At the instant shown the 100-lb block A is moving down the plane at 5 ft/s while being attached to the 50-lb block B. If the coefficient of kinetic friction is \( \mu_k = 0.2 \), determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.

Block A:

\[ \sum F_x = ma_x; \quad -T_A - 0.2N_A + 100\left( \frac{3}{5} \right) = \left( \frac{100}{32.2} \right)a_A \]

\[ \sum F_y = ma_y; \quad N_A - 100\left( \frac{4}{5} \right) = 0 \]

Thus,

\[ T_A - 44 = -3.1056a_A \quad (1) \]

Block B:

\[ \sum F_x = ma_x; \quad T_B - 50 = \left( \frac{50}{32.2} \right)a_B \]

\[ T_B - 50 = -1.553a_B \quad (2) \]

Pulleys at C and D:

\[ + \sum F = 0; \quad 2T_B - 2T_A = 0 \quad \text{Because the mass of Pulleys is zero} \]

\[ T_A = T_B \quad (3) \]

Kinematics:

\[ s_A + 2s_C = \ell \]

\[ s_D + (s_D - s_B) = \ell' \]

\[ s_C + d + s_D = d' \]

Thus,

\[ a_A = -2a_C \]

\[ 2a_B = a_B \]

\[ a_C = -a_B \]

so that

\[ a_A = a_B \quad (4) \]

Solving Eqs. (1) - (4):

\[ a_A = a_B = -1.288 \text{ ft/s}^2 \]

\[ T_A = T_B = 48.0 \text{ lb} \]

Thus,

\[ a_A = 1.29 \text{ ft/s}^2 \quad \text{Ans} \]

\[ v^2 = \dot{s} + 2a_s (s - s_0) \]

\[ 0 = (5)^2 + 2(-1.288)(s - 0) \]

\[ s = 9.70 \text{ ft} \quad \text{Ans} \]
13-34. The boy having a weight of 80 lb hangs uniformly from the bar. Determine the force in each of his arms in \( t = 2 \) s if the bar is moving upward with (a) a constant velocity of 3 ft/s, and (b) a speed of \( v = (4t^2) \) ft/s, where \( t \) is in seconds.

a) \( T = 40 \) lb  Ans

b) \( v = 4t^2 \)

\[
\alpha = 8t
\]

\[
\sum F_j = ma_j; \quad 2T - 80 = \frac{80}{32.2} (8t)
\]

At \( t = 2 \) s,

\( T = 59.9 \) lb  Ans

13-35. The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m on the plate when the system is released from rest.

Block A:

\[
\sum F_j = ma_j; \quad N_x - 10(9.81)\cos 30^\circ = 0 \quad N_x = 84.96 \text{ N}
\]

\[
\sum F_j = ma_j; \quad -T + 2(84.96) + 10(9.81)\sin 30^\circ = 10a_A
\]

\[
T - 66.04 = -10a_A \quad (1)
\]

Block B:

\[
\sum F_j = ma_j; \quad N_y - 84.96 - 50(9.81)\cos 30^\circ = 0
\]

\[
N_y = 509.7 \text{ N}
\]

\[
\sum F_j = ma_j; \quad -0.2(84.96) - 0.1(509.7) - 10 - 50(9.81)\sin 30^\circ = 50a_B
\]

\[
177.28 - T = 50a_B \quad (2)
\]

\( s_A = s_B = 0 \)

\( \Delta s_A = -\Delta s_B \)

\[
a_A = -a_B \quad (3)
\]

Solving Eqs. (1) - (3):

\[
a_B = 1.854 \text{ m/s}^2
\]

\[
a_A = -1.854 \text{ m/s}^2 \quad T = 84.58 \text{ N}
\]

In order to slide 0.5 m along the plate the block must move 0.25 m. Thus,

\[
\Delta s_B = 0.25 \text{ m}
\]

\[
\Delta s_A = -0.25 \text{ m}
\]

\[
s_A = s_0 + v_0t + \frac{1}{2}a_A t^2
\]

\[
-0.25 = 0 + 0 + \frac{1}{2}(-1.854)t^2
\]

\[
t = 0.519 \text{ s} \quad \text{Ans}
\]
Block B rest on a smooth surface. If the coefficients of static and kinetic friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$ respectively, determine the acceleration of each of each block if someone pushes horizontally on block A with a force of (a) $F = 6\text{ lb}$, (b) $F = 50\text{ lb}$.

To determine whether block A will slide on block B, we need to:
1) determine the maximum acceleration $a_{\text{max}}$ of block B that can be provided by the static friction between A and B;
2) determine the acceleration $a_{\text{together}}$ assuming A does not slide on B;
3) compare $a_{\text{max}}$ and $a_{\text{together}}$:
   - If $a_{\text{max}} \geq a_{\text{together}}$, block A will not slide on block B
   - If $a_{\text{max}} < a_{\text{together}}$, block A will slide on block B

The static friction $f_s = N \times \mu_s = 20 \times 0.4 = 8\text{ lb}$

The maximum acceleration of block B: $a_{\text{max}} = \frac{f_s}{m_B} = \frac{8}{30/32.2} = 8.587 \text{ ft/sec}^2$

a) $a_{\text{together}} = \frac{F}{m_A + m_B} = \frac{6}{(20 + 30)/32.2} = 3.864 \text{ ft/sec}^2 < a_{\text{max}} \Rightarrow A$ will not slide on B

b) $a_{\text{together}} = \frac{F}{m_A + m_B} = \frac{50}{(20 + 30)/32.2} = 32.2 \text{ ft/sec}^2 > a_{\text{max}} \Rightarrow A$ will slide on B

$f_s = N \times \mu_k = 20 \times 0.3 = 6\text{ lb}$

$a_B = \frac{f_k}{m_B} = \frac{6}{30/32.2} = 6.44 \text{ ft/sec}^2$

$a_A = \frac{F - f_k}{m_A} = \frac{50 - 6}{20/32.2} = 70.84 \text{ ft/sec}^2$
13-66. The 150-lb man lies against the cushion for which the coefficient of static friction is \( \mu_s = 0.5 \). If he rotates about the \( z \) axis with a constant speed \( v = 30 \) ft/s, determine the smallest angle \( \theta \) of the cushion at which he will begin to slip off.

\[
\begin{align*}
\text{Horizontal:} & \quad \sum F_x = m a_x : \quad 0.5N \cos \theta + N \sin \theta = \frac{150}{32.2} \left( \frac{30^2}{8} \right) \\
\text{Vertical:} & \quad \sum F_y = m a_y : \quad 150 + N \cos \theta - 0.5N \sin \theta = 0 \\
& \quad \text{can get } \tan \theta \text{ from this eqn.} \\
& \quad \theta = 47.5^\circ \\
\end{align*}
\]

13-67. Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at \( \theta = 30^\circ \) from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the \( n \), \( t \), and \( b \) directions which the chair exerts on a 50-kg passenger during the motion?

\[
\begin{align*}
\sum F_n = m a_n : & \quad T \sin 30^\circ = 80 \left( \frac{v^2}{4 + 6 \sin 30^\circ} \right) \\
\sum F_t = m a_t : & \quad T \cos 30^\circ + 80(9.81) = 0 \\
& \quad T = 966.2 \text{ N} \\
& \quad v = 6.30 \text{ m/s} \quad \text{Ans} \\
\sum F_b = m a_b : & \quad F_c = 50 \left( \frac{6(30)^2}{7} \right) = 283 \text{ N} \quad \text{Ans} \\
& \quad F_t = 0 \\
& \quad F_c = 490.5 = 0 \\
& \quad F_c = 490 \text{ N} \quad \text{Ans}
\end{align*}
\]
(3-82) The 5-lb packages ride on the surface of the conveyor belt. If the belt starts from rest and increases to a constant speed of 2 ft/s in 2 s, determine the maximum angle $\theta$ so that none of the packages slip on the inclined surface AB of the belt. The coefficient of static friction between the belt and a package is $\mu_s = 0.3$. At what angle $\phi$ do the packages first begin to slip off the surface of the belt after the belt is moving at its constant speed of 2 ft/s? Neglect the size of the packages.

\[ v = v_i + a \cdot t, \quad 2 = 0 + a(2); \quad a_i = 1 \text{ ft/s}^2 \]

\[ \Sigma F_x = ma_x; \quad N - 5\cos \theta = 0 \quad \text{(1)} \]
\[ \Sigma F_y = ma_y; \quad 0.3N - 5\sin \theta = \frac{5}{32.2} \quad \text{(2)} \]

Solving Eqs. (1) and (2) yields:

\[ \theta = 15.0^\circ \quad \text{Ans} \]

\[ N = 4.83 \text{ lb} \]

For circular motion

\[ \frac{\sqrt{\Sigma F_x}}{ma_x}; \quad 5\cos \phi - N = \frac{\sqrt{2}}{32.2} \quad \text{(3)} \]
\[ \sqrt{\Sigma F_y} = ma_y; \quad 5\sin \phi - 0.3N = 0 \quad \text{(4)} \]

(about how to solve Eqs (1) and (2))

Solve (1) \[ N = 5\cos \theta \]; substitute into (2)

\[ 0.3\times 5\cos \theta - 5\sin \theta = \frac{5}{32.2} \]

\[ \Rightarrow 1.5\cos \theta = 5\sin \theta + \frac{5}{32.2} \]

square both sides of the above Eq.

\[ 2.25\cos^2 \theta = 25\sin^2 \theta - 15\sin \theta + 0.024 \]

substitute \( 1 - \sin^2 \theta \) into \( \cos^2 \theta \)

\[ \Rightarrow 2.25 - 2.25\sin^2 \theta = 25\sin^2 \theta - 15\sin \theta + 0.024 \]

solve the quadratic equation about \( \sin \theta \)

\[ \sin \theta = 0.2587 \]

\[ \Rightarrow \theta = 15^\circ \]

\( \phi \) can be solved from equation (3) and (4) in the same way.
13-88. The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components \( x = 1.5 \text{ m}, \) \( y = 0, \) \( z = -0.5 \text{ m}, \) where \( t \) is in seconds. Determine the components of force \( F_x, F_y, \) and \( F_z \) which the slide exerts on him at the instant \( t = 2 \text{ s}. \) Neglect the size of the boy.

\[
\begin{align*}
\dot{r} &= 1.5 \quad \dot{\theta} = 0.7r \\
\dot{r} &= 0 \quad \dot{\phi} = 0.7 \\
\ddot{r} &= 0 \quad \dot{z} = -0.5 \\
\theta &= \dot{\theta} = 0 \\
\phi &= \dot{\phi} = 0 \\
\end{align*}
\]

\[
\begin{align*}
\alpha_{xy} &= r \dot{\theta} + 2r \dot{\phi} = 0 \\
\omega_r &= \dot{r} + \frac{\partial}{\partial r} (r \dot{\theta}) = 0 \\
\omega_{xz} &= \dot{z} + \frac{\partial}{\partial z} (z \dot{\theta}) = 0 \\
\end{align*}
\]

\[
\begin{align*}
\sum F_x &= ma_x; \quad F_x = 40(9.81) = 392 \text{ N} \quad \text{Ans} \\
\sum F_y &= ma_y; \quad F_y = 0 \quad \text{Ans} \\
\sum F_z &= ma_z; \quad F_z = 40(9.81) = 392 \text{ N} \\
\end{align*}
\]

13-89. The girl has a mass of 50 kg. She is seated on the horse of the merry-go-round which undergoes constant rotational motion \( \dot{\theta} = 1.5 \text{ rad/s}. \) If the path of the horse is defined by \( r = 4 \text{ m}, \) \( z = (0.5 \sin \theta) \text{ m}, \) determine the maximum and minimum force \( F_r \) the horse exerts on her during the motion.

\[
\begin{align*}
\dot{\theta} &= 1.5 \quad \dot{\theta} = 0 \\
\theta &= 0 \\
\phi &= 0 \\
\end{align*}
\]

\[
\begin{align*}
z &= 0.5 \sin \theta \\
\dot{z} &= 0.5 \cos \theta \dot{\theta} \\
\ddot{z} &= -0.5 \sin \theta \dot{\theta}^2 + 0.5 \cos \theta \\
\end{align*}
\]

\[
\begin{align*}
\dot{F}_x &= 50(9.81) = 490.5 \text{ N} \\
\dot{F}_y &= 56 \text{ N} \\
\end{align*}
\]

Max. when \( \sin \theta = 1, \quad (F_x)_{\text{max}} = 547 \text{ N} \quad \text{Ans} \\
Min. when \sin \theta = 1, \quad (F_x)_{\text{min}} = 434 \text{ N} \quad \text{Ans} \\

13-90. The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an angular velocity \( \dot{\theta} = 4 \text{ rad/s} \) and an angular acceleration \( \ddot{\theta} = 8 \text{ rad/s}^2 \) at the instant \( \theta = 30^\circ \), determine the force of the guide on the particle. Motion occurs in the horizontal plane.

\[
\begin{align*}
\theta &= 30^\circ \quad \dot{\theta} = 4 \text{ rad/s} \quad \ddot{\theta} = 8 \text{ rad/s}^2 \\
\end{align*}
\]

\[
\begin{align*}
r &= 2(0.5 \cos \theta) = 1 \cos \theta \\
\dot{r} &= -\sin \theta \dot{\theta} \\
\ddot{r} &= -\cos \theta \dot{\theta}^2 - \sin \theta \ddot{\theta} \\
\end{align*}
\]

At \( \theta = 30^\circ \), \( \dot{\theta} = 4 \text{ rad/s} \) and \( \ddot{\theta} = 8 \text{ rad/s}^2 \):

\[
\begin{align*}
r &= 1 \cos 30^\circ = 0.8660 \text{ ft} \\
\dot{r} &= -\sin 30^\circ (4) = -2 \text{ ft/s} \\
\ddot{r} &= -\cos 30^\circ (4)^2 - \sin 30^\circ (8) = -17.856 \text{ ft/s}^2 \\
a_r &= \dot{r} - \ddot{r} = 0.8660(4) - (-17.856) = 31.713 \text{ ft/s}^2 \\
\omega_r &= r \dot{\theta} + 2r \ddot{\theta} = 0.8660(8) + 2(-2)(4) = 9.072 \text{ ft/s}^2 \\
\end{align*}
\]

\[
\begin{align*}
\sum F = m \dot{r} - N \cos 30^\circ = 0.5 \text{ ft} \\
\sum F_x = m \omega_r = 0.5 \text{ ft} \\
\sum F_y = m a_r = 0.5 \text{ ft} \\
\sum F_z = ma_z = 0.5 \text{ ft} \\
F = 0.143 \text{ lb} \quad \text{Ans}
\end{align*}
\]
The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when \( \theta = 30^\circ \). The rod is rotating with a constant angular velocity \( \dot{\theta} = 2 \text{ rad/s} \). Assume the particle contacts only one side of the slot at any instant.

\[
r = \frac{0.5}{\cos \theta} = 0.5 \sec \theta \quad \text{(magnitude of position vector OA)}
\]

\[
r = 0.5 \sec \theta \tan \theta \dot{\theta}
\]

\[
\ddot{r} = 0.5 \left\{ \left[ \left( \sec \theta \tan \theta \dot{\theta} \right) \tan \theta + \sec \theta \left( \sec^2 \theta \dot{\theta} \right) \right] \dot{\theta} + \sec \theta \tan \theta \ddot{\theta} \right\}
\]

\[
= 0.5 \left[ \sec \theta \tan^2 \theta \dot{\theta}^2 + \sec^3 \theta \dot{\theta}^2 + \sec \theta \tan \theta \ddot{\theta} \right]
\]

When \( \theta = 30^\circ \), \( \dot{\theta} = 2 \text{ rad/s} \) and \( \ddot{\theta} = 0 \)

\[
r = 0.5 \sec 30^\circ = 0.5774 \text{ m}
\]

\[
r = 0.5 \sec 30^\circ \tan 30^\circ(2) = 0.6667 \text{ m/s}
\]

\[
\ddot{r} = 0.5 \left[ \sec 30^\circ \tan^2 30^\circ(2)^2 + \sec^3 30^\circ(2)^2 + \sec 30^\circ \tan 30^\circ(0) \right]
\]

\[
= 3.849 \text{ m/s}^2
\]

\[
a_r = \ddot{r} - r \dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2
\]

\[
a_\theta = r \ddot{\theta} + 2r \dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2
\]

\[\mathbf{\sum F}_r = m \ddot{r}; \quad N \cos 30^\circ - 0.5(9.81) \cos 30^\circ = 0.5(1.540)
\]

\[N = 5.79 \text{ N} \quad \text{Ans}
\]

\[\mathbf{\sum F}_r = m \ddot{\theta}; \quad F + 0.5(9.81) \sin 30^\circ - 5.79 \sin 30^\circ = 0.5(2.667)
\]

\[F = 1.78 \text{ N} \quad \text{Ans}
\]