by row!

Better to do by blocks — illustrate on board.

See also: Bill Gropp’s class at Illinois, Lecture 2

---

**Matrix transpose**

Suppose stride *s* divides *n*. Then can rewrite as:

**Strip mining:**

\[
\begin{align*}
  &j = 1, n, s \\
  &j = jj, jj + s - 1 \\
  &i = 1, n, s \\
  &i = ii, ii + s - 1 \\
  &b(j, i) = a(i, j)
\end{align*}
\]

**Loop reordering:**

\[
\begin{align*}
  &j = 1, n, s \\
  &i = 1, n, s \\
  &j = jj, jj + s - 1 \\
  &i = ii, ii + s - 1 \\
  &b(j, i) = a(i, j)
\end{align*}
\]

Loops over blocks in outer loops, within block in inner loops.

---

**Block matrix multiply**

Compute *C* = *AB*. Can partition into blocks:

\[
\begin{bmatrix}
  C_{11} & C_{12} \\
  C_{21} & C_{22}
\end{bmatrix} =
\begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
  B_{11} & B_{12} \\
  B_{21} & B_{22}
\end{bmatrix}
\]

where

\[
C_{ij} = A_{11}B_{1j} + A_{12}B_{2j}
\]

When blocks *A*\(_{11}\) and *B*\(_{11}\) are in cache can compute the *A*\(_{11}\)*\(_{11}\) *part* of *C*\(_{11}\) = *A*\(_{11}\)*\(_{11}\) + *A*\(_{12}\)*\(_{12}\)*\(_{11}\)

Might next bring in *B*\(_{12}\) and compute the *A*\(_{11}\)*\(_{12}\) *part* of

\[
C_{12} = A_{11}B_{12} + A_{12}B_{22}
\]
Flop rate for matrix multiply/add

\[ a, b \text{ each } 1000 \times 1000 \text{ matrices.} \]

Compare time of \( c = \text{matmul}(a, b) \text{ vs. } c = a+b \).

Compare megaflops per second:
\[ 1 \times 10^{-6} \times \text{nflops}/(t_2-t_1). \]

Add: CPU time (sec): 0.00687200
rate: \( 145.52 \text{ megaflops/sec} \)

Multiply: CPU time (sec): 2.38393500 slower
rate: 838.53 megaflops/sec higher

For addition: \( \text{nflops} = n^2 \)
For multiplication: \( \text{nflops} = (2n-1)n^2, \)
More flops, but each element is used \( n \) times,
\[ \Rightarrow \text{More flops per memory access} \Rightarrow \text{higher rate.} \]

Optimizing Fortran

See the examples at
$\text{CLASSHG/codes/fortran/optimize}$.
$\text{CLASSHG/codes/particles}$.

Developing programs to minimize bugs

• Start simple and add features slowly
  Tackle stripped-down version of problem first

• Modularize: break problem into pieces
  Subroutines or functions with
  well-defined inputs and outputs
  Develop and debug separately first

Unit tests: Test small pieces (early and often)

• Python has a unittest module to assist,
  Allows specification of test cases, test suites.

Regression testing:
Test that adding a new feature (or fixing a bug)
didn’t break old features.
Keep sample programs that test various features of the code,
Run these after making improvements or “fixing” a bug.
Debugging in Fortran

Need to compile with \textit{-g} flag, no optimization. (Runs slower, so recompile once debugged.)

\texttt{gdb} — command line debugger similar to \texttt{pdb}.

\texttt{ddd} — GUI front end for \texttt{gdb}, can be obtained on VM via:

\begin{verbatim}
$ sudo apt-get install ddd
\end{verbatim}

\texttt{Eclipse} — IDE that uses \texttt{gdb}.

Much better commercial debuggers available, e.g. \texttt{totalview}.

Segmentation faults

Sometimes running a program gives:

\begin{verbatim}
$ ./a.out
Segmentation Fault
\end{verbatim}

This generally means the code tried to write to a part of memory where it didn't have permission.

Or:

\begin{verbatim}
$ ./a.out
Bus error
\end{verbatim}

This generally means a bad address not even in memory.

Often these are a result of an array index out of bounds.

\begin{verbatim}
integer :: i
real(kind=8), dimension(10) :: x
do i=1,15
  x(i) = 20.d0
  print *, "i = ",i
  print *, x(i)
endo
\end{verbatim}

produces:

\begin{verbatim}
... i = 10 20.00000000000 00 i = 1077149696 Segmentation fault
\end{verbatim}

\texttt{Why?} \texttt{x(11)} points to memory where \texttt{i} is stored!
Overwriting variables

integer :: i
real(kind=8), dimension(10) :: x

do i=1,15
  x(i) = 0.d0
  print *, "i = ",i
  print *, x(i)
enddo

Goes into an infinite loop — i gets reset to 0.

Array bounds checking

$ gfortran -fbounds-check run1.f90

Gives:
...
i = 10
20.0000000000000
Fortran runtime error: Array reference out of bounds for array ’x’, upper bound of dimension 1 exceeded (in file ’demo1.f90’, at line 11)