Physics 122
Solutions to Chapter 25

25.1. Model: Use the charge model.
Solve: (a) In the process of charging by rubbing, electrons are removed from one material and transferred to
the other because they are relatively free to move. Protons, on the other hand, are tightly bound in nuclei. So,
electrons have been removed from the glass rod to make it positively charged.
(b) Because each electron has a charge of $1.60 \times 10^{-19} \text{ C}$, the number of electrons removed is
$$n = \frac{5 \times 10^{-9} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 3.13 \times 10^{10}$$

25.6. Model: Use the charge model.
Solve: (a) No, we cannot conclude that the wall is charged. Attractive electric forces occur between (i) two
opposite charges, or (ii) a charge and a neutral object that is polarized by the charge. Rubbing the balloon does
drive the charge. Since the balloon is rubber, its charge is negative. As the balloon is brought near the wall, the
wall becomes polarized. The positive side of the wall is closer to the balloon than the negative side, so there is a
net attractive electric force between the wall and the balloon. This causes the balloon to stick to the wall, with a
normal force balancing the attractive electric force and an upward frictional force balancing the very small
weight of the balloon.

(b)

25.11. Model: Model the charged masses as point charges.
Visualize:

Solve: (a) The charge $q_1$ exerts a force $\vec{F}_{1 \text{on} 2}$ on $q_2$ to the right, and the charge $q_2$ exerts a force $\vec{F}_{2 \text{on} 1}$ on $q_1$ to
the left. Using Coulomb’s law,
$$F_{\text{on} 2} = F_{\text{on} 1} = \frac{K q_1 q_2}{r_{12}^2} = \frac{9.0 \times 10^9 \text{ N m}^2/\text{C}^2 (1.0 \text{ C})(1.0 \text{ C})}{(1.0 \text{ m})^2} = 9.0 \times 10^9 \text{ N}$$
(b) Newton’s second law on either $q_1$ or $q_2$ is
$$F_{\text{on} 2} = m_i a_i \Rightarrow a_i = \frac{9.0 \times 10^9 \text{ N}}{1.0 \text{ kg}} = 9.0 \times 10^9 \text{ m/s}^2$$
Assess: Coulomb is a pretty “big” unit. That is why $F_{\text{on} 2}$ is such a large force.

25.20. Model: The gravitational field of an object depends on its mass and extends through all of space.
Solve: (a) The gravitational field strength due to a planet at the radius of its satellite’s orbit is 12 N/kg. That is,
$$\vec{g}_{\text{orbit}} = \left( \frac{G m_{\text{planet}}}{r_{\text{orbit}}^2} \right)_{\text{toward planet}} = (12 \text{ N/kg, toward planet})$$
When the radius of the orbit is doubled,
\[ \vec{g}_{\text{new \ orbit}} = \left( \frac{G m_{\text{planet}}}{(2r_{\text{orbit}})^2} \right), \text{toward planet} \] (b) When the planet’s density is doubled, then \( m_{\text{new}} = \rho_{\text{new}}V = 2\rho V = 2m_{\text{planet}} \). Thus, assuming that \( V \) remains the same,

\[ \vec{g}_{\text{orbit}} = \left( \frac{G2m_{\text{planet}}}{r_{\text{orbit}}^2} \right), \text{toward planet} \] = (24 N/kg, toward planet)

(c) \( \vec{g}_{\text{orbit}} \) does not depend on the satellite’s mass. Thus, \( \vec{g}_{\text{orbit}} = (12 \text{ N/kg, toward planet}) \).

25.26. Model: A field is the agent that exerts an electric force on a charge.

Visualize:

Solve: (a) To balance the weight of a proton \( \Sigma F_{\text{net}} = F_{\text{on \ p}} + \vec{w} = 0 \) N. This means

\[ F_{\text{on \ p}} = mg = \frac{mg}{|q|} = \frac{(1.67 \times 10^{-27} \text{ kg})(9.8 \text{ N/kg})}{1.60 \times 10^{-19} \text{ C}} = 1.02 \times 10^{-7} \text{ N/C} \]

Because \( F_{\text{on \ p}} \) must be upward and the proton charge is positive, the electric field at the location of the proton must also be pointing upward. Thus \( \vec{E} = (1.02 \times 10^{-7} \text{ N/C, upward}) \).

(b) In the case of the electron,

\[ E = \frac{mg}{|q|} = \frac{(9.11 \times 10^{-31} \text{ kg})(9.8 \text{ N/kg})}{1.60 \times 10^{-19} \text{ C}} = 5.58 \times 10^{-11} \text{ N/C} \]

Because \( F_{\text{on \ e}} \) must be upward and the electron has a negative charge, the electric field at the location of the electron must be pointing downward. Thus \( \vec{E} = (5.58 \times 10^{-11} \text{ N/C, downward}) \).

25.54. Model: The charges are point charges.

Visualize:

We must first identify the region of space where the third charge \( q_3 \) is located. You can see from the figure that the forces can’t possibly add to zero if \( q_3 \) is above or below the axis or outside the charges. However, at some point on the \( x \)-axis between the two charges the forces from the two charges will be oppositely directed.

Solve: The mathematical problem is to find the position for which the forces \( \vec{F}_{1 \text{ on } 3} \) and \( \vec{F}_{2 \text{ on } 3} \) are equal in magnitude. If \( q_3 \) is the distance \( x \) from \( q_1 \), it is the distance \( L - x \) from \( q_2 \). The magnitudes of the forces are
Equating the two forces,

\[ \frac{Kq_i |q|}{x^2} = \frac{K(4q)|q_3|}{(L-x)^2} \Rightarrow (L-x)^2 = 4x^2 \Rightarrow x = \frac{L}{3} \text{ and } -L \]

The solution \( x = -L \) is not allowed as you can see from the figure. To find the magnitude of the charge \( q_3 \), we apply the equilibrium condition to charge \( q_1 \):

\[ F_{2,x} = F_{1,x} \Rightarrow \frac{Kq_1 |q|}{L^2} = \frac{Kq_3 |q|}{(\frac{L}{3})^2} \Rightarrow 4q = 9|q_1| \Rightarrow |q_1| = \frac{4}{9}q \]

We are now able to check the static equilibrium condition for the charge \( 4q \) (or \( q_3 \)):

\[ F_{1,x} = F_{3,x} \Rightarrow K\frac{|q_2| |q_1|}{L^2} = K\frac{|q_3| |q_1|}{(L-x)^2} \Rightarrow q = \frac{4q}{L} \]

The sign of the third charge \( q_3 \) must be negative. A positive sign on \( q_3 \) will not have a net force of zero either on the charge \( q \) or the charge \( 4q \). In summary, a charge of \(-\frac{4}{9}q\) placed \( x = \frac{L}{3} \) from the charge \( q \) will cause the 3-charge system to be in static equilibrium.

25.58. Model: The charged plastic beads are point charges and the spring is an ideal spring that obeys Hooke’s law.

Solve: Let \( q \) be the charge on each plastic bead. The repulsive force between the beads pushes the beads apart. The spring is stretched until the restoring spring force on either bead is equal to the repulsive Coulomb force. That is,

\[ \frac{Kq^2}{r^2} = k\Delta x \Rightarrow q = \sqrt{\frac{k\Delta x r^2}{K}} \]

The spring constant \( k \) is obtained by noting that the weight of a 1.0 g mass stretches the spring 1.0 cm. Thus

\[ mg = k(1.0 \times 10^{-2} \text{ m}) \Rightarrow k = \frac{\left(1.0 \times 10^{-3} \text{ kg}\right)(9.8 \text{ N/kg})}{1.0 \times 10^{-2} \text{ m}} = 0.98 \text{ N/m} \]

\[ \Rightarrow q = \sqrt{\frac{(0.98 \text{ N/m})(4.5 \times 10^{-2} \text{ m} - 4.0 \times 10^{-2} \text{ m})}{9.0 \times 10^9 \text{ N m}^2/\text{C}^2}} = 33.2 \text{ nC} \]

25.59. Model: The charged spheres are point charges.

Visualize: Pictorial representation Free-body diagram

Each sphere is in static equilibrium and the string makes an angle \( \theta \) with the vertical. The three forces acting on each sphere are the electric force, the weight of the sphere, and the tension force.

Solve: In static equilibrium, Newton’s first law is \( \vec{F}_{\text{net}} = \vec{T} + \vec{w} + \vec{F}_e = \vec{0} \). In component form,
\[(F_{\text{net}})_x = T_x + w_x + (F_x)_y = 0 \text{ N} \quad (F_{\text{net}})_y = T_y + w_y + (F_y)_y = 0 \text{ N}\]

\[\Rightarrow -T \sin \theta + 0 \text{ N} + \frac{Kq^2}{d^2} = 0 \text{ N} \quad T \cos \theta - mg + 0 \text{ N} = 0 \text{ N}\]

\[\Rightarrow T \sin \theta = \frac{Kq^2}{d^2} = \frac{Kq^2}{(2L \sin \theta)^2}, \quad T \cos \theta = +mg\]

Dividing the two equations,

\[
\sin^2 \theta \tan \theta = \frac{Kq^2}{4E^2 mg} = \left(\frac{9.0 \times 10^9 \text{ N m}^2/\text{C}^2}{(100 \times 10^{-9} \text{ C})^2}\right) \left(\frac{5 \times 10^{-3} \text{ kg}}{9.8 \text{ N/kg}}\right) = 4.59 \times 10^{-4}
\]

For small-angles, \(\tan \theta = \sin \theta\). With this approximation we obtain \(\sin \theta = 0.07714 \text{ rad} \text{ and } \theta = 4.42^\circ\).

**25.63. Model:** The electric field is that of a positive point charge located at the origin.

**Visualize:** Please refer to Figure P25.63. Place the 5 nC charge at the origin.

**Solve:** The electric field is

\[
\vec{E} = \left(\frac{1}{4\pi \varepsilon_0} \frac{q}{r^2}, \text{ away from } q\right) = \left(\frac{9 \times 10^9 \text{ N m}^2/\text{C}^2}{r^2}, \text{ away from } q\right)
\]

\[
= \left(45.0 \text{ N m}^2/\text{C}, \text{ away from } q\right)
\]

At each of the three points,

\[
\vec{E}_1 = \left(45.0 \text{ N m}^2/\text{C}, \text{ away from } q\right) = \left(9 \times 10^9 \text{ N/C} \left(\cos \theta \hat{i} + \sin \theta \hat{j}\right)\right)
\]

\[
= \left(9 \times 10^9 \text{ N/C} \left(\frac{1}{\sqrt{5}} \hat{i} + \frac{2}{\sqrt{5}} \hat{j}\right)\right) = \left(4.02 \times 10^4 \hat{i} + 8.05 \times 10^4 \hat{j}\right) \text{ N/C}
\]

\[
\vec{E}_2 = \left(45.0 \text{ N m}^2/\text{C}, \text{ away from } q\right) = 4.5 \times 10^4 \hat{j} \text{ N/C}
\]

\[
\vec{E}_3 = \left(45.0 \text{ N m}^2/\text{C}, \text{ away from } q\right) = \left(4.02 \times 10^4 \hat{i} - 8.05 \times 10^4 \hat{j}\right) \text{ N/C}
\]

**25.66. Model:** The electric field is that of three point charges.

**Visualize:**

\[q_1 = 1 \text{ nC} \quad q_2 = 1 \text{ nC} \quad q_3 = 1 \text{ nC}\]

\[r_1 = r_2 = \sqrt[(3)]{(1 \text{ cm})^2 + (3 \text{ cm})^2} = 3.162 \text{ cm} \text{ and the angle is } \theta = \tan^{-1}(1/3) = 18.43^\circ\]. Using the equation for the field of a point charge,

\[
\vec{E}_1 = \left(\frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1^2}, \text{ away from } q_1\right) = \left(\frac{9 \times 10^9 \text{ N m}^2/\text{C}^2}{(3.162 \text{ cm})^2}, \text{ away from } q_1\right)
\]

\[
= \left(4.5 \times 10^4 \hat{j}, \text{ away from } q_1\right) \text{ N/C}
\]

\[
\vec{E}_2 = \left(\frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2^2}, \text{ away from } q_2\right) = \left(\frac{9 \times 10^9 \text{ N m}^2/\text{C}^2}{(3.162 \text{ cm})^2}, \text{ away from } q_2\right)
\]

\[
= \left(4.5 \times 10^4 \hat{j}, \text{ away from } q_2\right) \text{ N/C}
\]

\[
\vec{E}_3 = \left(\frac{1}{4\pi \varepsilon_0} \frac{q_3}{r_3^2}, \text{ away from } q_3\right) = \left(\frac{9 \times 10^9 \text{ N m}^2/\text{C}^2}{(3.162 \text{ cm})^2}, \text{ away from } q_3\right)
\]

\[
= \left(4.5 \times 10^4 \hat{j}, \text{ away from } q_3\right) \text{ N/C}
\]
\[ E_i = E_x = \frac{K|q|}{r_i^2} = \left( \frac{9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \cdot (1.0 \times 10^{-9} \text{ C})}{(0.03162 \text{ m})^2} \right) = 9000 \text{ N/C} \]

We now use the angle \( \theta \) to find the components of the field vectors:

\[
\vec{E}_i = E_i \cos \theta \hat{i} - E_i \sin \theta \hat{j} = \left( 8540 \hat{i} - 2840 \hat{j} \right) \text{ N/C} \quad \vec{E}_x = E_x \cos \theta \hat{i} + E_x \sin \theta \hat{j} = \left( 8540 \hat{i} + 2840 \hat{j} \right) \text{ N/C}
\]

\( \vec{E}_2 \) is easier since it has only an \( x \)-component. Its magnitude is

\[
E_2 = \frac{K|q|}{r_2^2} = \left( \frac{9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \cdot (1.0 \times 10^{-9} \text{ C})}{(0.03000 \text{ m})^2} \right) = 10,000 \text{ N/C} \Rightarrow \vec{E}_2 = E_2 \hat{i} = 10,000 \text{ N/C}
\]

(b) The electric field is defined in terms of an electric force acting on charge \( q \):

\[ \vec{E} = \vec{F}/q \text{.} \]

Since forces obey a principle of superposition (\( \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \cdots \)) it follows that the electric field due to several charges also obeys a principle of superposition.

(c) The net electric field at a point 3 cm to the right of \( q_2 \) is

\[ \vec{E}_{\text{net}} = \vec{E}_i + \vec{E}_2 + \vec{E}_1 = 27,100 \hat{i} \text{ N/C} \text{.} \]

The \( y \)-components of \( \vec{E}_i \) and \( \vec{E}_2 \) cancel, giving a net field pointing along the \( x \)-axis.

25.68. Model: The charged ball attached to the string is a point charge.

Visualize:

The ball is in static equilibrium in the external electric field when the string makes an angle \( \theta = 20^\circ \) with the vertical. The three forces acting on the charged ball are the electric force due to the field, the weight of the ball, and the tension force.

Solve: In static equilibrium, Newton’s second law for the ball is

\[ \vec{F}_{\text{net}} = \vec{T} + \vec{w} + \vec{F}_e = \vec{0} \text{.} \]

In component form,

\[ (F_{\text{net}})_x = T_x + 0 \text{ N} + qE = 0 \text{ N} \quad (F_{\text{net}})_y = T_y - mg + 0 \text{ N} = 0 \text{ N} \]

The above two equations simplify to

\[ T \sin \theta = qE \quad T \cos \theta = mg \]

Dividing both equations, we get

\[ \tan \theta = \frac{qE}{mg} \Rightarrow q = \frac{mg \tan \theta}{E} = \frac{(5.0 \times 10^{-3} \text{ kg})(9.8 \text{ N/kg}) \tan 20^\circ}{100,000 \text{ N/C}} = 1.78 \times 10^{-7} \text{ C} = 178 \text{ nC} \]

25.76. Solve: (a) Kinetic energy is \( K = \frac{1}{2}mv^2 \), so the velocity squared is \( v^2 = 2K/m \). From kinematics, a particle moving through distance \( \Delta x \) with acceleration \( a \), starting from rest, finishes with \( v^2 = 2a\Delta x \). To gain \( K = 2 \times 10^{-18} \text{ J} \) of kinetic energy in \( \Delta x = 2.0 \mu \text{m} \) requires an acceleration

\[ a = \frac{v^2}{2\Delta x} = \frac{2K/m}{2\Delta x} = \frac{K}{m\Delta x} = \frac{2.0 \times 10^{-18} \text{ J}}{(9.11\times 10^{-31} \text{ kg})(2.0 \times 10^{-6} \text{ m})} = 1.10 \times 10^{18} \text{ m/s}^2 \]
(b) The force that produces this acceleration is

\[ F = ma = \left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.10 \times 10^{18} \text{ m/s}^2\right) = 1.0 \times 10^{-12} \text{ N} \]

(c) The electric field is

\[ E = \frac{F}{e} = \frac{1.0 \times 10^{-12} \text{ N}}{1.6 \times 10^{-19} \text{ C}} = 6.25 \times 10^6 \text{ N/C} \]

(d) The force on an electron due to charge \( q \) is \( F = \frac{K|q|e}{r^2} \). To have a breakdown, the force on the electron must be at least \( 1.0 \times 10^{-12} \text{ N} \). The minimum charge that could cause a breakdown will be the charge that causes exactly a force of \( 1.0 \times 10^{-12} \text{ N} \):

\[
|F| = \frac{K|q|e}{r^2} = 1.0 \times 10^{-12} \text{ N} \Rightarrow |q| = \frac{r^2 F}{Ke} = \frac{(0.01 \text{ m})^2 \left(1.0 \times 10^{-12} \text{ N}\right)}{9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \left(1.6 \times 10^{-19} \text{ C}\right)} = 6.9 \times 10^{-8} \text{ C} = 68 \text{ nC}
\]