1. i. Since \( p = A \cos(kx) \cos(ly) \)

\[ -\nabla p = \left( -\frac{\partial p}{\partial x}, -\frac{\partial p}{\partial y} \right) = \left( A k \sin(kx) \cos(ly), A l \cos(kx) \sin(ly) \right) \]

![Diagram showing vectors and pressures](image)

This arrow is \( Ak \)

and this (larger) arrow is \( Al \)

1 > k as drawn

Hall erode the initial pressure field

1. ii. The divergence is

\[ -\nabla \cdot \nabla p = \left( -\frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial y^2} \right) = - (p_{xx} + p_{yy}) = -\nabla^2 p \]

\[ = A (k^2 + l^2) \cos(kx) \cos(ly) \]

1. iii. Clearly this would not be incompressible if \( \eta \propto -\nabla p \)
2. i. \( \frac{dT}{Dt} = 0 \)

2. ii. Zero

2. iii. Note that \( \frac{dT}{dt} + u \frac{dT}{dx} = 0 \) (*)

Does \( \frac{dT}{dx} \) change with time? No, you can show this by taking \( \frac{\partial(\text{*)}}{\partial x} \) to form

\[
\frac{\partial}{\partial t} \left( \frac{dT}{dx} \right) + \frac{\partial x}{\partial x} \frac{dT}{dx} + u \frac{\partial^2 T}{\partial x^2} = 0
\]

\( \frac{dT}{dx} \) is constant for all time, and \( \frac{dT}{dx} = \frac{T_0}{L} \)

so (*) can be written as

\[
\frac{dT}{dt} + \frac{U}{H} \frac{T_0}{L} z = 0 \quad \Rightarrow \quad T_t = -\frac{U T_0}{H L} z
\]
This has solution

$$T = -\frac{2UT_0}{HL} \approx t + T(x, z)$$

constant of integration

so the full solution is

$$T = \frac{T_0}{L} x + \frac{T_0}{H} z \left(1 - \frac{2u}{L} t\right)$$

at $z = \frac{L}{2}$:

$$\frac{dT}{dt} = -\frac{T_0 u}{L}$$

$T_0$ $0$ $T_0$

$T_0$ $0$ $T_0$

$t \gg \frac{L}{u}$

$L \approx \frac{L}{u}$

$2.10.$

$$\nabla T = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial z}\right) = \left[\frac{T_0}{L}, \frac{T_0}{H} \left(1 - \frac{2u}{L} t\right)\right]$$

constant growing even more negative in time (after $t = \frac{L}{u}$)
\[ E = m g \]

\[ -mg + A \left( \frac{H}{2} + \eta \right) (e - \Delta e) g + A \left( \frac{H}{2} - \eta \right) \rho_0 g = m \frac{\partial \eta}{\partial t} \]

\[ \text{gravity: } \quad \text{Buoyancy: } \quad \int_{V} \rho \, dV \]

For steady state, \( \eta = \eta_{tt} = 0 \)

\[ m = A \rho_0 - A \rho_0 \Delta e = A \rho_0 \quad (\text{for } \Delta e \ll \rho_1) \]

and then it is easy to simplify (t) to find

\[ \eta_{tt} + \frac{g'}{H} \eta = 0 \]

where \( g' = \frac{g \Delta e}{\rho_0} \) is the "reduced gravity".

\[ \eta = \eta_0 \cos \omega t \quad \text{where} \quad \eta = \sqrt{\frac{g'}{H}} \]
3. ii. You can do this most easily by rotating your coordinate system.

\[ g \cos \theta \]

and the solution is the same as in 3. i.

except \( \omega = \sqrt{\frac{g' \cos \theta}{H}} \)

3. iii. In layer (1) \( m \gamma_{tt} = (\rho_0 - \Delta \rho) V g - mg \)

and in (2) \( m \gamma_{tt} = \rho_0 V g - mg \)

and \( m = V \rho_0 - V \frac{\Delta \rho}{2} \)
so the equations became

1. \(\ddot{\eta} = -g/2\)
2. \(\dddot{\eta} = g/2\)

which has solution

\[
\eta_1 = \eta_0 - \frac{g}{2} t^2
\]

\[
\eta_2 = \frac{g}{2} (t - \frac{T}{2})^2 - \eta_0
\]

where \(T = 4 \sqrt{\frac{2 \eta_0}{g'}} = 80\) sec. for \(\eta_0 = 2\) m.