Comparative advantage with many goods

In this case, we can’t use relative prices to determine CA (there is one between any 2 goods).
Instead we can focus on the relative productivity for each good.
The cost of producing 1 unit of good $i$ is equal to the wage $w$ multiplied by the number of hours required $a_{Li}$ i.e. $wa_{Li}$
At home the cost is $wa_{Li}$ and abroad it is $w*a_{Li}$
If $wa_{Li} < w^*a^*_{Li}$ or $a^*_{Li}/a_{Li} > w/w^* \> a^*_{Li}/a_{Li} < w/w^*$
good i should be produced in the home country

If $wa_{Li} > w^*a^*_{Li}$ or $a^*_{Li}/a_{Li} < w/w^*$
good i should be produced in the foreign country

Rationale: $a^*_{Li}/a_{Li} = \frac{1/a_{Li}}{1/a^*_{Li}}$

is the relative productivity which is compared to the relative wage (i.e. cost) $w/w^*$. 
Example:
France and Germany
Calculators, bread, cheese, wine and apples
we only need to know the relative wage to figure out which good each country will export.
Assume that $w/w^* = 2.5$
<table>
<thead>
<tr>
<th>Good</th>
<th>$a_{Li}$ (Fr)</th>
<th>$a^*_{Li}$ (G)</th>
<th>$a^*<em>{Li}/a</em>{Li}$</th>
<th>w/w*</th>
<th>CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculator</td>
<td>50</td>
<td>25</td>
<td>0.5</td>
<td></td>
<td>G</td>
</tr>
<tr>
<td>Bread</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
<td>G</td>
</tr>
<tr>
<td>Cheese</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td></td>
<td>F</td>
</tr>
<tr>
<td>Wine</td>
<td>5</td>
<td>20</td>
<td>4</td>
<td></td>
<td>F</td>
</tr>
<tr>
<td>Apples</td>
<td>3</td>
<td>30</td>
<td>10</td>
<td></td>
<td>F</td>
</tr>
</tbody>
</table>

For calculators, the relative productivity in France (0.5) is lower than the relative wage (2.5), so France is not competitive and Germany has the comparative advantage.
Similarly for bread, France faces wages that are higher than the relative productivity (2.5>2) so France has a comparative disadvantage in both computers and bread.

However in the case of cheese, wine and apple, France’s relative productivity is higher than the relative wage so France has the comparative advantage in these 3 goods and will export them to Germany while Germany will export computers and bread to France.
Comparative advantage with many goods - with transportation cost.

Let’s assume that the cost of the good doubles when shipped abroad - all the a’s double for each country’s export good.

In the previous analysis we have determined that calculators and bread are Germany’s export goods while cheese, wine, and apples are France’s export goods when shipping costs are neglected.
The relative productivity for apple from France is still greater than the relative wage for France (5 > 2.5) so France still has a CA in apple and export them.
The relative productivity for computer from Germany is also greater than the relative wage for Germany \((1/1 > 1/2.5 \text{ or } 1 > .4)\) so Germany still has a CA in computers and export them.

When we consider the other three goods that were previously traded (bread, cheese and wine), we find the following:

the relative productivity has become smaller than the relative wage due to the inclusion of the shipping costs.

It is now prohibitive to export these goods. These are categorized as \textit{non-tradables}. 