27.1. **Visualize:**

As discussed in Section 27.1, the symmetry of the electric field must match the symmetry of the charge distribution. In particular, the electric field of a cylindrically symmetric charge distribution cannot have a component parallel to the cylinder axis. Also, the electric field of a cylindrically symmetric charge distribution cannot have a component tangent to the circular cross section. The only shape for the electric field that matches the symmetry of the charge distribution with respect to (i) translation parallel to the cylinder axis, (ii) rotation by an angle about the cylinder axis, and (iii) reflections in any plane containing or perpendicular to the cylinder axis is the one shown in the figure.

27.5. **Model:** The electric flux “flows” out of a closed surface around a region of space containing a net positive charge and into a closed surface surrounding a net negative charge.

**Visualize:** Please refer to Figure Ex27.5. Let \( A \) be the area of each of the six faces of the cube.

**Solve:** The electric flux is defined as \( \Phi = \vec{E} \cdot \vec{A} = EA \cos \theta \), where \( \theta \) is the angle between the electric field and a line perpendicular to the plane of the surface. The electric flux out of the closed cube surface is

\[
\Phi_{\text{out}} = (10 \text{ N/C} + 10 \text{ N/C} + 10 \text{ N/C} + 5 \text{ N/C}) A \cos 0^\circ = (35A) \text{ N m}^2/\text{C}
\]

Similarly, the electric flux into the closed cube surface is

\[
\Phi_{\text{in}} = (15 \text{ N/C} + 20 \text{ N/C}) A \cos 180^\circ = -(35A) \text{ N m}^2/\text{C}
\]

Hence, \( \Phi_{\text{out}} + \Phi_{\text{in}} = 0 \text{ N m}^2/\text{C} \). Since the net electric flux is zero, the closed box contains no charge.

27.9. **Model:** The electric field is uniform over the entire surface.

**Visualize:** Please refer to Figure Ex27.9. The electric field vectors make an angle of 30° below the surface. Because the normal \( \hat{n} \) to the planar surface is at an angle of 90° relative to the surface, the angle between \( \hat{n} \) and \( E \) is \( \theta = 120^\circ \).

**Solve:** The electric flux is

\[
\Phi = \vec{E} \cdot \vec{A} = EA \cos \theta = (200 \text{ N/C})(1.0 \times 10^{-2} \text{ m}^2) \cos 120^\circ = -1.0 \text{ N m}^2/\text{C}
\]

27.21. **Visualize:**

For any closed surface that encloses a total charge \( Q_{\text{enc}} \), the net electric flux through the closed surface is \( \Phi = Q_{\text{enc}}/\epsilon_0 \). 
27.22. **Visualize:** Please refer to Figure Ex27.22. For any closed surface that encloses a total charge \( Q_{in} \), the net electric flux through the closed surface is \( \Phi = \frac{Q_{in}}{\varepsilon_0} \). For the closed surface of the torus, \( Q_{in} \) includes only the \(-1\) nC charge. So, the net flux through the torus is due to this charge:

\[
\Phi = \frac{-1 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2} = -113 \text{ N m}^2/\text{C}
\]

This is inward flux.

27.26. **Model:** The excess charge on a conductor resides on the outer surface.  
**Visualize:** Please refer to Figure P27.26.  
**Solve:** Point 1 is at the surface of a charged conductor, hence

\[
\vec{E}_{surface} = \left( \frac{q}{\varepsilon_0} \right), \text{ perpendicular to surface} \Rightarrow E_{surface} = \frac{(5.0 \times 10^{-10})(1.60 \times 10^{-19} \text{ C/m}^2)}{8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2} = 904 \text{ N/C}
\]

At point 2 the electric field strength is zero because this point lies inside the conductor. The electric field strength at point 3 is zero because there is no excess charge on the interior surface of the box. This can be quickly seen by considering a Gaussian surface just inside the interior surface of the box as shown in Figure 27.31.

27.30. **Visualize:** Please refer to Figure P 27.30.  
**Solve:** For any closed surface that encloses a total charge \( Q_{in} \), the net electric flux through the surface is \( \Phi = \frac{Q_{in}}{\varepsilon_0} \). We can write three equations from the three closed surfaces in the figure:

\[
\Phi_A = \frac{-q}{\varepsilon_0} = \frac{q_1 + q_3}{\varepsilon_0} \Rightarrow q_1 + q_3 = -q \\
\Phi_B = \frac{3q}{\varepsilon_0} = \frac{q_1 + q_2}{\varepsilon_0} \Rightarrow q_1 + q_2 = 3q
\]

\[
\Phi_C = \frac{-2q}{\varepsilon_0} = \frac{q_2 + q_1}{\varepsilon_0} \Rightarrow q_2 + q_1 = -2q
\]

Subtracting third equation from the first,

\[q_1 - q_2 = +q\]

Adding second equation to this equation,

\[2q_1 = +4q \Rightarrow q_1 = 2q\]

That is, \( q_1 = +2q \), \( q_2 = +q \), and \( q_3 = -3q \).
27.38. **Model:** The excess charge on a conductor resides on the outer surface. The field inside, outside, and within the hollow metal sphere has spherical symmetry.

**Visualize:**

The figure shows spherical Gaussian surfaces with radii $r \leq a$, $a < r < b$, and $r \geq b$. These surfaces match the symmetry of the charge distribution.

**Solve:**

(a) For $r \leq a$, Gauss’s law is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0} = \frac{+Q}{\varepsilon_0}$$

Notice that the electric field is everywhere perpendicular to the spherical surface. Because of the spherical symmetry of the charge, the electric field magnitude $E$ is the same at all points on the Gaussian surface. Thus,

$$\Phi_e = E A_{\text{sphere}} = E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi \varepsilon_0 r^2} \Rightarrow \vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{Q_r}{r^2}$$

where we made use of the fact that $E$ is directed radially outward. The field depends only on the enclosed charge, not on the charge on the outer sphere.

For $a < r < b$, Gauss’s law is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = E A_{\text{sphere}} = E\left(4\pi r^2\right) = \frac{Q_{\text{in}}}{\varepsilon_0}$$

Here $Q_{\text{in}} = 0 \text{ C}$. It is not $+Q$, because the charge in the cavity polarizes the metal sphere in such a way that $E = 0$ in the metal. Thus a charge $-Q$ moves to the inner surface. Because the hollow sphere has a net charge of $+2Q$, the exterior surface now has a charge of $+3Q$. Thus, the electric field $E = 0 \text{ N/C}$.

For $r \geq b$,

$$Q_{\text{in}} = Q_{\text{exterior}} + Q_{\text{interior}} + Q_{\text{cavity}} = +3Q + (-Q) + (+Q) = +3Q$$

Gauss’s law applied to the Gaussian surface at $r \geq b$ yields:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = E A_{\text{sphere}} = E\left(4\pi r^2\right) = \frac{Q_{\text{in}}}{\varepsilon_0} = \frac{+3Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi \varepsilon_0} \left(\frac{3Q}{r^2}\right) \Rightarrow \vec{E} = \frac{1}{4\pi \varepsilon_0} \left(\frac{3Q}{r^2}\right) \hat{r}$$

(b) As determined in part (a), the inside surface of the hollow sphere has a charge of $-Q$, and the exterior surface of the hollow sphere has a charge of $+3Q$.

**Assess:** The hollow sphere still has the same charge $+2Q$ as given in the problem, although the sphere is polarized.
27.45. **Model:** The charge distributions of the ball and the metal shell are assumed to have spherical symmetry.

**Visualize:**

The spherical symmetry of the charge distribution tells us that the electric field points radially inward or outward. We will therefore choose Gaussian surfaces to match the spherical symmetry of the charge distribution and the field. The figure shows four Gaussian surfaces in the four regions: \( r \leq a \), \( a < r < b \), \( b \leq r \leq c \) and \( r > c \).

**Solve:**  
(a) Gauss’s law is \( \Phi_e = \oint E \cdot dA = Q_{\text{in}} / \varepsilon_0 \). Applying it to the region \( r \leq a \), where the charge is negative so \( \vec{E} \) points inward, we get

\[
-\varepsilon_0 E A_{\text{sphere}} = -E \left( 4\pi r^2 \right) = -\frac{\rho \left( \frac{4}{3} \pi r^3 \right)}{\varepsilon_0} \Rightarrow E = -\frac{\rho r}{3\varepsilon_0}
\]

Here \( \rho = -Q / \frac{4}{3} \pi a^3 \) is the charge density (C/m^3). Thus

\[
\vec{E} = \left( \frac{1}{4\pi \varepsilon_0} \frac{Q r}{a^3}, \text{ inward} \right) = -\frac{1}{4\pi \varepsilon_0} \frac{Q r}{a^3} \hat{r}
\]

Applying Gauss’s law to the region \( a < r < b \),

\[
-\varepsilon_0 E A_{\text{sphere}} = -\frac{Q}{\varepsilon_0} 
\Rightarrow E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \Rightarrow \vec{E} = -\frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \hat{r}
\]

\( \vec{E} = \vec{0} \) in the region \( b \leq r \leq c \) because this is a conductor in electrostatic equilibrium.

To apply Gauss’s to the region \( r > c \), we use \( Q_{\text{in}} = -Q + 2Q = +Q \). Thus,

\[
\varepsilon_0 E A_{\text{sphere}} = \frac{Q}{\varepsilon_0} 
\Rightarrow E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \Rightarrow \vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \hat{r}
\]
27.46. **Model:** The three planes of charge are infinite planes.

**Visualize:**

From planar symmetry the electric field can point straight toward or away from the plane. The three planes are labeled as P (top), P', and P'' (bottom).

**Solve:** From Example 27.6, the electric field of an infinite charged plane of charge density $\eta$ is

$$E_{\text{plane}} = \frac{\eta}{2\varepsilon_0} \Rightarrow E_p = E_{p'} = \frac{\eta}{4\varepsilon_0} = \frac{E_{p''}}{2}$$

In region 1 the three electric fields are

$$\vec{E}_p = -\frac{\eta}{4\varepsilon_0} \hat{j} \quad \vec{E}_{p'} = \frac{\eta}{2\varepsilon_0} \hat{j} \quad \vec{E}_{p''} = -\frac{\eta}{4\varepsilon_0} \hat{j}$$

Adding the three contributions, we get $\vec{E}_{\text{net}} = 0 \, \text{N/C}$.

In region 2 the three electric fields are

$$\vec{E}_p = -\frac{\eta}{4\varepsilon_0} \hat{j} \quad \vec{E}_{p'} = \frac{\eta}{2\varepsilon_0} \hat{j} \quad \vec{E}_{p''} = -\frac{\eta}{4\varepsilon_0} \hat{j}$$

Thus, $\vec{E}_{\text{net}} = (\eta/2\varepsilon_0) \hat{j}$.

In region 3,

$$\vec{E}_p = \frac{\eta}{4\varepsilon_0} \hat{j} \quad \vec{E}_{p'} = -\frac{\eta}{2\varepsilon_0} \hat{j} \quad \vec{E}_{p''} = -\frac{\eta}{4\varepsilon_0} \hat{j}$$

Thus, $\vec{E}_{\text{net}} = -(\eta/2\varepsilon_0) \hat{j}$.

In region 4,

$$\vec{E}_p = \frac{\eta}{4\varepsilon_0} \hat{j} \quad \vec{E}_{p'} = -\frac{\eta}{2\varepsilon_0} \hat{j} \quad \vec{E}_{p''} = \frac{\eta}{4\varepsilon_0} \hat{j}$$

Thus $\vec{E}_{\text{net}} = 0 \, \text{N/C}$.
27.47. **Model:** The charge has planar symmetry, so the electric field must point toward or away from the slab. Furthermore, the field strength must be the same at equal distances on either side of the center of the slab.

**Visualize:**

Choose Gaussian surfaces to be cylinders of length $2z$ centered on the $z = 0$ plane. The ends of the cylinders have area $A$.

**Solve:**

(a) For the Gaussian cylinder inside the slab, with $z < z_0$, Gauss’s law is

$$
\oint \mathbf{E} \cdot d\mathbf{A} = \oint_{\text{top}} \mathbf{E} \cdot d\mathbf{A} + \oint_{\text{bottom}} \mathbf{E} \cdot d\mathbf{A} + \oint_{\text{sides}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0}
$$

The field is parallel to the sides, so the third integral is zero. The field emerges from both ends, so the first two integrals are the same. The charge enclosed is the volume of the cylinder multiplied by the charge density, or $Q_{\text{in}} = \rho_0 V = \rho_0 (2zA)$. Thus

$$
\frac{Q_{\text{in}}}{\varepsilon_0} = \frac{\rho_0 (2zA)}{\varepsilon_0} = \oint_{\text{top}} \mathbf{E} \cdot d\mathbf{A} + \oint_{\text{bottom}} \mathbf{E} \cdot d\mathbf{A} + 0 = 2EA \Rightarrow E = \frac{\rho_0 z}{\varepsilon_0}
$$

The field increases linearly with distance from the center.

(b) The analysis is the same for the cylinder that extends outside the slab, with $z > z_0$, except that the enclosed charge $Q = \rho_0 (2z_0 A)$ is that within a cylinder of length $2z_0$ rather than $2z$. Thus

$$
\frac{Q_{\text{out}}}{\varepsilon_0} = \frac{\rho_0 (2z_0 A)}{\varepsilon_0} = \oint_{\text{top}} \mathbf{E} \cdot d\mathbf{A} + \oint_{\text{bottom}} \mathbf{E} \cdot d\mathbf{A} + 0 = 2EA \Rightarrow E = \frac{\rho_0 z_0}{\varepsilon_0}
$$

The field strength outside the slab is constant, and it matches the result of part (a) at the boundary.

(c)
27.48. **Model:** The infinitely wide plane of charge with surface charge density $\eta$ polarizes the infinitely wide conductor.

**Visualize:**

Because $\vec{E} = \vec{0}$ in the metal there will be an induced charge polarization. The face of the conductor adjacent to the plane of charge is negatively charged. This makes the other face of the conductor positively charged. We thus have three infinite planes of charge. These are P (top conducting face), P’ (bottom conducting face), and P″ (plane of charge).

**Solve:** Let $\eta_1$, $\eta_2$, and $\eta_3$ be the surface charge densities of the three surfaces with $\eta_2$ a negative number. The electric field due to a plane of charge with surface charge density $\eta$ is $E = \eta/2\varepsilon_0$. Because the electric field inside a conductor is zero (region 2),

$$\vec{E}_p + \vec{E}_p + \vec{E}_p = \vec{0} \text{ N/C} \Rightarrow -\frac{\eta_1}{2\varepsilon_0} \hat{j} + \frac{\eta_2}{2\varepsilon_0} \hat{j} + \frac{\eta_3}{2\varepsilon_0} \hat{j} = \vec{0} \text{ N/C} \Rightarrow -\eta_1 + \eta_2 + \eta_3 = 0 \text{ C/m}^2$$

We have made the substitution $\eta_3 = \eta$. Also note that the field inside the conductor is downward from planes P and P’ and upward from P″. Because $\eta_1 + \eta_2 = 0$ C/m$^2$, because the conductor is neutral, $\eta_2 = -\eta_1$. The above equation becomes

$$-\eta_1 - \eta_1 + \eta = 0 \text{ C/m}^2 \Rightarrow \eta_1 = \frac{1}{2} \eta \Rightarrow \eta_2 = -\frac{1}{2} \eta$$

We are now in a position to find electric field in regions 1–4.

For region 1,

$$\vec{E}_p = \frac{\eta}{4\varepsilon_0} \hat{j}, \quad \vec{E}_p = -\frac{\eta}{4\varepsilon_0} \hat{j}, \quad \vec{E}_p = \frac{\eta}{2\varepsilon_0} \hat{j}$$

The electric field is $\vec{E}_{\text{net}} = \vec{E}_p + \vec{E}_p + \vec{E}_p = \left(\frac{\eta}{2\varepsilon_0}\right) \hat{j}$.

In region 2, $\vec{E}_{\text{net}} = 0$ N/C. In region 3,

$$\vec{E}_p = -\frac{\eta}{4\varepsilon_0} \hat{j}, \quad \vec{E}_p = \frac{\eta}{4\varepsilon_0} \hat{j}, \quad \vec{E}_p = \frac{\eta}{2\varepsilon_0} \hat{j}$$

The electric field is $\vec{E}_{\text{net}} = \left(\frac{\eta}{2\varepsilon_0}\right) \hat{j}$.

In region 4,

$$\vec{E}_p = -\frac{\eta}{4\varepsilon_0} \hat{j}, \quad \vec{E}_p = \frac{\eta}{4\varepsilon_0} \hat{j}, \quad \vec{E}_p = -\frac{\eta}{2\varepsilon_0} \hat{j}$$

The electric field is $\vec{E}_{\text{net}} = -\left(\frac{\eta}{2\varepsilon_0}\right) \hat{j}$. 
27.50. **Model:** A long, charged wire can be modeled as an infinitely long line of charge. 

**Visualize:**

The figure shows an infinitely long line of charge that is surrounded by a hollow metal cylinder of radius $R$. The symmetry of the situation indicates that the only possible shape of the electric field is to point straight in or out from the wire. The shape of the field suggests that we choose our Gaussian surface to be a cylinder of radius $r$ and length $L$, centered on the wire.

**Solve:** (a) For the region $r < R$, Gauss’s law is

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \Rightarrow \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = \frac{\lambda L}{\varepsilon_0}$$

$$\Rightarrow 0 \text{ N m}^2/C + 0 \text{ N m}^2/C + \vec{E} \cdot \vec{A}_{\text{side}} = \frac{\lambda L}{\varepsilon_0} \Rightarrow E(2\pi r)L = \frac{\lambda L}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi \varepsilon_0 r} \Rightarrow \vec{E} = \left( \frac{\lambda}{2\pi \varepsilon_0 r}, \text{ outward} \right) = \frac{\lambda}{2\pi \varepsilon_0 r} \hat{r}$$

(b) Applying Gauss’s law to the Gaussian surface at $r > R$,

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = \frac{Q_m}{\varepsilon_0}$$

$$\Rightarrow 0 \text{ N m}^2/C + 0 \text{ N m}^2/C + \vec{E} \cdot \vec{A}_{\text{wall}} = \frac{Q_m}{\varepsilon_0}$$

$$\Rightarrow E(2\pi rL) = \frac{Q_{\text{dip}} + Q_{\text{cylinder}}}{\varepsilon_0} = \frac{\lambda L + 2\lambda L}{\varepsilon_0} \Rightarrow E = \frac{3\lambda}{2\pi \varepsilon_0 r} \Rightarrow \vec{E} = \frac{3\lambda}{2\pi \varepsilon_0 r} \hat{r}$$
Assume that the negative charge uniformly distributed in the atom has spherical symmetry.

The nucleus is a positive point charge \( +Ze \) at the center of a sphere of radius \( R \). The spherical symmetry of the charge distribution tells us that the electric field must be radial. We choose a spherical Gaussian surface to match the spherical symmetry of the charge distribution and the field. The Gaussian surface is at \( r < R \), which means that we will calculate the amount of charge contained in this surface.

**Solve:**

**a)** Gauss’s law is \( \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \). The amount of charge inside is

\[
Q_{in} = \rho \left( \frac{4\pi r^3}{3} \right) + Ze = \left( \frac{-Ze}{4\pi R^2} \right) \left( \frac{4\pi r^3}{3} \right) + Ze = \left( \frac{-Ze}{R^2} \right) r^3 + Ze = Ze \left[ 1 - \frac{r^3}{R^3} \right]
\]

\[\Rightarrow E_{in} \left( 4\pi r^2 \right) = \frac{Ze}{\varepsilon_0} \left[ 1 - \frac{r^3}{R^3} \right] \Rightarrow E_{in} = \frac{Ze}{4\pi \varepsilon_0} \left[ \frac{1}{r^2} - \frac{r}{R^3} \right] \]

\[\text{(b)} \text{ At the surface of the atom, } r = R. \text{ Thus,}\]

\[
E_{in} = \frac{Ze}{4\pi \varepsilon_0} \left[ \frac{1}{R^2} - \frac{R}{R^2} \right] = 0 \text{ N/C}
\]

This is an expected result, which can be quickly obtained from Gauss’s law. Applying Gauss’s law to a Gaussian surface just outside \( r = R \). Because the atom is electrically neutral, \( Q_{in} = 0 \). Thus

\[
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = 0 \Rightarrow E = 0 \text{ N/C}
\]

\[\text{(c)} \text{ The electric field strength at } r = \frac{1}{2} R = 0.05 \text{ nm is}\]

\[
E_{in} = 92 \left( 1.60 \times 10^{-19} \text{ C} \right) \left( 9.0 \times 10^9 \text{ C}^2/\text{Nm}^2 \right) \left[ \frac{1}{\left( 0.05 \text{ nm} \right)^2} - \left( \frac{0.05 \text{ nm}}{0.10 \text{ nm}} \right)^3 \right] = 4.64 \times 10^{13} \text{ N/C}
\]