26 The Electric Field

26.1 Electric Field Models

26.2 The Field of Multiple Point Charges

1. You’ve been assigned the task of determining the magnitude and direction of the electric field at a point in space. Give a step-by-step procedure of how you will do so. List any objects you will use, any measurements you will make, and any calculations you will need to perform. Make sure that your measurements do not disturb the charges that are creating the field.

A tiny, positive test charge will be placed at the point in space and the force will be measured. From the force measurement and the charge the electric field will be calculated using \( E = \frac{F}{q} \). The direction of the field will be the same as the direction of the force since \( q \) is positive.

2. Is there an electric field at the position of the dot? If so, draw the electric field vector on the figure. If not, what would you need to do to create an electric field at this point?

Yes there is an electric field at this position.

3. This is the electric field in a region of space.
   a. Explain the information that is portrayed in this diagram.

   There is some source of positive charge on the left side.

   The longer vectors indicate a relatively larger electric field on the left, closest to the charge.

   b. If field vectors were drawn at the same six points but each was only half as long, would the picture represent the same electric field or a different electric field? Explain.

   It could represent the same electric field because the length of the field vector only represents relative magnitude of the electric field at that point.
4. Each figure shows two vectors. Can a point charge create an electric field that looks like this at these two points? If so, add the charge to the figure. If not, why not?

Note: The dots are the points to which the vectors are attached. There are no charges at these points.

a. \[ \text{Yes. Point 2 is further away from charge.} \]

b. \[ \text{No. A point charge creates a radial field.} \]

c. \[ \text{No. Field lines never cross.} \]

d. \[ \text{No. The field due to a pt charge cannot go in 2 directions.} \]

5. At each of the dots, use a **black** pen or pencil to draw and label the electric fields \( \vec{E}_1 \) and \( \vec{E}_2 \) due to the two point charges. Make sure that the **relative** lengths of your vectors indicate the strength of each electric field. Then use a **red** pen or pencil to draw and label the net electric field \( \vec{E}_{\text{net}} \).

a.

b.

c.

d.

6. For each of the figures, use dots to mark any point or points (other than infinity) where \( \vec{E} = \vec{0} \).

a.

b.
7. Each figure shows two points near to two charges. Compare the electric field strengths $E_1$ and $E_2$ at these two points. Is $E_1 > E_2$, $E_1 = E_2$, or $E_1 < E_2$?

a. $E_1 < E_2$

b. $E_1 > E_2$

c. $E_1 = E_2$

d. $E_1 < E_2$

e. $E_1 < E_2$

f. $E_1 > E_2$

8. For each figure, draw and label the net electric field vector $\vec{E}_{\text{net}}$ at each of the points marked with a dot or, if appropriate, label the dot $\vec{E}_{\text{net}} = 0$. The lengths of your vectors should indicate the magnitude of $\vec{E}$ at each point.

9. At the position of the dot, draw fields $\vec{E}_1$ and $\vec{E}_2$ due to $q_1$ and $q_2$, and the net electric field $\vec{E}_{\text{net}}$. Then, in the blanks, state whether the x- and y-components of $\vec{E}_{\text{net}}$ are positive or negative.

a. $q_1$, $q_2$:

$E_{\text{net}}$:

$E_1$:

$E_2$:

b. $q_1$, $q_2$:

$E_{\text{net}}$:

$E_1$:

$E_2$:
10. Use a black pen or pencil to draw the two electric fields $\vec{E}_1$ and $\vec{E}_2$ at each dot. Then use a red pen or pencil to draw $\vec{E}_{\text{net}}$. The lengths of your vectors should indicate the magnitude of $\vec{E}$ at each point.

![Diagrams of electric fields](image)

11. Draw the electric field vector at the three points marked with a dot. Hint: Think of the charges as horizontal positive/negative pairs, then use superposition.

12. The figure shows the electric field lines in a region of space. Draw the electric field vectors at the three dots.

13. The figure shows the electric field lines in a region of space. Rank in order, from largest to smallest, the electric field strengths $E_1$ to $E_4$ at points 1 to 4.

Order: $E_3 = E_4 > E_2 > E_1$

Explanation:

The electric field strength is larger in the region where the field lines are closer together ($E_3$ and $E_4$) and smaller where the field lines are farther apart.
26.3 The Electric Field of a Continuous Charge Distribution


a. The segment is shrunk to one-third of its original length. What is the ratio \( \lambda_f / \lambda_i \), where \( \lambda_i \) and \( \lambda_f \) are the initial and final linear charge densities?

\[
\frac{\lambda_f}{\lambda_i} = \frac{(Q_f/L_f)}{(Q_i/L_i)} \quad \text{But} \quad Q_i = Q_f \quad \Rightarrow \quad \frac{\lambda_f}{\lambda_i} = \frac{L_i}{L_f} = 3
\]

b. A proton is very far from the wire. What is the ratio \( F_f / F_i \) of the electric force on the proton after the segment is shrunk to the force on the proton before the segment is shrunk?

\[
F_f < \lambda \quad \Rightarrow \quad \frac{F_f}{F_i} = 3
\]

c. Suppose the original segment of wire is stretched to 10 times its original length. How much charge must be added to the wire to keep the linear charge density unchanged?

10 times the original amount of charge would give a constant linear charge density. So the amount of charge to add to the original is 9 times the original charge.

15. A wire has initial linear charge density \( \lambda_i \). The wire is stretched in length by 50%, and one-third of the charge is removed. What is the ratio \( \lambda_f / \lambda_i \), where \( \lambda_f \) is the final linear charge density?

\[
\lambda_i = \frac{Q_i}{L_i} \quad \text{and} \quad L_f = 1.5 L_i \quad \Rightarrow \quad Q_f = \frac{Q_i}{3}
\]

\[
\lambda_f = \frac{Q_f}{L_f} = \frac{(\frac{Q_i}{3})}{(1.5 L_i)} = \frac{1}{(3)(1.5)} \frac{Q_i}{L_i} = \frac{1}{4.5} \lambda_i
\]

\[
\frac{\lambda_f}{\lambda_i} = \frac{1}{4.5}
\]

16. The figure shows a uniformly charged positive wire. Five small, equally-spaced segments of charge are shown. Use these five segments to estimate the wire’s electric field—both magnitude and direction—at each point in space marked with a dot. Draw each \( \vec{E} \) on the figure.
17. Equal-length, equally charged positive and negative wires are placed end-to-end. Draw the electric field at each of the dots. 
Hint: Think about the superposition of the fields of a positive and a negative wire.

18. Two rings of charge face each other. The total charge on each ring is indicated beneath it. 
Draw the electric field vector on the axis of the rings at the midpoint between them (at the dot), or label the point \( E = 0 \).

a. \( E = 0 \)  
\( Q = + \)  
\( Q = + \)

b. \( E \)  
\( Q = - \)  
\( Q = + \)

c. \( E \)  
\( Q = + \)  
\( Q = +++ \)

19. The figure shows two charged rods bent into a semicircle. For each, draw the electric field vector at the "center" of the semicircle.

a.  

b.  

20. A hollow soda straw is uniformly charged. What is the electric field at the center (inside) of the straw? Explain.

The electric field at the center is zero.
We can think of the straw as being made up of many rings of positive charge. At the center of the ring adding all field vectors gives a resultant electric field equal to zero.

21. An electron experiences a force of magnitude \( F \) when it is 1 cm from a very long charged wire with linear charge density \( \lambda \). If the charge density is doubled, at what distance from the wire will a proton experience a force of the same magnitude \( F \)?

\[
F = eE = e\left(\frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{r}\right)
\]

If the charge density \( \lambda \) is doubled, then the distance \( r \) from the wire must also be doubled for the force to be the same. \( r = 2 \text{cm} \)
26.4 The Electric Field of Rings, Planes, and Spheres

22. An irregularly-shaped area of charge has surface charge density $\eta_i$.
   Each dimension ($x$ and $y$) of the area is reduced by a factor of 3.16.
   a. What is the ratio $\frac{\eta_f}{\eta_i}$, where $\eta_f$ is the final surface charge density?

\[
A_f = \frac{A_i}{3.16^2} \quad \frac{\eta_f}{\eta_i} = \frac{(\frac{\theta}{A_f})}{(\frac{\theta}{A_i})} = \frac{A_i}{A_f} = \frac{3.16^2}{9.99}
\]

b. Compare the final force on a electron very far away to the initial force on the same electron.

\[
F_f = eE_f = e\frac{\eta_f}{\epsilon_0} \quad F_i = eE_i = e\frac{\eta_i}{\epsilon_0} \quad \frac{F_f}{F_i} = \frac{\eta_f}{\eta_i} = 9.99 \quad F_f = 9.99 F_i
\]

23. A circular disk has surface charge density $8$ nC/cm$^2$. What will be the surface charge density if the radius of the disk is doubled?

\[
\eta_1 = \frac{\Phi}{A_1} = \frac{\Phi}{\pi r_1^2} = 8 \quad \text{nC/cm}^2
\]

\[
\eta_2 = \frac{\Phi}{A_2} = \frac{\Phi}{\pi (2r_1)^2} = \frac{\Phi}{4\pi r_1^2} = \frac{1}{4} \eta_1 = 2 \quad \text{nC/cm}^2
\]

24. Rank in order, from largest to smallest, the surface charge densities $\eta_1$ to $\eta_4$ of surfaces 1 to 4.

Order: $\eta_1 = \eta_3 > \eta_4 > \eta_2$

Explanation:

\[
\eta_1 = \frac{\Phi_1}{A_1} = \frac{10 \text{nC}}{(1 \text{cm} \times 1 \text{cm})} = 10 \quad \text{nC/cm}^2
\]

\[
\eta_2 = \frac{20 \text{nC}}{(2 \text{cm} \times 2 \text{cm})} = 5 \quad \text{nC/cm}^2
\]

\[
\eta_3 = \frac{20 \text{nC}}{(1 \text{cm} \times 2 \text{cm})} = 10 \quad \text{nC/cm}^2
\]

\[
\eta_4 = \frac{15 \text{nC}}{(2 \text{cm} \times 2 \text{cm})} = 7.5 \quad \text{nC/cm}^2
\]

25. The figure shows an edge view of a plane of negative charge. Draw the electric field diagram.
26. A sphere of radius \( R_i \) has charge \( Q_i \). What happens to the electric field strength at \( r = 2R_i \)? if:
   a. The quantity of charge is halved?
      \[
      E = \frac{Q}{4\pi \varepsilon_0 r^2}
      \]
      If \( Q \) is halved, then \( E \) is also halved.
   b. The radius is halved? The field outside a sphere is the same as that of a point charge \( Q \) located at the center of the sphere. So if the radius of the sphere changes, the field remains the same outside the sphere at the distance \( r = 2R_i \).

27. A pendulum is made with a ball of mass \( m \) and positive charge \( q \). It is suspended from a large, uniformly charged positive plate. Gravity is not negligible.
   a. Draw a free-body diagram of the ball when the string is at an angle of 45°.
   b. Would discharging the ball cause the tension in the string to increase, decrease, or stay the same? Explain.
      Discharging the ball will cause the electric force to decrease. Therefore the tension will also decrease.
      \[
      T \cos 45° = mg - qE = 0
      \]
      \[
      T \cos 45° = mg - qE
      \]
      c. Would discharging the ball cause the period of the pendulum to increase, decrease, or stay the same? Explain.
      Discharging the ball will cause the restoring force
      \[
      F = (mg + qE) \sin 45°
      \]
      to decrease. Therefore the period of the pendulum will increase.
26.5 The Parallel-Plate Capacitor

28. Rank in order, from largest to smallest, the electric field strengths $E_1$ to $E_5$ at each of these points.

Order: $E_1 = E_2 = E_3 = E_4 = E_5$

Explanation:

The electric field is constant everywhere between the plates. This is indicated by the electric field vectors which are all the same length and in the same direction.

29. A parallel-plate capacitor is constructed of two square plates, size \( L \times L \), separated by distance \( d \). The plates are given charge \( \pm Q \). What is the ratio \( E_f/E_i \) of the final electric field strength \( E_f \) to the initial electric field strength \( E_i \) if:

a. \( Q \) is doubled?

\[
\frac{E_f}{E_i} = \frac{\frac{Q_f}{A_f}}{\frac{Q_i}{A_i}} = \frac{\frac{2Q}{A}}{\frac{Q}{A}} = 2
\]

b. \( L \) is doubled?

If \( L \) is doubled then \( A_f = 4A_i \)

\[
\frac{E_f}{E_i} = \frac{\frac{A_i}{A_f}}{\frac{A_i}{4A_i}} = \frac{1}{4}
\]

c. \( d \) is doubled?

$E$ does not depend on \( d \).

\[
\frac{E_f}{E_i} = 1
\]

30. A ball hangs from a thread between two vertical capacitor plates. Initially the ball is neutral. Before the capacitor is charged, the ball hangs exactly in the center.

a. After the capacitor is charged, is the ball’s equilibrium position to the right, to the left, or in the center? Explain.

In the center. Since there is neutral charge on the ball the electric field exerts zero force on the ball.

b. A charged rod is used to give the ball a negative charge. Afterward, is the ball’s equilibrium position to the right, to the left, or in the center? Explain.

To the right. The electric field points to the left so the force on a negative charge is to the right.
c. Draw a free-body diagram of the negatively charged ball in static equilibrium.

31. A neutral metal rod is suspended in the center of a parallel-plate capacitor. Then the capacitor is charged as shown.

a. Is the rod now positive, negative, or neutral? Explain.

The rod is still neutrally charged. No charges have been added to or removed from the metal rod.

b. Is the rod polarized? If so, draw plusses and minuses on the figure to show the charge distribution. If not, why not?

The rod is polarized.

c. Does the rod swing toward one of the plates, or does it remain in the center? If it swings, which way? Explain.

The rod remains in the center because there are equal forces pulling the left end of the rod to the left and pulling the right end of the rod to the right.
26.6 Motion of a Charged Particle in an Electric Field

26.7 Motion of a Dipole in an Electric Field

32. A small positive charge $q$ experiences a force of magnitude $F_1$ when placed at point 1. In terms of $F_1$:

a. What is the force on charge $q$ at point 3? $3F_1$

b. What is the force on a charge $3q$ at point 1? $3F_1$

c. What is the force on a charge $2q$ at point 2? $4F_1$

d. What is the force on a charge $-2q$ at point 2? $-4F_1$

33. A small object is released in the center of the capacitor. For each situation, does the object move to the right, to the left, or remain in place? If it moves, does it accelerate or move at constant speed?

a. Positive object released from rest.
   Accelerates to the right.

b. Negative object released from rest.
   Accelerates to the left.

c. Neutral object released from rest.
   Remains in place.

34. Positively and negatively charged objects, with equal masses and equal quantities of charge, enter the capacitor in the directions shown.

a. Use solid lines to draw their trajectories on the figure if their initial velocities are fast.

b. Use dotted lines to draw their trajectories on the figure if their initial velocities are slow.

35. An electron is launched from the positive plate at a 45° angle. It does not have sufficient speed to make it to the negative plate. Draw its trajectory on the figure.
36. A proton and an electron are released from rest in the center of a capacitor.
   a. Compare the forces on the two charges. Are they equal, or is one larger? Explain.
   
   The forces on the two charges are equal: \( F = qE \)
   They each have the same amount of charge and are placed in the same field.

   b. Compare the accelerations of the two charges. Are they equal, or is one larger? Explain.
   
   The acceleration of the electron is larger because the electron has smaller mass. \( a = \frac{F}{m} \)

37. The figure shows an electron orbiting a proton in a hydrogen atom.
   a. What force or forces act on the electron?
   
   The electric force.

   b. Draw and label the following vectors on the figure: the electron’s velocity \( \vec{v} \) and acceleration \( \vec{a} \), the net force \( \vec{F}_{\text{net}} \) on the electron, and the electric field \( \vec{E} \) at the position of the electron.

38. Does a charged particle always move in the direction of the electric field? If so, explain why.
    If not, give an example that is otherwise.

   No. If the charged particle has an initial velocity then it could travel in a trajectory that is not in the direction of the field. Also a charge could move in circular motion as in problem # 37 above.

39. Three charges are placed at the corners of a triangle. The ++ charge has twice the quantity of charge of the two – charges; the net charge is zero.
   a. Draw the force vectors on each of the charges.
   
   b. Is the triangle in equilibrium? If not, draw the equilibrium orientation directly beneath the triangle that is shown. No.
   
   c. In equilibrium, will the triangle move to the right, move to the left, or remain in place? Explain.

   In equilibrium the triangle will remain in place because the net force is zero and the net torque is zero.