**Problem 1.** Thin spherical shells. The inner thin shell has a radius of a and the outer thin shell has a radius b. We assume the magnitude of Q is greater than the magnitude of -q.

We want to find the potential in all of the regions. First we start with **region III**, \( r > b \). We find the field using Gauss’ law and then calculate the potential difference using a line integral:

\[
V_\infty - V(r) = -\int E.dr
\]

\[
= -\int \frac{k(Q - q)}{r^2} dr
\]

\[
= \frac{k(Q - q)}{r}
\]

Since \( V \) at infinity is zero we get:

\[
V_\infty(r) = \frac{k(Q - q)}{r}
\]

Now we look in **region II**. The charge enclosed in a Gaussian surface is now \(-q\), so we will integrate from the radius \( r \) to the shell at \( b \).
\[ V(b) - V(r) = \frac{1}{\varepsilon} E \, dr \]
\[ = \frac{1}{\varepsilon} kq \, dr \]
\[ = - \frac{kq}{b} + \frac{kq}{r} \]

Since we know \( V(b) \) we get:

\[ V_a(r) = \frac{k(Q - q)}{b} + \frac{kq}{b} - \frac{kq}{r} \]
\[ = \frac{kQ}{b} - \frac{kq}{r} \]

Finally we turn to region I. Inside there is no electric field, so the potential is constant and the same as the potential at \( V(a) \).

\[ V_I(r) = \frac{k(Q - q)}{b} + \frac{kq}{b} - \frac{kq}{a} \]

We can double check by taking the gradient of the above functions to get the proper electric fields back. Let us make an approximate plot to see the behavior of the potential.

The potential increases until it gets to \( b \), then decreases because of the negative charge at \( a \). Once inside \( a \), there is no electric field so the potential difference is zero.
**Problem 2. Thin Cylinder and Line Charge.** A very thin cylindrical shell of radius $a$ and length $L$ carries a charge of $+q$. Inside there is a line charge of length $L$ and charge $+Q$.

We start by calculating $E$ in region II. This is a cylinder so we will use Gauss’ law again and find $V$ by calculating a line integral.

$$V(r) - V(a) = -\int_r^a Edr$$

$$= -\int_r^a \frac{(q+Q)}{2\pi\varepsilon_0 r}dr$$

$$= -\frac{(q+Q)}{2\pi\varepsilon_0 a} \ln\left(\frac{r}{a}\right)$$

Since $V(a)$ is arbitrary we can make it zero, thus:

$$V_a(r) = -\frac{(q+Q)}{2\pi\varepsilon_0 a} \ln\left(\frac{r}{a}\right), \quad r>a$$

Now we will calculate the potential in region I, for $r<a$.

$$V(a) - V(r) = \int_a^r Edr$$

$$= \int_a^r \frac{Q}{2\pi\varepsilon_0 r} dr$$

$$= -\frac{Q}{2\pi\varepsilon_0 a} \ln\left(\frac{a}{r}\right)$$

We know $V(a)=0$ so we get:
\[ V_i(r) = \frac{Q}{2\pi \varepsilon_0} \ln\left(\frac{a}{r}\right), \; r < a \]

We can sketch roughly how the potential looks below:

The potential decreases as we increase our radius, and becomes negative past \( r = a \).