Today:

- Cache considerations
- Optimizing Fortran codes
- Debugging Fortran

Wednesday:

- Software packages
- LAPACK and BLAS

Read: Class notes and references
There are several new sections!
We will look at some basics worth keeping in mind.

However:

• Also important to consider programmer time.

• Writing readable code is very important in getting program correct.

• Some optimizations not worth spending time on.

• Often best to first get code working properly and then determine whether optimization is necessary. “Premature optimization is the root of all evil” (Don Knuth)

• If so, determine which parts of code need to be improved and spend effort on these sections.

• Use optimized software such as BLAS, LAPACK.
Between registers and memory there are 2 or 3 levels of cache, each larger but slower.

**Registers:** access time 1 cycle

**L1 cache:** a few cycles

**L2 cache:** \( \sim 10 \text{ cycles} \)

(Main) **Memory:** \( \sim 250 \text{ cycles} \)

**Hard drive:** 1000s of cycles
Array ordering — which loop is faster?

integer, parameter :: m = 4097, n = 10000
real(kind=8), dimension(m,n) :: a

do i = 1,m
    do j=1,n
        a(i,j) = 0.d0
    enddo
enddo

do j = 1,n
    do i=1,m
        a(i,j) = 0.d0
    enddo
enddo

R.J. LeVeque, University of Washington
AMath 483/583, Lecture 10, April 18, 2011
Array ordering — which loop is faster?

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do j = 1,n
    do i=1,m
        a(i,j) = 0.d0
        enddo
    enddo

First: 0.72 seconds, Second: 0.19 seconds
Much worse if $m$ is high power of 2

```fortran
integer, parameter :: m = 4096, n = 10000
real(kind=8), dimension(m,n) :: a

do i = 1,m
    do j=1,n
        a(i,j) = 0.d0
    enddo
enddo

doj = 1,n
    do i=1,m
        a(i,j) = 0.d0
    enddo
enddo
```

First: 2.4 seconds, Second: 0.19 seconds
More about cache

Simplified model of one level direct mapped cache.

32-bit memory address: \(4.3 \times 10^9\) addresses

Suppose cache holds \(512 = 2^9\) cache lines (9-bit address)

A given memory location cannot go anywhere in cache.
9 low order bits of memory address determine cache address.
More about cache

Simplified model of one level direct mapped cache.

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Suppose cache holds 512 = \( 2^9 \) cache lines (9-bit address)

A given memory location cannot go anywhere in cache. 9 low order bits of memory address determine cache address.

For a memory fetch:

- Determine cache address, check if this holds desired words from memory.
- If so, use it.
- If not, check “dirty bit” to see if has been modified since load.
- If so, write to memory before loading new cache line.
Return to example where matrix has $4096 = 2^{12}$ rows.

Cache line holds 64 bytes = 8 floats. $4096/8 = 512$ cache lines per column of matrix.

Loading one column of matrix will fill up cache lines $0, 1, 2, \ldots, 511$.

Second column will go back to cache line 0. But all elements in cache have been used before this happens, prefetching can be done by optimizing compiler.
Cache collisions

Return to example where matrix has $4096 = 2^{12}$ rows.

Cache line holds 64 bytes = 8 floats. $4096/8 = 512$ cache lines per column of matrix.

Loading one column of matrix will fill up cache lines $0, 1, 2, \ldots, 511$.

Second column will go back to cache line 0. But all elements in cache have been used before this happens, prefetching can be done by optimizing compiler.

Worse — Going across the rows:

The first 8 elements of column 1 go to cache line 0.

The first 8 elements of column 2 also map to cache line 0.

Similarly for all columns. The rest of cache stays empty.
More about cache

If cache holds more lines:

1024 lines

first 8 bytes of column 1 go to cache line 0,
first 8 bytes of column 2 go to cache line 512,
first 8 bytes of column 3 go to cache line 0,
first 8 bytes of column 4 go to cache line 512.

Still only using $1/512$ of cache.
If cache holds more lines:

1024 lines $\rightarrow$

- first 8 bytes of column 1 go to cache line 0,
- first 8 bytes of column 2 go to cache line 512,
- first 8 bytes of column 3 go to cache line 0,
- first 8 bytes of column 4 go to cache line 512.

Still only using 1/512 of cache.

In practice cache is often set associative: small number of cache addresses for each memory address.
Padding

Matrix dimensions that are high powers of 2 should usually be avoided.

Even though natural for some algorithms such as FFTs
May be worth declaring larger arrays and only using part of it.
do  j=1,n
   do  i=1,n
      b(j,i) = a(i,j)
   enddo
endo

Accessing $a$ by column but $b$ by row!

Better to do by blocks — illustrate on board.

See also: Bill Gropp’s class at Illinois, Lecture 2
Suppose stride $s$ divides $n$. Then can rewrite as:

**Strip mining:**

```fortran
    do jj=1,n,s
        do j=jj,jj+s-1
            do ii=1,n,s
                do i=ii,ii+s-1
                    b(j,i) = a(i,j)
            enddo
        enddo
    enddo
```

**Loop reordering:**

```fortran
    do jj=1,n,s
        do ii=1,n,s
            do j=jj,jj+s-1
                do i=ii,ii+s-1
                    b(j,i) = a(i,j)
            enddo
        enddo
    enddo
```

Loops over blocks in outer loops, within block in inner loops.
Compute $C = AB$. Can partition into blocks:

$$
\begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix}
= 
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
$$

where

$$C_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j}$$

When blocks $A_{11}$ and $B_{11}$ are in cache can compute the $A_{11}B_{11}$ part of $C_{11} = A_{11}B_{11} + A_{12}B_{21}$
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where

$$C_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j}$$

When blocks $A_{11}$ and $B_{11}$ are in cache can compute the $A_{11}B_{11}$ part of $C_{11} = A_{11}B_{11} + A_{12}B_{21}$

Might next bring in $B_{12}$ and compute the $A_{11}B_{12}$ part of $C_{12} = A_{11}B_{12} + A_{12}B_{22}$
Flop rate for matrix multiply/add

\(a, b \text{ each } 1000 \times 1000 \text{ matrices.}\)

Compare time of \(c = \text{matmul}(a,b)\) vs. \(c = a+b\).

Compare megaflops per second: \(1e-6*\text{nflops}/(t2-t1)\).

Add: CPU time (sec): 0.00687200
rate: 145.52 megaflops/sec

Multiply: CPU time (sec): 2.38393500 slower
rate: 838.53 megaflops/sec higher
Flop rate for matrix multiply/add

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For addition: \(\text{nflops} = n^{**2}\)
For multiplication: \(\text{nflops} = (2n-1) \times n^{**2},\)
More flops, but each element is used \(n\) times,
\(\implies\) More flops per memory access \(\implies\) higher rate.
See the examples at

$\text{CLASSHG/codes/fortran/optimize}$.

$\text{CLASSHG/codes/particles}$.
Developing programs to minimize bugs

- Start simple and add features slowly
  Tackle stripped-down version of problem first
Developing programs to minimize bugs

- Start simple and add features slowly
  Tackle stripped-down version of problem first

- Modularize: break problem into pieces
  Subroutines or functions with well-defined inputs and outputs
  Develop and debug separately first
Developing programs to minimize bugs

Unit tests: Test small pieces (early and often)

- Python has a unittest module to assist,
- Allows specification of test cases, test suites.
Developing programs to minimize bugs

Unit tests: Test small pieces (early and often)

- Python has a unittest module to assist,
- Allows specification of test cases, test suites.

Regression testing:

Test that adding a new feature (or fixing a bug) didn’t break old features.

Keep sample programs that test various features of the code, Run these after making improvements or “fixing” a bug.
Debugging in Fortran

Need to compile with \(-g\) flag, no optimization. (Runs slower, so recompile once debugged.)

**gdb** — command line debugger similar to pdb.

**ddd** — GUI front end for gdb, can be obtained on VM via:

```
$ sudo apt-get install ddd
```

**Eclipse** — IDE that uses gdb.

Much better commercial debuggers available, e.g. **totalview**.
See the examples at

$CLASSHG/codes/fortran/debug.
Sometimes running a program gives:

$ ./a.out
Segmentation Fault

This generally means the code tried to write to a part of memory where it didn’t have permission.

Or:

$ ./a.out
Bus error

This generally means a bad address not even in memory.

Often these are a result of an array index out of bounds.
Segmentation faults

integer :: i
real(kind=8), dimension(10) :: x

do i=1,15
  x(i) = 20.d0
  print *, "i = ",i
  print *, x(i)
endo

produces:

...  
i = 10
   20.00000000000000
i = 1077149696
Segmentation fault

Why? x(11) points to memory where i is stored!
integer :: i
real(kind=8), dimension(10) :: x

do i=1,15
  x(i) = 0.d0
  print *, "i = ",i
  print *, x(i)
enddo

Goes into an infinite loop — i gets reset to 0.
Array bounds checking

$ gfortran -fbounds-check run1.f90

Gives:

...  
i = 10  
20.0000000000000  
Fortran runtime error: Array reference out of bound for array 'x', upper bound of dimension 1 exceeded  
(in file 'demo1.f90', at line 11)