Chapter 17
Temperature and the Kinetic Theory of Gases

39  •  [SSM]  A constant-volume gas thermometer reads 50.0 torr at the triple point of water. (a) Sketch a graph of pressure vs. absolute temperature for this thermometer. (b) What will be the pressure when the thermometer measures a temperature of 300 K? (c) What ideal-gas temperature corresponds to a pressure of 678 torr?

**Picture the Problem** We can use the equation of the graph plotted in (a) to (b) find the pressure when the temperature is 300 K and (c) the ideal-gas temperature when the pressure is 678 torr.

(a) A graph of pressure (in torr) versus temperature (in kelvins) for this thermometer is shown to the right. The equation of this graph is:

\[ P = \left( \frac{50.0 \text{ torr}}{273 K} \right) T \]  

(b) Evaluate \( P \) when \( T = 300 \text{ K} \):

\[ P(300 \text{ K}) = \left( \frac{50.0 \text{ torr}}{273 K} \right)(300 \text{ K}) = 54.9 \text{ torr} \]

(c) Solve equation (1) for \( T \) to obtain:

\[ T = \left( \frac{273 K}{50.0 \text{ torr}} \right) P \]

Evaluate \( T \) (678 torr):

\[ T(678 \text{ torr}) = \left( \frac{273 K}{50.0 \text{ torr}} \right)(678 \text{ torr}) = 3.70 \times 10^3 \text{ K} \]

The Ideal-Gas Law

46  •  An ideal gas in a cylinder fitted with a piston (Figure 17-20) is held at fixed pressure. If the temperature of the gas increases from 50° to 100°C, by what factor does the volume change?

**Picture the Problem** Let the subscript 50 refer to the gas at 50°C and the subscript 100 to the gas at 100°C. We can apply the ideal-gas law for a fixed
amount of gas to find the ratio of the final and initial volumes.

Apply the ideal-gas law for a fixed amount of gas:

\[
\frac{P_{100}V_{100}}{T_{100}} = \frac{P_{50}V_{50}}{T_{50}}
\]

or, because \(P_{100} = P_{50}\),

\[
\frac{V_{100}}{V_{50}} = \frac{T_{100}}{T_{50}}
\]

Substitute numerical values and evaluate \(\frac{V_{100}}{V_{50}}\):

\[
\frac{V_{100}}{V_{50}} = \frac{(273.15 + 100)K}{(273.15 + 50)K} = \frac{1.15}{1.15} = 1.15
\]

or a 15% increase in volume.

48  A pressure as low as \(1.00 \times 10^{-8}\) torr can be achieved using an oil diffusion pump. How many molecules are there in \(1.00\) cm\(^3\) of a gas at this pressure if its temperature is 300 K?

**Picture the Problem** We can use the ideal-gas law to relate the number of molecules in the gas to its pressure, volume, and temperature.

Solve the ideal-gas law for the number of molecules in a gas as a function of its pressure, volume, and temperature:

\[
N = \frac{PV}{kT}
\]

Substitute numerical values and evaluate \(N\):

\[
N = \frac{(1.00 \times 10^{-8} \text{ torr})(133.32 \text{ Pa/torr})(1.00 \times 10^{-6} \text{ m}^3)}{(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 3.22 \times 10^8
\]

56  A scuba diver is 40 m below the surface of a lake, where the temperature is 5.0ºC. He releases an air bubble that has a volume of 15 cm\(^3\). The bubble rises to the surface, where the temperature is 25ºC. Assume that the air in the bubble is always in thermal equilibrium with the surrounding water, and assume that there is no exchange of molecules between the bubble and the surrounding water. What is the volume of the bubble right before it breaks the surface? Hint: Remember that the pressure also changes.

**Picture the Problem** Let the subscript 1 refer to the conditions at the bottom of the lake and the subscript 2 to the surface of the lake and apply the ideal-gas law for a fixed amount of gas.
Apply the ideal-gas law for a fixed amount of gas:

\[
\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \Rightarrow V_2 = \frac{V_1T_2P_1}{T_1P_2}
\]

The pressure at the bottom of the lake is the sum of the pressure at its surface (atmospheric) and the pressure due to the depth of the lake:

\[P_1 = P_{\text{atm}} + \rho gh\]

Substituting for \(P_1\) yields:

\[V_2 = \frac{V_1T_2(P_{\text{atm}} + \rho gh)}{T_1P_2}\]

Substitute numerical values and evaluate \(V_2\):

\[V_2 = \frac{(15 \, \text{cm}^3)(298 \, \text{K})[101.325 \, \text{kPa} + (1.00 \times 10^3 \, \text{kg/m}^3)(9.81 \, \text{m/s}^2)(40 \, \text{m})]}{(278 \, \text{K})(101.325 \, \text{kPa})}\]

\[= 78 \, \text{cm}^3\]

Estimate the rms speed and the average kinetic energy of a hydrogen atom in a gas at a temperature of \(1.0 \times 10^7 \, \text{K}\). (At this temperature, which is approximately the temperature in the interior of a star, hydrogen atoms are ionized and become protons.)

**Picture the Problem** Because we’re given the temperature of the hydrogen atom and know its molar mass, we can find its rms speed using \(v_{\text{rms}} = \sqrt{3RT/M}\) and its average kinetic energy from \(K_{av} = \frac{1}{2}kT\). See Appendix C for the molar mass of hydrogen.

Relate the rms speed of a hydrogen atom to its temperature and molar mass:

\[v_{\text{rms}} = \sqrt{\frac{3RT}{M}}\]

Substitute numerical values and evaluate \(v_{\text{rms}}\):

\[v_{\text{rms}} = \sqrt{\frac{3(8.314 \, \text{J/mol} \cdot \text{K})(1.0 \times 10^7 \, \text{K})}{1.0079 \times 10^{-3} \, \text{kg/mol}}}\]

\[= 5.0 \times 10^5 \, \text{m/s}\]

Express the average kinetic energy of the hydrogen atom as a function of

\[K_{av} = \frac{1}{2}kT\]
its temperature:

Substitute numerical values and evaluate \( K_{av} \):

\[
K_{av} = \frac{1}{2} \left(1.381 \times 10^{-23} \text{ J/K} \right) \left(1.0 \times 10^7 \text{ K} \right) = 2.1 \times 10^{-16} \text{ J}
\]

65 [SSM] Oxygen (O\(_2\)) is confined to a cube-shaped container 15 cm on an edge at a temperature of 300 K. Compare the average kinetic energy of a molecule of the gas to the change in its gravitational potential energy if it falls 15 cm (the height of the container).

**Picture the Problem** We can use \( K = \frac{1}{2} kT \) and \( \Delta U = mgh = Mgh/N_A \) to express the ratio of the average kinetic energy of a molecule of the gas to the change in its gravitational potential energy if it falls from the top of the container to the bottom. See Appendix C for the molar mass of oxygen.

Express the average kinetic energy of a molecule of the gas as a function of its temperature:

\[
K_{av} = \frac{1}{2} kT
\]

Letting \( h \) represent the height of the container, express the change in the potential energy of a molecule as it falls from the top of the container to the bottom:

\[
\Delta U = mgh = \frac{M_{O_2} gh}{N_A}
\]

Express the ratio of \( K_{av} \) to \( \Delta U \) and simplify to obtain:

\[
\frac{K_{av}}{\Delta U} = \frac{\frac{1}{2} kT}{M_{O_2} gh} = \frac{3 N_A kT}{2 M_{O_2} gh}
\]

Substitute numerical values and evaluate \( K_{av}/\Delta U \):

\[
\frac{K_{av}}{\Delta U} = \frac{3 (6.022 \times 10^{23} \text{ particles/mol}) \left(1.381 \times 10^{-23} \text{ J/K} \right) \left(300 \text{ K} \right)}{2 \left(32.0 \times 10^{-3} \text{ kg/mol} \right) \left(9.81 \text{ m/s}^2 \right) \left(0.15 \text{ m} \right)} = 7.9 \times 10^4
\]