International Trade and Factor Mobility under External Economies of Scale

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Abstract

This paper analyzes international trade in goods and capital movement in the basic model of external economies of scale developed in Wong (2000b). It is shown that externality, even if it is a mild one, can have important implications on goods trade and capital movement. Many of the results derived in the neoclassical framework may not be valid once externality exists.

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1 Introduction

With the rapid growth of capital movement and foreign direct investment among countries, the relationship between international trade in goods and capital movement has been gaining more and more attention and analysis in the literature. However, most of the existing analysis on trade and capital movement is done within the neoclassical paradigm.

Trade theorists have made a lot of painstaking efforts in extending the neoclassical framework. One direction of extension is to introduce externality into the neoclassical framework. A very common approach is the so-called Marshallian externality, with economies of scale external to firms but internal to the industry (Marshall, 1879, 1890).\footnote{For discussion of some of the fundamental concepts of externality and how it is modelled in the theory of international trade, see Wong (2000a)} Despite the efforts made in the past decades, most work so far is still concentrating on international trade while keeping capital (and other factors) of a country bounded by the borders of the country. In other words, it is assumed that international trade exists while international factor movement is not.\footnote{Assuming no international factor movement in the neoclassical framework is sometimes justified by two arguments. First, it is argued that in the neoclassical framework factors of production can be regarded as negative outputs so that there is some symmetry between factor inputs and outputs. Thus allowing movement of one factor can be regarded as allowing one more tradable goods. Second, under certain conditions free trade in goods leads to factor price equalisation. This means that under free trade factors have no incentive to move to another country even if it is allowed.}

The purpose of this paper is to analyze some of the issues related to international trade in goods and capital movement in a framework that exhibits external economies of scale. In particular, it wants to analyze whether it still makes sense to ignore movement of factors between countries when externality is present. It is also interesting to ask and answer the following questions: Will factors have incentives to move from one country to another under free trade in goods? Will countries remain diversified in production when both goods and capital can move internationally? Can we still have the Law of Comparative Advantage? Can we predict the patterns of trade and investment of countries?

To answer the above questions, we use the basic model of external economies of scale described in Wong (2000b).\footnote{The basic model is extended in Wong (2000c).} The model has two sectors, with increasing returns in one of the sectors but with no cross-sector externality. The rest of the model has the characteristics of the neoclassical framework. As a result, the model allows us to better identify the roles of external economics of scale in international trade and capital movement.
The analysis provided in this paper shows that externality has important implications on closed and open economies. It is no longer true that capital movement is regarded as the movement of one more good. In fact, international capital movement in a model of externality should be analyzed explicitly. This paper shows that introducing externality to the neoclassical framework, even if in only one sector, can significantly alter the relationship between international trade in goods and capital movement.

Section 2 of this paper describes the basic model of external economies of scale, with special attention paid to the properties of some of the variables of a closed economy. Section 3 derives the autarkic equilibrium of an economy and shows how the equilibrium may be affected by an increase in the size of the economy. Section 4 analyzes free trade in goods with exogenously given level of international investment in a two-country model, while section 5 turns to the case in which free movement of capital is allowed with autarky in trade in goods. The analysis and results in these two sections are then combined together in Section 6 in a case in which both goods and capital can move freely between two countries. The last section concludes.

2 The Basic Model

The present paper makes use of the basic model of external economies of scale developed in Wong (2000b). The model is briefly described as follows. Consider an economy called home, in which capital and labor are used to produce two homogeneous goods labeled 1 and 2. The technologies of the sectors can be described by the following production functions:

\[ Q_1 = h_1(Q_1)F_1(K_1, L_1) \]
\[ Q_2 = F_2(K_2, L_2), \]

where \( Q_i \) is the output of good \( i, i = 1, 2 \), and \( K_i \) and \( L_i \) are respectively the capital and labor inputs in sector \( i \). Function \( F_i(K_i, L_i) \) is increasing, linearly homogeneous, concave and differentiable in factor inputs. There is an asymmetry between the two sectors. For sector 1, the technology factor \( h_1(Q_1) \) describes the presence of externality: It is regarded as a constant by all the firms in the sector, but its value actually depends on the output level of the sector. The function is assumed to have the following properties: \( h_1(Q_1) > 0 \) and \( h_{11}(Q_1) \equiv \frac{dh_1}{dQ_1} > 0 \) for \( Q_1 > 0 \). The sign of \( h_{11}(Q_1) \) means that the sector is subject to increasing returns. On the other hand, sector 2 is subject to constant returns: \( F_2(K_2, L_2) \) represents the relationship between output and inputs. Both sectors are competitive. The rest of the economy is
characterized by the neoclassical features, including perfect sectoral factor mobility and perfect price flexibility. We add the further assumption that sector 1 is capital intensive at all factor prices.

The present model is the same as the neoclassical framework except that one of the sectors is subject to external economies of scale.\(^4\) The assumption of no cross-sector externality and externality in one sector only is to avoid “contaminating” the effects of externality so that we can see more clearly how the presence of external economies of scale may affect international trade and factor mobility.\(^5\)

For positive outputs, the rate of variable returns to scale (VRS) of sector 1 is defined as \(\varepsilon_{11} \equiv Q_1 h_{11}(Q_1)/h_1(Q_1) > 0\). To get positive social marginal products of factors in sector 1, it is assumed that \(\varepsilon_{11} < 1\). As a matter of fact, this paper focuses on the case in which externality in sector 1 is only mild, meaning that \(\varepsilon_{11}\) is not too close to unity. We will explain the implications of this assumption later. Choosing good 2 as the numeraire, denote the supply price of good 1 by \(p^s\).\(^6\)

The capital and labor endowments of the economy are fixed and denoted by \(\bar{K}\) and \(\bar{L}\), respectively. With the possibility of international capital movement, denote the amount of foreign capital working in the economy by \(k\).\(^7\) The available amounts of capital and labor in the economy are given by

\[
\begin{align*}
K &= \bar{K} + k \\
L &= \bar{L}.
\end{align*}
\]

Following the approach in Wong (2000b), define two virtual outputs: \(\hat{Q}_i = F_i(K_i, L_i)\), \(i = 1, 2\). Since function \(F_i(K_i, L_i)\) has the properties of a neoclassical production function, we can see that the virtual outputs with the rest of the framework define a virtual system, which behaves like a neoclassical framework. Of course, since externality is absence in sector 2, \(\hat{Q}_2 = Q_2\). The corresponding virtual supply price of good 1 is defined as \(\hat{p}^s \equiv h_1p^s\).

In the virtual system, we can define the virtual GDP (gross domestic function) function, \(g(\hat{p}^s, K, L)\).\(^8\) Differentiation of the GDP function with respect to commodity

\(^4\)We keep other assumptions of a neoclassical framework such as price flexibility, perfect competition, and perfect factor mobility across sectors.
\(^5\)A more general model is considered in Wong (2000a, 2000c).
\(^6\)The supply price of a good is defined as the minimum price of the good that will make the profits of the firms in the sector non-negative. In this paper, supply price of good 1 is the same as the relative supply price of good 1 as good 2 is the numeraire.
\(^7\)A negative \(k\) means that home capital flows out to another country.
\(^8\)For the properties of a GDP function, which is also called GNP function or revenue function, see Wong (1995).
prices yields the virtual outputs, $\tilde{Q}_i(p^s, K, L)$. Using the definition of $\tilde{Q}_i$, we have the relationship between the virtual and real systems:

$$Q_1 = h_1(Q_1)\tilde{Q}_1(p^s, K, L) \quad (3)$$
$$Q_2 = \tilde{Q}_2(p^s, K, L). \quad (4)$$

### 2.1 Output Responses

Equations (3) and (4) provide the link between the virtual and real systems. Differentiate these equations, using the definitions of $\tilde{p}^s$ and rearranging the terms to yield:

$$\Phi \begin{bmatrix} \frac{\partial}{\partial Q_1} & 0 \\ -p^s h_{11} \tilde{Q}_{2p} & 1 \end{bmatrix} \begin{bmatrix} dQ_1 \\ dQ_2 \end{bmatrix} = \begin{bmatrix} h_{11} \tilde{Q}_{1p} \\ h_{11} \tilde{Q}_{2p} \end{bmatrix} dp^s + \begin{bmatrix} h_{11} \tilde{Q}_{1K} \\ \tilde{Q}_{2K} \end{bmatrix} dK$$
$$+ \begin{bmatrix} h_{11} \tilde{Q}_{1L} \\ \tilde{Q}_{2L} \end{bmatrix} dL, \quad (5)$$

where $\Phi = 1 - \varepsilon_{11} - \varepsilon_{11}\eta_{1p}$, $\eta_{1p} \equiv \tilde{p}^s \tilde{Q}_{1p}/\tilde{Q}_i$, and $\tilde{Q}_{1p} \equiv \partial \tilde{Q}_i/\partial \tilde{p}^s$. Note that $\eta_{1p}$ is the elasticity of the virtual supply of good $i$ with respect to the supply price of good 1; $\eta_{1p} \equiv \tilde{p}^s \tilde{Q}_{1p}/\tilde{Q}_i$. Assuming a strictly convex virtual production possibility frontier, we have $\tilde{Q}_{1p}, \eta_{1p} > 0$ and $\tilde{Q}_{2p}, \eta_{2p} < 0$. Equation (5) can be solved for the output levels:

$$\hat{Q}_1 = \frac{\eta_{1p}}{\Phi} \hat{p}^s + \frac{\eta_{1K}}{\Phi} \hat{K} + \frac{\eta_{1L}}{\Phi} \hat{L} \quad (6)$$
$$\hat{Q}_2 = \left[ \frac{1}{\Phi} - \varepsilon_{11} \eta_{2p} \eta_{1K} \right] \hat{p}^s + \left[ \frac{\varepsilon_{11} \eta_{1p} \eta_{1K}}{\Phi} \right] \hat{K} + \left[ \frac{\eta_{2K} + \varepsilon_{11} \eta_{2p} \eta_{1L}}{\Phi} \right] \hat{L}, \quad (7)$$

where “hat” represents the proportional change of a variable; for example, $\hat{Q}_1 = dQ_1/Q_1$. Equations (6) and (7) show how the outputs of the sectors are dependent on the supply price and factor endowments. It is clear that whether the responses of outputs to prices and factor endowments are normal depends on the sign and magnitude of the externality effect, $\varepsilon_{11}$. For example, equation (6) suggests that the price-output response is normal if and only if $\Phi > 0$. It can also be shown that according to an adjustment rule commonly found in the literature, an equilibrium is stable if and only if $\Phi > 0$. In the present paper, we assume that the external

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Note: The adjustment rule is that the good-1-to-good-2 output ratio increases if the demand price is higher than the supply price. See Wong (1995, 2000b) for more details.
economies of scale is mild. We suppose that the following condition is satisfied:

\[
\varepsilon_{11} < \frac{1}{1 + \eta_{1p}},
\]

which implies that \( \Phi > 0 \). This condition is assumed in the rest of this paper.\(^{10}\)

**Assumption A:** Condition (8) holds, implying that \( \Phi > 0 \).

Define \( z \equiv Q_1/Q_2 \) as the output ratio, and combine (6) and (7) to give

\[
\hat{z} = \frac{\mu}{\Phi} \hat{p} + \sigma \hat{K} + \frac{\zeta}{\Phi} \hat{L},
\]

where

\[
\begin{align*}
\mu &= \eta_{1p} - (1 - \varepsilon_{11}) \eta_{2p} > 0 \\
\sigma &= \eta_{1K}(1 - \varepsilon_{11} \eta_{2p}) - \eta_{2K} \Phi > 0 \\
\zeta &= \eta_{1L}(1 - \varepsilon_{11} \eta_{2p}) - \eta_{2L} \Phi < 0 \\
\sigma + \zeta &= \varepsilon_{11} \left[ 1 + \frac{g\eta_{1p}}{Q_2} \right] > 0.
\end{align*}
\]

The signs of these variables is based on the assumed factor intensity ranking and the assumption that \( \Phi > 0 \).

Wong (2000b) argues that equations (6) to (9), which describe the comparative-static properties of the basic model, give only *local* responses of outputs to price and factor endowments and that the responses of outputs to price and factor endowments are always normal in a *global* sense. Thus Wong (2000b) concludes that in the present basic model of externality, Rybcznski Theorem is always valid, even if at an equilibrium \( \Phi < 0 \). He further shows that if global changes are allowed, worries about unstable equilibria are usually misplaced. First, unstable equilibria nearly always do not exist. Second, even if the economy is currently at an unstable equilibrium, a comparative-static shock will shift the equilibrium to a new, stable one. At this new equilibrium, \( \Phi > 0 \). This allows us to focus on the cases in which \( \Phi > 0 \), an assumption we make in the present paper.

\(^{10}\)With \( \Phi > 0 \), (local) responses of outputs to prices and factor endowments are normal, and a production equilibrium is Marshallian stable. This assumption is stronger than what is usually needed because, as Wong (2000b) argues, even if a production equilibrium is unstable when \( \Phi < 0 \), a small shock will shift the equilibrium to a stable point with \( \Phi > 0 \).
Since the focus of the present model is international trade and factor mobility, two variables will play very important roles: the world price ratio $p$, and the level of capital movement (into the home economy) $k$. For this reason, condition (3) is inverted to yield the reduced-form output function of good $i$, $Q_i = Q_i(p, k)$, $i = 1, 2$: for simplicity the given factor endowments are not shown in the function. Using (6), the partial derivatives of the output function are

\[ Q_{1p} = \frac{h_1^2 \dot{Q}_{1p}}{\Phi} > 0 \]  
\[ Q_{1k} = \frac{h_1 \dot{Q}_{1k}}{\Phi} > 0. \]

where $Q_{1p} \equiv \partial Q_i / \partial p$ and $Q_{1k} \equiv \partial Q_i / \partial k$. The sign in the above equations is due to the assumption that $\Phi > 0$.

Using the output function $Q_1 = Q_1(p, k)$, we can have a similar form for the virtual GDP function, $g = G(p, k) \equiv g(h_1(Q_1(p, k))p, K + k, L)$, where for simplicity the given factor endowments are not given in the function. This function has the following derivatives:

\[ G_p = \frac{Q_1(1 - \varepsilon_{11})}{\Phi} > 0 \]
\[ G_k = \frac{h_1 \varepsilon_{11} \eta_{1k} p Q_1}{K \Phi} + r > 0. \]

Note that if $Q_1 = 0$, then $G_p = 0$ and $G_k = r$. Of course, with no production of good 1, no externality exists.

### 2.2 Factor Prices

Competitive factor prices in the virtual system can be derived from the virtual GDP function:

\[ r \equiv \bar{r}(ph_1(Q_1), K, L) = \frac{\partial g}{\partial K} \]  
\[ w \equiv \bar{w}(ph_1(Q_1), K, L) = \frac{\partial g}{\partial L}, \]

where $r$ is the rental rate and $w$ the wage rate.\(^{11}\) Conditions (12) and (13) show the dependence of the factor prices on the price ratio and factor endowments.

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\(^{11}\) Note that the factor prices in the virtual system are the same as the factor prices in the real system. For the proof, see Wong (2000b).
Differentiate the factor prices in (12) and (13) with respect to \( p \) and rearrange terms to give\(^\text{12}\)

\[
\begin{align*}
\dot{r} &= \frac{\eta_{rp}(1 - \varepsilon_{11})}{\Phi} \hat{p}, \\
\dot{w} &= \frac{\eta_{wp}(1 - \varepsilon_{11})}{\Phi} \hat{p},
\end{align*}
\]

(14)  

(15)

where

\[\eta_{jp} = \frac{\partial j \hat{p}}{\partial \hat{p} j},\]

is the price elasticity of factor price \( j = r, w \) in the virtual system. With the assumed factor intensity ranking of the sectors, \( \eta_{rp} > 1 \) and \( \eta_{wp} < 0 \). Whether the Stolper-Samuelson Theorem is valid locally with externality, an issue that has received a lot of attention in the literature, boils down to whether \( \Phi \) is positive or negative. However, Wong (2000b) argues that in a *global* sense the theorem is always valid because after a shock the economy will shift to a stable equilibrium, with the factor prices reacting normally to a change in commodity prices. Since the final equilibrium is stable, \( \Phi > 0 \). Furthermore, \( \Phi < (1 - \varepsilon_{11}) \). Condition (14) implies that for an increase in \( p \), \( \dot{r}/\hat{p} > 1 \) and \( \dot{w}/\hat{p} < 0 \). This is the Stolper-Samuelson Theorem.

In the present paper, it is more useful to express factor prices in terms of the commodity price ratio and the level of international capital movement. Recall that the output function of sector 1 can be expressed as the reduced-form, \( Q_1 = Q_1(p, k) \), the rental rate can be expressed as\(^\text{13}\)

\[
\mathcal{R}(p, k) \equiv \tilde{r}(ph_1(Q_1(p, k)), \bar{K} + k, \bar{L}).
\]

(16)

The derivatives of the rental rate in (16) are

\[
\begin{align*}
\mathcal{R}_p &= r_p = \frac{\tilde{r}_p h_1(1 - \varepsilon_{11})}{\Phi} > 0, \\
\mathcal{R}_k &= \frac{r}{K} \left[ \frac{\varepsilon_{11} \eta_{rp} \eta_{1K}}{\Phi} + \eta_{rK} \right],
\end{align*}
\]

(17)  

(18)

where \( \eta_{rK} \) is the elasticity of the rental rate with respect to capital endowment. If the economy is diversified, \( \eta_{rK} = 0 \) because with diversification factor prices are

\(^\text{12}\)We have used \( dr = \tilde{r}_p (h_1 + ph_{11} Q_{1p}) dp \).

\(^\text{13}\)A similar expression for the wage rate can be derived. Since the present paper is about international capital movement, we focus more on the rental rate than on the wage rate.
independent of factor endowments. The sign of $\mathcal{R}_p$ in (17) is true with mild external economies of scale. In (18), the impact of capital inflow on the rental rate exists through two channels: the direct effect, $\eta_{rK}$, which is zero if the economy is diversified because with diversification factor prices are independent of factor endowments, and the indirect effect, which is due to an increase in the output of good 1. The increase in good 1 output causes a rise in the virtual price ratio, and through the Stolper-Samuelson effect, a rise in the rental rate.

2.3 National Income

We assume that foreign capital working at home is paid its market rate in terms of good 2, $r$, free of any income taxes. The total income earned by foreign capital is equal to $rk = \mathcal{R}(p, k)k$, which is remitted out of the country. As a result, the home GNP is equal to $I = \mathcal{I}(p, k) \equiv \mathcal{G}(p, k) - \mathcal{R}(p, k)k$. The derivatives of this function is

$$I_p = \mathcal{G}_p - \mathcal{R}_p k = \frac{h_1 \Theta (1 - \varepsilon_{11})}{\Phi}$$

(19)

$$I_k = \mathcal{G}_k - \mathcal{R}_k k + r = \frac{p \Theta \varepsilon_{11} \eta_1 K}{K \Phi} - \tilde{r}_K k;$$

(20)

where $\Theta = (\tilde{Q}_1 - \tilde{r}_p k)$. If $k$ is negative or is not great, both $\Theta$ and $I_p$ are positive. The sign of $I_k$ can be determined in some cases. For example, if the economy is diversified so that $\tilde{r}_K = 0$, or if $k \geq 0$, then $I_k > 0$.

3 Autarkic Equilibrium

We now introduce the demand side of the economy. Assume that the preferences of the economy can be described by a social utility function, which is increasing, differentiable, quasi-concave, and homothetic. From the social utility function, the consumption demand for good $i$ can be written as $C_i = C_i(p, I(p, k))$. Define $m \equiv p \partial C_1 / \partial I$ as the marginal propensity to consume good 1. Given homothetic preferences, $0 < m < 1$. Furthermore, homotheticity of the social utility function means that the ratio of the consumption of the goods, $z$, can be expressed as a decreasing function of the demand price, $p^d$. Define the price elasticity of demand as $\gamma \equiv -\dot{z} / \dot{p}^d$.

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14If $k$ is negative, i.e., when home capital flows out, the amount of capital income earned in the foreign economy will be added to the GDP to get the GNP. If remittance is free of any foreign taxes, the amount of capital income should be written as $-r^*k$, but for simplicity we still keep the expression of $-rk$ to represent the income of the moving capital. This is not a crucial point in the present paper as eventually under free capital movement $r = r^*$. 

---
3.1 Autarkic Price Ratio and Rental Rate

The autarkic equilibrium of the economy is an output ratio \( z \) that equilibrates the supply price and the demand price under no international trade in goods or capital movement, i.e.,

\[
p^s = p^d = p^a,
\]

where \( p^a \) is the autarkic price ratio.\(^{15}\) Define \( \theta = \gamma \Phi + \mu \). If \( \Phi > 0 \), so is \( \theta \). It has been shown that \( \theta > 0 \) is sufficient for a locally stable autarkic equilibrium.\(^{16}\) However, as argued in Wong (2000b), after a comparative static shock, even if the initial equilibrium is an unstable one, the economy will adjust to a new, stable equilibrium, thus maintaining the normality of a comparative static shock. This allows us to justify the assumption that \( \theta > 0 \). Using the corresponding meaning of the variables, this assumption can be stated in an alternative way

\[
\gamma(1 - \varepsilon_{11} - \varepsilon_{11}\eta_{1p}) + \eta_{1p} - \eta_{2p} + \varepsilon_{11}\eta_{2p} > 0,
\]

or,

\[
\varepsilon_{11} < \frac{\gamma + \eta_{1p} - \eta_{2p}}{\gamma(1 + \eta_{1p}) - \eta_{2p}}.
\]

Note that \( \varepsilon_{11} \) is assumed to be less than unity, condition (22) is satisfied if either (a) \( \gamma \) is sufficiently small, or (b) \( \eta_{1p} \) is sufficiently small. Of course, if \( \Phi > 0 \), then condition (22) is valid.

3.2 Effects of An Expansion of the Economy

Before we turn to international trade and factor mobility, let us evaluate the effects of a uniform increase in the size of the economy on the autarkic equilibrium. Substitute \( \hat{z} = -\gamma \hat{p}^d \) and (21) into (9) and rearrange terms to give

\[
-\gamma \Phi \hat{p}^a = \mu \hat{p}^a + \sigma \hat{K} + \zeta \hat{L}.
\]

Let us consider a case in which the economy expands uniformly, i.e., \( \hat{K} = \hat{L} \), and (23) reduces to

\[
\hat{p}^a = -\theta(\sigma + \zeta) \hat{K}.
\]

\(^{15}\)Another autarkic equilibrium condition is \( Q_1(p, 0) = C_1(p, G(p, 0)) \), which can be solved for the autarkic price ratio \( p^a \).

\(^{16}\)This is based on the adjustment rule suggested in Ide and Takayama (1991) and Wong (2000b).
Condition (24) means that a uniform expansion of the economy will lower its autarkic price ratio.\textsuperscript{17} To determine the effect of an increase in the economy’s size on the rental rate, we differentiate (16) to give the percentage change in the autarkic rental rate:

$$\hat{r}^a = \eta_{rp} \left[ 1 + \varepsilon_{11} \hat{Q} / \hat{p}_p \right] \hat{p}^a + \eta_{rp} \varepsilon_{11} \left[ \hat{Q} / \hat{K} + \hat{Q} / \hat{L} \right] \hat{K}. \tag{25}$$

In evaluating (25), we have used the fact that with diversification, changes in the factor endowments will have only indirect effects, but no direct effects, on factor prices. The indirect effects exist through a change in the output of good 1. Using (6), (24), and the fact that $\eta_{1K} + \eta_{1L} = 1$, (25) reduces to

$$\hat{r}^a = \frac{\eta_{rp}}{\Phi} \left[ \varepsilon_{11} - \theta (1 - \varepsilon_{11})(\sigma + \zeta) \right] \hat{K}$$

$$= \frac{\varepsilon_{11} \eta_{rp}}{\Phi} \left[ 1 - \theta (1 - \varepsilon_{11})(1 + \eta_{1p}/s_2) \right] \hat{K}, \tag{26}$$

where $s_2 \equiv Q_2 / g$ is the share of the output of sector in the GDP and where the value of $\sigma + \zeta$ has been used. Let us consider the following condition

$$\theta (1 - \varepsilon_{11})(1 + \eta_{1p}/s_2) > 1,$$

or

$$\theta (1 - \varepsilon_{11}) > \frac{s_2}{s_2 + \eta_{1p}}. \tag{27}$$

If condition (27) is satisfied, the term within the brackets in (26) is negative. This means, from (26), that the rental rate drops after an expansion of the economy if $\Phi > 0$ and if condition (27) is satisfied. Given the assumption that $\Phi > 0$ (so that $\theta > 0$), (27) can be stated in an alternative way, after arranging the terms,

$$\varepsilon_{11} < 1 - \frac{s_2}{(s_2 + \eta_{1p})\theta}. \tag{28}$$

Condition (28) gives a limit on the value of $\varepsilon_{11}$ that is linked to a drop in the rental rate in an expanded economy.\textsuperscript{18}

Since this paper focuses on an economy with a mild externality in sector 1, we assume that $\Phi > 0$ and that (28) is satisfied, meaning that an economy that expands uniformly will experience a drop in its rental rate.

\textsuperscript{17}Similar results have been derived in Markusen and Melvin (1981), Tawada (1989), Ide and Takayama (1993), and Wong (1995).

\textsuperscript{18}Note that condition (28) is a non-linear equation of $\varepsilon_{11}$.
Lemma 1. Given mild external economies of scale as defined above, a uniform expansion of the economy will lower both the autarkic price ratio and the autarkic rental rate.

3.3 Export Supply Function

We now make use of the supply and demand functions of good 1 to define its export supply function, \( E_1(p, k) \equiv Q_1(p, k) - C_1(p, T(p, k)) \). Using conditions (10), (11), (19), and (20), the derivatives of the export supply function are equal to

\[
E_{1p} = \frac{h_1^2 \hat{Q}_{1p}}{\Phi} - C_{1p} - \frac{mh_1 \Theta (1 - \varepsilon_{11})}{p\Phi}
\]

\[
E_{1k} = \frac{h_1 \hat{Q}_{1K}(1 - m\varepsilon_{11})}{\Phi} + \frac{mh_1 k\varepsilon_{11} \hat{r}_K \eta_{1K}}{K\Phi} + \frac{m\hat{r}_K k}{p}.
\]

Lemma 2. (a) Given that \( Q_1 > 0 \), if either (i) \( k \) is sufficiently small in magnitude, or (ii) \( k > 0 \) and the economy is diversified, then \( E_{1k} > 0 \). (b) When \( Q_1 = 0 \), \( E_{1k} > 0 \) if and only if \( k < 0 \). (c) \( E_{1p} > 0 \) for small export levels of good 1.

Proof. (a) Since sector 1 is capital intensive, \( \hat{Q}_{1K} > 0 \) and \( \eta_{1K} > 1 \). By assumption, \( 0 < m, \varepsilon_{11} < 1, \Phi > 0 \). If \( k \) is sufficiently small, (30) implies that \( E_{1k} > 0 \). If the economy is diversified, \( \hat{r}_K = 0 \). If \( k > 0 \) (home being a host country), then (30) again implies that \( E_{1k} > 0 \). (b) If \( Q_1 = 0 \), the economy produces constant-returns good 2 only. Condition (30) reduces to \( E_{1k} = m\hat{r}_K k/p \). Since \( \hat{r}_K < 0 \), condition (30) gives the results. (c) This part is proved in Wong (2000b), noting that \( \Phi \) is assumed to be positive. ■ ■

Note that part (c) of the lemma does not rule out the possibility that \( E_{1p} \) could be negative when the export levels of good 1 are large.

4 Free Trade in Goods with Exogenous Capital Flow

We now analyze international trade in goods and capital movement. To do that, we introduce another economy, which we call foreign. Foreign variables are denoted by asterisks while home variables have no asterisks. These two economies satisfy the
following assumptions: (a) Both economies have the same technologies and preferences; (b) Both economies have the same capital-labor ratio, while home is bigger than foreign.\footnote{As Markusen and Melvin (1981) and Ide and Takayama (1993) note, the assumption of identical technologies is to “neutralize” the Ricardian basis for trade, while the assumption of identical capital-labor ratio is to “neutralize” the Heckscher-Ohlin basis for trade. The assumption of identical and homothetic preferences is to avoid any demand biases.}

Our goal is to analyze issues related to free movements of goods and capital between the two economies. However, we will analyze two special cases first: the first case with free trade in goods but exogenous capital flow (as in the case of a quantitative restriction on capital movement), and the second case with free movement of capital while movement of goods are given exogenously, except that free remittance of capital income is allowed (as in the case with a quota). In this section, we concentrate on the first case. In other words, we allow free trade in goods while capital movement is given exogenously.

### 4.1 Free Trade in Goods with No Capital Movement

To consider the case of free trade in goods with exogenous capital movement, let us consider first the subcase in which there is no capital movement, i.e., \( k = k^* = 0 \).

The equilibrium conditions for free trade are described by

\[
\mathcal{E}_1(p, 0) + \mathcal{E}^*_1(p^*, 0) = 0 \tag{31}
\]

\[
p = p^*. \tag{32}
\]

Condition (31) describes the equality between supply and demand for good 1 in the world, and condition (32) gives the equality between home and foreign price ratio, assuming no trade restrictions or transport costs. Since this case has been analyzed in depth in Wong (2000b), we just state one result here: the home economy, which is bigger, has a comparative advantage in good 1, and if a trade point adjusts in a Marshallian way as described in Wong (2000b), home exports good 1. Denote home’s export level of good 1 by \( E_1^0 > 0 \).\footnote{See Tawada (1989), Ide and Takayama (1993), and Wong (2000b). Wong (2000b) shows that under certain adjustment rules, a modified Law of Comparative Advantage exists in the present model of external economies of scale.}

We showed earlier that the bigger home economy produces a higher output level of good 1 under autarky. So home must produce a higher output level of good 1 under free trade in goods with no capital movement, since it exports good 1. From (16), when the two countries are facing the same commodity price ratio, \( p \), home that
produces a higher good-1 output must experience a higher virtual price ratio and thus a higher rental rate. We thus have:

**Lemma 3.** Under free trade in goods with no capital movement, the bigger country has a higher rental rate.

### 4.2 Free Trade in Goods with Possibly Non-zero Capital Movement

We now turn to a more general model, with \( k \) and \( k^* \) given exogenously, as in the case in which one of the countries imposes a binding quantitative restriction on international capital movement. The equilibrium conditions are

\[
\begin{align*}
E_1(p, k) + E_1^*(p^*, k^*) &= 0 \quad (33) \\
k + k^* &= 0 \quad (34) \\
p &= p^*. \quad (35)
\end{align*}
\]

By Walras’s Law, equilibrium of the good-2 market is implied by conditions (33) to (35). These conditions can be combined to give

\[
E_1(p, k) + E_1^*(p, -k) = 0. \quad (36)
\]

Condition (36) can be solved for the price ratio that, at the given capital movement \( k \), equilibrates the commodity markets, \( p = \rho(k) \). To obtain the dependence of the price ratio on the capital movement, we differentiate condition (36) and rearrange terms to give

\[
\rho' \equiv \frac{dp}{dk} = \frac{E_{1k^*} - E_{1k}}{E_{1p} + E_{1p^*}}. \quad (37)
\]

The denominator in (37) is positive to satisfy the usual Marshallian stability condition. It is satisfied if both \( E_{1p} \) and \( E_{1p^*} \) are positive or if one of them is mildly negative while the other one is sufficiently positive. We regard a positive denominator as a normal condition and assume that it holds. Thus an increase in \( k \) raises the free-trade price ratio if and only if it causes a decrease in the excess supply of good 1 in the world.

Substitute \( p = \rho(k) \) into home’s export supply function of good 1 to yield

\[
E_1 = E_1(\rho(k), k). \quad (38)
\]
Condition (38) describes home’s equilibrium export level of good 1 at various levels of international capital movement. Such dependence can be illustrated by schedule GT in Figure 1. The schedule cuts the vertical axis at a level of $E^0_1 > 0$, which is home export of good 1 with no international capital movement.

The slope of schedule GT can be obtained by differentiating equation (38), making use of (37) and rearranging the terms:

$$\frac{dE_1}{dk}\bigg|_{GT} = \frac{\mathcal{E}_{1p^*} E_{1k^*} + \mathcal{E}_{1k} E_{1p^*}}{\mathcal{E}_{1p} + \mathcal{E}_{1p^*}}.$$  \hspace{1cm} (39)

The sign of the slope of the schedule is in general ambiguous. Using the terminology in Wong (1986), we can say that capital movement augments (or diminishes) good trade if the schedule is positively (or negatively) sloped. Consider the following condition that describes a particular pattern of production, trade, and capital movement.

**Condition W.** Home is a host country to foreign capital. Home is diversified in production while foreign is completely specialized in producing good 2.

As will be shown later, under free movement of goods and capital, home exports good 1 and receives capital, and at least one of the countries must be completely specialized. This implies that condition W is likely to be satisfied under free trade in goods and capital movement. From Lemma 2, we note that $\mathcal{E}_{1k}$ and $\mathcal{E}_{1k^*}$ are positive if $k$ is sufficiently small or if condition W is satisfied. Using this result and condition (39), we have:

**Proposition 1** For small movements of capital from foreign to home or under condition W, capital movement augments goods trade.

The special case with small, exogenous movements of capital has been analyzed in Markusen and Melvin (1981) and Markusen (1983). The model considered in the present model is more general because it allows finite movements of capital and homothetic production functions.

We now examine the stability of a free-trade equilibrium. Assume that home export of good 1 adjusts according to the following equation:

$$\dot{E}_1 = A(p^* - p),$$  \hspace{1cm} (40)

where $A$ is a positive constant. The rationale behind condition (40) is that home has incentives to sell more good 1 to foreign if $p^* > p$. Differentiate both sides of (40) to
give

\[ d\dot{E}_1 = -A \left( \frac{1}{\xi_{1p}^*} + \frac{1}{\xi_{1p}} \right) dE_1. \]  

(41)

For a stable free-trade equilibrium, we require and assume that the term inside the parentheses in (41) be positive. Note that the adjustment rule in (41) is sometimes called Marshallian adjustment, as it is based on quantity adjustment.

**Condition E.** \( \xi_{1p}^*, \xi_{1p} > 0 \).

**Lemma 4.** Condition E is sufficient for a Marshallian stable free-trade equilibrium.

Adjustment of the home export level of good 1 can be illustrated in Figure 1. Assuming condition E, in the region above schedule GT, such as point M, \( p > p^* \). By (40), \( E_1 \) drops, and point M moves down toward the schedule. Similarly, in the region below the schedule, such as point N, \( p < p^* \), and point N shifts up.\(^{21}\)

5 **Endogenous International Capital Movements**

We now turn to the cases in which endogenous international capital movement is allowed. In these cases, we treat home’s export of good 1 as exogenously given (as under a binding export/import quota imposed by one of the countries on good 1), and allow capital to move internationally free of any restrictions.

The equilibrium conditions are

\[ \xi_1(p, k) + \xi_1^*(p^*, k^*) = 0 \]  
\[ k + k^* = 0 \]  
\[ R(p, k) = R^*(p^*, k^*). \]  

(42)
(43)
(44)

In (42) and (44), \( p \) is the price ratio that solves \( E_1 = \xi_1(p, k) \), with \( E_1 \) and \( k \) being treated as parameters; i.e., \( p = \phi(E_1, k) \). The derivatives of the price function are: \( \phi_E \equiv \partial \phi / \partial E_1 = 1/\xi_{1p} \) and \( \phi_k \equiv \partial \phi / \partial k = -\xi_{1k}/\xi_{1p} \). A similar function for the foreign 

\(^{21}\)Note that in the trade literature stability of a trade equilibrium is usually given in terms of price adjustment, which is also called Walrasian adjustment. The well-known Marshall-Lerner condition is derived based on the Walrasian adjustment. See Wong (1995, Chapter 2 for more details.)
price ratio can be defined: \( p^* = \phi^*(-E_1, -k) \), where (42) and (43) have been used. Using these two price functions, (44) reduces to
\[
\mathcal{R}(\phi(E_1, k), k) = \mathcal{R}^*(\phi^*(-E_1, -k), -k).
\] (45)
Condition (45) represents different combinations of \( E_1 \) and \( k \) that equilibrate the capital market. These combinations are illustrated by schedule KM in Figure 2.

To derive the properties of schedule KM, let us begin with the special case in which no trade in goods but remittance of capital income in terms of good 2 is allowed. This means that \( E_1 = E_1^* = 0 \). Condition (45) reduces to
\[
\mathcal{R}^*(\phi^*(0, -k), -k). \tag{46}
\]
Condition (46) can be solved for the equilibrium level of capital movement under autarky in trade in goods. The natural question is whether home is a source or a host country of capital movement, i.e., whether \( k \) is positive or negative at the equilibrium point when trade in goods is not allowed.

To answer that question, we can note from Lemma 1 that with mild external economies of scale the bigger home economy has a lower autarkic rental rate. Thus domestic capital tends to flow out until equation (46) is satisfied.\(^{22}\) Denote the corresponding value of capital movement by \( k^0 \), which is negative. This point is shown in Figure 2.

We now consider a more general case in which trade in goods is not prohibited so that we can consider other parts of schedule KM. The slope of schedule KM is obtained by differentiating both sides of (45) making use of the derivatives of functions \( \phi \) and \( \phi^* \); i.e.,
\[
\left. \frac{dE_1}{dk} \right|_{\text{KM}} = \frac{\mathcal{R}_p E_{1k} E_{1p}^* + \mathcal{R}_p^* E_{1k}^* E_{1p} - (\mathcal{R}_k + \mathcal{R}_k^*) E_{1p} E_{1p}^*}{\mathcal{R}_p^* E_{1p} + \mathcal{R}_p E_{1p}^*}. \tag{47}
\]
In general, the sign of the slope of schedule KM given in (47) is ambiguous. We now show that in order to have a stable capital market equilibrium for any given value of \( E_1 \), schedule KM has to be positively sloped.

Assume that international capital movement adjusts according to the following condition:
\[
\dot{k} = B(r - r^*), \tag{48}
\]
\(^{22}\)If there are multiple equilibria with international capital movement with autarky in trade in goods, the equilibrium reached is the one with the greatest value of \( k \) (negative), as capital flows according to the gap between the rental rates in the countries.
where $B$ is a positive constant. Condition (48), which can be called the Marshallian adjustment, states that more foreign capital flows to home if home rental rate is higher than the foreign one. Differentiate both sides of (48) to give

$$d\hat{k} = B \left( \frac{\partial r}{\partial k} + \frac{\partial r^*}{\partial k^*} \right) dk,$$

(49)

where the partial derivatives are evaluated under a constant export level. Using function $\phi(E_1, k)$, (49) reduces to

$$d\hat{k} = -B \left( \frac{R_p E_{1k}}{E_{1p}} + \frac{R_p^* E_{1k}^*}{E_{1p}^*} - R_k - R_k^* \right) dk.$$

(50)

A Marshallian stable capital-movement equilibrium requires that the term inside the parentheses in (50) be positive, but the term is equal to the slope of schedule KM, as (47) shows.

**Lemma 5.** A sufficient and necessary condition for a stable capital-market equilibrium based on the adjustment rule given in (48) is that schedule KM is positively sloped.

In the rest of this paper, we assume that schedule KM is positively sloped. Denote the vertical intercept of schedule KM by $E_1^k$. Making use of Proposition 1, we have

**Proposition 2** Assume a stable world capital market and either small values of $k$ or condition W. (a) Home’s export of good 1 augments international capital movement, and (b) international goods trade and capital movement are complements in the quantitative-relationship sense.

### 6 Endogenous Trade in Goods and Capital Movement

We now permit free movements of both goods and capital. The equilibrium conditions are:

$$E_1(p, k) + \mathcal{E}_1^*(p^*, k^*) = 0 \quad (51)$$

$$k + k^* = 0 \quad (52)$$

$$p = p^* \quad (53)$$

$$\mathcal{R}(p, k) = \mathcal{R}^*(p^*, k^*). \quad (54)$$

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These four conditions describe equilibrium of both good 1 and capital markets, and can be solved for $p$, $p^*$, $k$, and $k^*$. Given the equilibrium values of these variables, the export levels of the countries can be obtained from the export supply functions $E_1(p, k)$ and $E^*_1(p^*, k^*)$.

Note that conditions (51) to (53) represent endogenous trade in goods and can be illustrated graphically by schedule GT in Figure 1, while conditions (51), (52), and (54) denote the equilibrium under endogenous capital movement and can be illustrated by schedule KM in Figure 2. The two schedules are now shown in Figure 3. The intersection point between them, point W, shows the values of $k$ and $E_1$ that satisfy conditions (51) to (54); i.e., their values under free goods trade and capital movement.

Let us derive some important properties of these schedules and the equilibrium under free goods trade and capital movement. As noted earlier, under free trade in goods home has a higher rental rate. This means that under free trade foreign capital tends to flow to home. In terms of Figure 3, this means that the vertical intercept of schedule GT, $E_1^0$, is on the left-hand side of schedule KM. This point has implications on the direction of movement of capital. Recall that under certain conditions home has a lower rental rate under autarky, but if only free trade in goods is allowed, home’s rental rate is higher than that in foreign. So starting from the autarkic point, if free trade in goods takes place gradually, then home will switch from a source country to a host country. This phenomenon is due to the fact that before goods trade takes place, the size of home leads to a lower relative price of good 1 and a lower rental rate as caused by a strong price effect. When free trade is allowed, the bigger country will increase its production of good 1, thus increasing its demand for capital, making home rental rate higher than the foreign rate.

**Proposition 3** If goods trade is not allowed, home is a source country of capital movement. When free trade exists, foreign capital will tend to flow in, making home a host country.

Figure 3 brings out one important feature: international trade in goods alone, international capital movement with autarky in goods trade, and free capital movement and goods trade in general leads to different world equilibria. Using the terminology in Wong (1986), we say that goods trade and capital movement are complements in the price-equalization sense, meaning that both of them are needed for production efficiency. Thus we have

**Proposition 4** International goods trade and capital movement are complements in the price-equalization sense.
Let us now examine the pattern of production at the free trade and capital movement point, W. In the neoclassical framework, factor price equalization implies that there is a flat part of the world production possibility frontier (PPF), sometimes called the Chipman Flat. (Jones, 1967; Kemp and Inada, 1969; Chipman, 1971; Uekawa, 1973; and Brecher and Feenstra, 1983). Diversification in both countries means that the world production point occurs at a point on the flat portion of the PPF. In the present framework with external economies of scale, we have the following proposition:

**Proposition 5** Under free goods trade and capital movement, the world PPF does not have a Chipman flat.

This proposition can be explained intuitively. The earlier analysis shows that if a country is diversified, its rental rate depends only on the virtual price ratio but not on factor endowments. This means that if both countries have the same rental rate and are diversified, they must have the same virtual price ratio. Since the countries under free trade have the same commodity prices, to have the same virtual price ratio requires that the countries produce the same output level of good 1. However, the last condition in general will not be satisfied because home is bigger than foreign.

**Corollary.** Under free trade and capital movement, at least one of the countries is completely specialized.

The corollary follows directly the proposition. This is an important result. It shows not only the roles of external economies of scale but also the implications of international capital movement. In the neoclassical framework, international trade in goods and capital movement do not necessarily lead to complete specialization. In the presence of external economies of scale, if a big country trades with a small country, as long as their factor endowments are not too different from each other, it is possible that both countries remain diversified. In the present model with external economies of scale and international capital movement, the equilibrium is not compatible with diversification in both countries.

To get more properties of the world equilibrium, let us examine its stability. Using the approach introduced earlier, we postulate that the export volume and capital movement adjust in the following way:

\[
\dot{E}_1 = A(p^* - p) \tag{55}
\]

\[
\dot{k} = B(r - r^*) \tag{56}
\]

\[23\text{ A rigorous proof of this proposition is left to the reader.}\]
where $A$ and $B$ are positive constants. Under the conditions specified earlier, various trade points will adjust according to the directions of the arrows shown in the diagram. Based on these adjustment rules, the equilibrium point $W$ is stable if schedule $KM$ cuts schedule $GT$ from below.

In Figure 3, the curve with arrows shows one possible adjustment path, with the locus of $k$ and $E_1$ moving along the curve in the direction suggested by the arrows. When the trade point shifts along this path, home initially sends out capital ($k < 0$). At some point, it switches from a source country to a host country for international capital movement. The pattern of trade, however, remains unchanged, with home exporting good 1.

The above adjustment path, which is due to the fact that the bigger country tends to export good 1 and receive capital from the smaller country under free goods trade and capital movement. However, when both countries are under autarky, the bigger country can have a lower rental rate. This means that when goods trade and capital movement are first allowed, capital in the bigger country will tend to flow out, but this movement of capital will not last forever because through trade and as more capital flows out, the bigger country will experience a rise in its rental rate, and eventually it starts to draw back its capital that went out before, and even to attract the inflow of capital from the smaller country.

**Proposition 6** Given adjustment rules (55) and (56), under free movements of goods and capital, the bigger country exports good 1 and is a host country for capital from foreign.

By this proposition, the bigger country not only has a comparative advantage in good 1 but also a higher rental rate to attract foreign capital. By the previous two propositions, the smaller country (foreign) likely is completely specialized in good 2 and sends out capital, as described by condition $W$.

In the neoclassical framework with movements of more than two goods/factors, the Law of Comparative Advantage can be extended to give the General Law of Comparative Advantage. In the presence of external economies of scale, we have shown that a modified Law of Comparative Advantage holds. The question now is whether a certain version of the General Law of Comparative Advantage can be stated in the present framework. If we denote the free-trade equilibrium values of

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24 This proposition can be proved rigorously by linearizing the two schedules in a region near the equilibrium point. The proof is left to the reader.


26 See Wong (2000b).
home export of good 1 and capital inflow by $E^w_1$ and $k^w$, respectively, then we want to see whether the following condition holds:

$$-(p^a - p^s^a)E^w_1 + (r^a - r^{*a})k^w \geq 0. \quad (57)$$

The above analysis shows that good 1 is relatively cheaper and rental rate lower in home under autarky, i.e., $p^a < p^s^a$ and $r^a < r^{*a}$. We also know that $E^w_1, k^w > 0$. Therefore the sign in expression in (57) is ambiguous. Since the world equilibrium has no direct relation with the autarkic conditions in the two countries, it is possible that $k^w$ is sufficiently large, and that $(r^a - r^{*a})k^w$ is sufficiently negative, so that the condition is violated. Thus we have

**Proposition 7** With free trade in goods and capital movement, the General Law of Comparative Advantage does not hold in the present framework.

### 7 Concluding Remarks

In this paper, we examined the relationship between international trade in goods and capital movement and their impacts on resources allocation and welfare of the countries involved. By using the basic model of externality, we can see how the existence of external economies of scale may change the results that are derived in the neoclassical framework with which trade theorists are so familiar. One message that this paper sends is that if the present model is a good description of the technologies and relative factor endowments in two countries, then the practice of using the neoclassical framework to predict the effects of international trade in goods and capital movement could be misleading.

There is no doubt that the basic model considered in the present paper is a special one. In particular, it assumes externality in one sector only, and ignores cross-sector externality.\footnote{Well, it is more general than the one-factor models used widely in the literature, and it uses a homothetic production function instead of a homogeneous production function used in many other papers.} How well it describes the world or how well it can be applied to a pair of countries chosen by a researcher is not known. However, the simplicity of the model does provide us much insight into the roles of externality in the theory of international trade and factor movement.
Figure 1

Schedule GT
Figure 2

Schedule KM
Figure 3

Free Trade and Capital Movement
References


