Introduction to Fluid Dynamics
Problem Set 2, 10/12/2007, Due at the start of class 10/19/2007

1. Consider 2-dimensional flow in the x-y plane where the velocity is given by 
   \((u,v) = D(x,-y)\)
   where \(D\) is a constant with units \(s^{-1}\).

   a.[5] Sketch the velocity field. Is this field divergent?
   b.[5] Find the mathematical expression for a streamline in the form \(y = f(x)\).
   c.[10] Find the expression for a parcel path as a function of time for a parcel that passes through \((x_0^L, y_0^L)\) at \(t = 0\). This will be equations for \(x^L(t)\) and \(y^L(t)\).
   d.[5] Find the expressions for the \(u\) and \(v\) velocity for this same parcel, again as functions of time.
   e.[10] Say you have dye along the line \(y = x\) at \(t = 0\). What happens to this line over time? Find the expression for the evolution of the slope of the line as a function of time. Does the distance between two parcels on the line change over time (i.e. does the line “stretch”)?

2. Consider a 2-dimensional flow in the x-y plane where the velocity is given by 
   \((u,v) = [U, V \cos(\omega t)]\)
   where \(U\) and \(V\) are constants with units of velocity, and \(\omega\) is a constant with units [rad \(s^{-1}\)].

   a.[5] What is the equation for a parcel path for a parcel that is at the origin at \(t = 0\)?
   b.[10] Develop the mathematical expression for the “streakline” that would result from a continuous injection of dye at the origin. Hint: the solution is not steady, but at any time it must pass through the origin. Sketch your answer at a couple of different times.