Specified acceleration

a) \( a = a(t) \)
   simply integrate with respect to time to get \( v \) and \( s \).

b) \( a = a(v) \)
   then
   \[ \int_{v_0}^{v} \frac{dv}{a(v)} = \int_{t_0}^{t} dt \]
   and
   \[ \int_{v_0}^{v} \frac{v \cdot dv}{a(v)} = \int_{s_0}^{s} ds \]

c) \( a = a(s) \)
   then
   \[ \int_{s_0}^{s} \frac{ds}{v(s)} = \int_{t_0}^{t} dt \]
   where \( v(s) \) is from
   \[ \int_{v_0}^{v} v \cdot dv = \int_{s_0}^{s} a(s) \cdot ds \]
Curvilinear Motion

Position $r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

Velocity $v = \frac{dr}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$

Acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k}$

2.68 The velocity of a point is $v = 2\mathbf{i} + 3t\mathbf{j}$ (ft/s). At $t = 0$ its position is $r = -\mathbf{i} + 2\mathbf{j}$ (ft). What is its position at $t = 2$ s?
A zoology graduate student is provided with a bow and an arrow tipped with a syringe of sedative and is assigned to measure the temperature of a black rhinoceros (*Diceros bicornis*). The range of his bow when it is fully drawn and aimed 45° above the horizontal is 100 m. A truculent rhino suddenly charges straight ahead at 30 km/hr. If he fully draws his bow and aims 20° above the horizontal, how far away should the rhino be when he releases the arrow?
If \( y = 150 \text{ mm} \), \( \frac{dy}{dt} = 300 \text{ mm/s} \), and \( \frac{d^2y}{dt^2} = 0 \), what are the magnitudes of the velocity and acceleration of point \( P \)?
Angular Motion

y. Angular speed (velocity)

\[ \omega = \frac{d\theta}{dt} \]

\[ \omega = \frac{d^2 \theta}{dt^2} \]

y. Angular acceleration

The unit vector is rotating with \( \omega = \frac{d\theta}{dt} \)

What is \( \frac{d\theta}{dt} \)?

\[ \frac{d\theta}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \]

where \( \eta \) is the unit vector

\[ \frac{d\theta}{dt} = \frac{d\theta}{dt} \cdot \eta = \omega \cdot \eta \]
Normal and Tangential Components

Consider:

\[ \mathbf{e}_t \text{ is a unit vector tangential to path} \]

**Velocity**

\[ \mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \cdot \mathbf{e}_t = \frac{ds}{dt} \cdot \mathbf{e}_t \]

\[ \mathbf{v} = \frac{ds}{dt} \cdot \mathbf{e}_t \quad \text{or} \quad \mathbf{v} = \mathbf{v} \cdot \mathbf{e}_t \]

**Acceleration**

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2s}{dt^2} \cdot \mathbf{e}_t + \frac{ds}{dt} \cdot \frac{d}{dt} \cdot \mathbf{e}_t \]

\[ = \frac{d^2s}{dt^2} \cdot \mathbf{e}_t + \mathbf{v} \cdot \frac{d\theta}{dt} \cdot \mathbf{e}_n \]

\[ \Delta \mathbf{e}_t = \Delta \theta \cdot \mathbf{e}_n \]
\[ a = \frac{dv}{dt} e_t + v \frac{ds}{dt} e_n \]

If we use the concept of instantaneous radius:

Then:

\[ v = v_t e_t = \frac{ds}{dt} e_t \]

\[ a = \frac{dv}{dt} e_t + \frac{v^2}{\rho} e_n \]

where \( \rho = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \).
Circular Motion

**Velocity**

\[ v = \frac{ds}{dt} e_\theta = v e_\theta \]

Here \( v = \frac{ds}{dt} = \frac{d(R \cdot \theta)}{dt} = R \cdot \frac{d\theta}{dt} \)

\[ v = R \cdot \omega \]

**Acceleration**

\[ a = \frac{dv}{dt} e_\theta + v \cdot \frac{d\theta}{dt} e_\theta = a_t + a_n \]

Where \( a_t = \frac{dv}{dt} = \frac{d}{dt} (R \cdot \omega) = R \cdot \alpha \)

and \( a_n = \frac{v^2}{R} \)