Two plates w/ fluid between
Top plate moving at speed \( U \)
("Couette Flow")

For real fluids we observe:

\[ u(0) = 0 \quad \text{"No slip condition"} \]
\[ u(H) = U \]

Force required to keep upper plate in motion = \( \mu \frac{U}{H} \) = \( \mu \frac{\partial u}{\partial z} \) = shear stress

\( \mu = \text{dynamic viscosity} \quad \left[ \frac{\text{kg}}{\text{m s}} \right] \)

This defines a "Newtonian Fluid"

Considering just a fluid parcel (with just \( \partial u/\partial z \))

\[ x\text{-force} = \mu \frac{\partial u}{\partial z} \, dy \, dx \quad \text{pull parcel to right} \]

\[ x\text{-force} = -\mu \frac{\partial u}{\partial z} \, dy \, dx \quad \text{pull parcel to left} \]

\[ \text{Viscous Force}^x = \lim_{\Delta V \to 0} \left\{ \frac{1}{\Delta V} \int_{\Delta z} \left( \mu \frac{\partial u}{\partial z} \right) \, dx \, dy \, dz \right\} = \frac{\partial}{\partial z} \mu \frac{\partial u}{\partial z} \]

due to \( \mu \partial^2 u/\partial z^2 \) only
- In general we may neglect spatial variation of \( \mu \).
- The relation stress = \( \mu \cdot \text{shear} \) with \( \mu = \text{const.} \) defines a "Newtonian Fluid".
- Non-Newtonian fluids: \( \mu \) may be a function of shear.
  - Shear thinning fluids: ball point pen ink, ketchup
  - "Thickening": Silly Putty

- Can have eq. \( \mu = \mu(\text{temperature}) \) and still be "Newtonian".

Generalizing the viscous force on a fluid parcel:

\[
\text{Visc. Force} = \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = \mu \nabla \cdot \mathbf{u} = \mu \nabla \cdot (\nabla \mathbf{u})
\]

(same expression \( \gamma + \tau \))