The Impact of Trade Openness
on the Real Exchange Rate and the Trade Balance:
Closed-Form Solutions and Some Evidence

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Abstract
When agents across countries have home-biased preferences in a world of incomplete
financial markets, an increase in the relative price of home-produced goods resulted
from a permanent adverse home output shock makes the cost of living of home
agents higher than that of foreign agents [appreciation of real exchange rate], reducing
home relative consumption. Intertemporal utility maximizing agents with the degree
of relative risk aversion greater than unity in the CES utility function would like to
smooth their relative consumption by reducing exports and increasing imports,
resulting trade deficit.

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1. Introduction

The effect of terms-of-trade shocks on current account was first studied by Harberger [1950], and Laursen and Metzler [1950] in a static Keynesian open-economy framework. The famous classical Harberger-Laursen-Metzler effect postulates that households reduce their savings and worsen the current account when they experience a decrease in real income due to the worsening of terms of trade. In an intertemporal framework of Uzawa [1968] type utility function, Obstfeld [1982] challenged this postulate by pointing out that this effect works only when the shocks have a temporary nature. Further, he argued that for a permanent worsening of the terms of trade, households would increase their savings, and therefore the current account would improve. Svensson and Razin [1983] clarified this argument by examining the role of the time preference rate in the determination of current account. They attributed the result of Obstfeld [1982]’s to the increasing rate of time preference to the wealth, generalizing their results that for a permanent worsening of the terms of trade, savings and current account fall with decreasing rate of time preference, while rising with the increasing rate of time preference. Ultimately, Persson and Svensson [1985] demonstrated that the effect of terms of trade deterioration on current account would have any sign, if restrictions on the rate of time preference are not imposed. Later, Edwards [1989], and Ostry and Reinhart [1992] incorporated nontradables into consumption, and examined the additional effect of terms-of-trade shocks that may work via the changes in the relative price of tradables and nontradables. In a dynamic general equilibrium model with home and foreign produced goods, Backus, Kehoe, and Kydland [1995] investigated the effect of the relation between the intertemporal and intratemporal elasticity of substitution\(^1\) on the current account. They showed that if a household’s relative risk aversion is greater than the elasticity of substitution across home and foreign produced goods,

\(^1\)The intertemporal elasticity of substitution is defined as the inverse of the coefficient of risk aversion, while the intratemporal elasticity of substitution is as the elasticity of substitution across home and foreign produced goods.
he would try to smooth consumption over time with respect to adverse terms-of-trade shocks by running current account deficit. This result has been the standard postulate on the current account in the intertemporal approach to current account determination so far.

However, for developed countries whose economies are relatively open and more synchronized with the world economy and where financial markets are well-developed, Tesar [1995] found that the possible gains from international risk sharing via trade in goods and services or trade in financial assets are minimal, while the gains for developing countries are possibly significant. The implication of her result is that for developed countries that are more integrated with the world economy, further trade in goods and services or trade in financial assets for intertemporal consumption risk sharing may not be necessary. Frankel and Rose [1998] and Clark and van Wincoop [2001] found that closer international trade links result in more close business cycle correlations of consumption with respect to country specific shocks for industrialized countries. This result implies that consumption across countries is affected by similar degrees by country specific shocks when trading economies are integrated in terms of trade in goods and services. On the current account, utilizing the dataset of 18 industrial and 71 developing countries from 1971 through 1995, Chinn and Prasad [2003] found that the degree of openness of an economy is negatively related to its current account position. That is, as the degree of trade openness rises, current account deviations get smaller. On the other hand, on the relationship between trade openness and real exchange rates, Hau [2000, 2002] examined the OECD country data, and documented that as the weight of imports in GDP gets larger, the size of real exchange rate fluctuations becomes smaller. Betts and Kehoe [2006] and Drozd [2006] also found the negative link between trade intensity and the real exchange rate volatility in a cross-section of 132 bilateral country pairs between 12 major OECD countries. Both in closed

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2 Obstfeld and Rogoff [1996], Hussein and de Mello [1999], and Ito and Chinn [2007] provided the evidence that countries with a very high degree of capital mobility would have relatively smooth current accounts.
forms and empirically, Ahn and Kim [2007] showed that as trade openness defined as the share of imports in GDP rises, stabilization monetary policy under flexible exchange rates would not have much effectiveness in intertemporal consumption risk sharing since the extent of real depreciation resulted from a monetary expansion would be reduced by the rises in import prices and the cost of living, weakening the expenditure switching effect of monetary policy. Since the expenditure switching effect of monetary policy works in increasing exports and decreasing imports, their work also has an important implication on the determination of trade balance: the increase in trade openness would reduce the impact of country specific shocks on trade balance via the reduction in the extent of real exchange rate changes because of the changes in import prices and in the cost of living. However, there has been no full blown intertemporal utility optimizing model explaining the impact of trade openness on the trade balance via the real exchange rate changes.

The objective of this paper is twofold: one is to show that in closed forms, the magnitudes of the real exchange rate changes and trade balance deviations from their steady state values by country specific output and money shocks depend on the parameters of the household’s CRRA utility function such as the degree of relative risk aversion and the share of home-produced goods in their consumption baskets in the 2 country dynamic stochastic general equilibrium model of intertemporal utility maximization. This model features incomplete financial markets, sticky prices, and heterogeneous preferences across countries. Especially, uncovered interest rate parity is assumed to focus solely on the channel through which the real exchange rate and current account are affected by country specific shocks. In our intertemporal framework, for the households with relative risk aversion of CRRA utility function greater than 1, adverse country-specific output and money shocks appreciate the real exchange rate,

\[^3\text{In our model, the intratemporal elasticity of substitution between home-produced goods and imports is assumed to be 1.}\]
worsening trade balance when consumption across countries is home-biased. This is because with home bias in consumption, adverse country specific output and money shocks raise home cost of living by more than foreign cost of living, appreciating the real exchange rate, which, in turn, worsens trade balance of home country. We find that as trade openness across trading counterparts rises, the extent of real exchange rate appreciation by the adverse shocks decreases so that the magnitude of trade deficits decreases as well. As trading countries trade more, country specific adverse shocks affect both countries’ cost of living more equally so that the real exchange rate would not change much with respect to idiosyncratic shocks, and therefore the trade deficit would not change much, either. The other is to test empirically the implications of our theoretic results. We investigate the responses of the real exchange rate and trade balance to monetary policy and real output innovations within the context of a PVAR model of monetary policy, real output, real exchange rates and trade balance for a panel data set of 47 countries under flexible exchange rate regimes from 1973:Q1 through 2006:Q4. As Bernanke and Blinder [1992] contend that the federal funds rate is a good monetary policy measure, short run interest rates such as federal funds rates and money market interest rates are used to represent monetary policy in the estimation. For real output, real GDP index is used. Following Broda [2001, 2004], we proceed in 2 stages. In the first stage, we estimate the reduced form coefficients and reduced form residuals of the panel VAR of short run interest rate, real exchange rates, real output, and trade balance, using the Seemingly Unrelated Regression (SUR). The impact of trade openness on the monetary policy and real output effects on real exchange rates and trade balance are measured by estimating the coefficients of the interaction terms constructed by multiplying trade openness to the variables of monetary policy and real output. If the coefficient on the interaction term shows the opposite sign of those on the short run interest rate and real output, it means that the increase in trade openness weakens the monetary policy and real output effects. On the other hand, if it shows the same sign, the increase in trade openness reinforces the
monetary policy and real output effects. In the second stage, from the reduced form coefficients and reduced form residuals of the panel VAR, we recover the structural coefficients by using the short run identifying assumptions such that in the current period, monetary policy and real output affect the real exchange rate, which, in turn, affects trade balance, not vice versa. We obtain the impulse responses of the real exchange rate and trade balance with respect to the one period one hundred basis point decrease in the short run interest rate and the one period one hundred basis point increase in real output in two cases where trade openness is considered and where it is not considered. Comparison of impulse responses in two cases shows that the increase in trade openness appreciates the real exchange rate, and worsens trade balance given the effects of monetary policy and real output innovations on each variables.

The following is the organization of this paper. Section 2 describes the structure of our model. Section 3 examines the relation between real exchange rates and country specific output and money shocks. In Section 4, trade balance is derived as a function of the real exchange rate. Section 5 carries out our empirical experiment. Section 6 concludes.

2. The Model

Preferences

In the world economy, there are two countries of the same economic size, Home and Foreign. In Home and Foreign, there are continuums of identical households, $0 \leq v \leq 1$ and $1 \leq v \leq 2$ respectively, each of who specializes in a single differentiated product indexed by $v$. The representative household $v$ in Home is assumed to maximize his lifetime utility given by

$$U_i = E_t \left\{ \sum_{i=0}^{\infty} B^{-t} \left[ \frac{C_t^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left( \frac{M}{P_i} \right)^{1-\varepsilon} - \eta_i L_i(v) \right] \right\}, 0 \leq \beta \leq 1, 0 \leq \rho < \infty, 0 < \varepsilon < \infty [1]$$

where $L_i(v)$ denotes the amount of labor supplied by the representative household, $v$. $\beta$
denotes the rate of time discount, and $\rho$ is the degree of relative risk aversion of CRRA utility function. $C$ shows the index of per capita consumption. $M/P$ represents real money holdings that provides a liquidity service via the reduction of transaction costs of goods and assets. $\varepsilon$ is the inverse of the elasticity of money demand with respect to consumption, and $\chi$ is some constant. Technology shows constant returns to scale, so that $Y(v) = L(v)$ where $Y(v)$ denotes the output of the variety $v$ produced by the representative household, $v$. $\eta$ is an expected adverse output shock arising in the home country that adversely affects home households' utility.

Representative Households’ preferences across countries are asymmetrically identical since the weights on domestic goods and imports, $\gamma$ and $1 - \gamma$, are the same. Per capita consumption indices of home and foreign countries take the following forms.

$$C = \frac{C_H^{1 - \gamma} C_V^{1 - \gamma}}{\gamma^{1 - \gamma} (1 - \gamma)^{1 - \gamma}}; \quad C^* = \frac{(C_{H}^*)^{1 - \gamma} (C_{F}^*)^{1 - \gamma}}{\gamma^{1 - \gamma} (1 - \gamma)^{1 - \gamma}}, \quad 0 \leq \gamma \leq 1 \quad [2]$$

where $C_H$ and $C_F$ show the representative home household’s consumption of home and foreign produced goods respectively, and $C_H^*$ and $C_F^*$ the representative foreign household’s consumption of home and foreign produced goods respectively.

The sub-indices of per capita consumption of home and foreign goods in home and foreign countries, $C_{H}^i, C_{F}^i, C_{H}^*,$ and $C_{F}^*$ are respectively,

$$C_{H}^i = \left[ \int_0^1 C_H(v) \frac{d\nu}{\nu} \right]^{\frac{1}{\gamma}}; \quad C_{F}^i = \left[ \int_0^1 C_F(v) \frac{d\nu}{\nu} \right]^{\frac{1}{\gamma}} \quad [3]$$

$$C_{H}^* = \left[ \int_0^1 C_H^*(v) \frac{d\nu}{\nu} \right]^{\frac{1}{\gamma}}; \quad C_{F}^* = \left[ \int_0^1 C_F^*(v) \frac{d\nu}{\nu} \right]^{\frac{1}{\gamma}} \quad [4]$$

where $C_H(v)$ and $C_F(v)$ show the representative home household’s consumption of home and foreign produced goods respectively, and $C_H^*(v), C_F^*(v)$ the representative foreign
household’s consumption of home and foreign produced goods respectively. The elasticity of substitution between goods produced within the same country is $\theta$ that is assumed greater than 1, while the elasticity of substitution across goods produced in Home and Foreign, $\sigma$ is assumed 1.

**Cost of Living of the Representative Households in Home and Foreign**

The consumption-based price indices of home and foreign countries are as follows.

$$P \equiv (P_{H}^*)^{\gamma} (P_{F}^*)^{1-\gamma} ; P^* \equiv (P_{H}^*)^{1-\gamma} (P_{F}^*)^{\gamma}$$ \[5\]

where $P_{H}^*$ and $P_{F}^*$ show the home country’s price indexes for the goods produced in home and foreign countries respectively, and $P_{H}^*$ and $P_{F}^*$ the foreign country’s price indexes for the goods produced in home and foreign countries respectively.

The sub-price indices for home and foreign goods are respectively,

$$P_{H} = \left[ \int_{0}^{1} P_{H}(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}} \quad ; \quad P_{F} = \left[ \int_{1}^{2} P_{F}(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}} \quad \quad \quad [6]$$

$$P_{H}^* = \left[ \int_{0}^{1} P_{H}^*(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}} \quad ; \quad P_{F}^* = \left[ \int_{1}^{2} P_{F}^*(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}} \quad \quad \quad [7]$$

where $P_{H}(v)$ and $P_{F}(v)$ represent the home country prices of each varieties produced in home and foreign countries respectively, while $P_{H}^*(v)$ and $P_{F}^*(v)$ the foreign country prices of each varieties produced in home and foreign countries respectively. Since 100% pass-through of the exchange rate changes onto the goods’ prices is assumed, the law of one price holds for each individual variety so that $P(v) = S P^*(v)$, $\forall v \in [0,2]$, where $S$ is the spot exchange rate of home currency to foreign currency. For the sub-price indices of $P_{H}$, and $P_{F}$, consumption-based purchasing power parity holds as well so that $P_{H} = S P_{H}^*$, and $P_{F} = S P_{F}^*$. However, since home
and foreign households do not have an identical preference for home and foreign-produced goods, consumption-based purchasing parity for overall consumer price indices, \( P \neq S P^* \), does not hold.

**Goods Market Equilibrium**

Under the sub-demand equations [3] and [4], optimal intratemporal consumption choices for each differentiated goods are determined as follows.

\[
C_{H'}(v) = \frac{P_{H}(v)}{P_{H}} \left[ \frac{P_{H}(v)}{P_{H}} \right]^{-\theta} C_{H'}; \quad C_{F'}(v) = \frac{P_{F}(v)}{P_{F}} \left[ \frac{P_{F}(v)}{P_{F}} \right]^{-\theta} C_{F'} \quad [8]
\]

\[
C'_{H'}(v) = \frac{P'_{H}(v)}{P'_{H}} \left[ \frac{P'_{H}(v)}{P'_{H}} \right]^{-\theta} C'_{H'}; \quad C'_{F'}(v) = \frac{P'_{F}(v)}{P'_{F}} \left[ \frac{P'_{F}(v)}{P'_{F}} \right]^{-\theta} C'_{F'} \quad [9]
\]

where \( C_{H'}(v) \) and \( C_{F'}(v) \) show the demand for each home and foreign varieties of the home household respectively, while \( C'_{H'}(v) \) and \( C'_{F'}(v) \) the demand for each home and foreign varieties of the foreign household respectively.

The Cobb-Douglas type consumption indices imply that the demand for home and foreign goods, \( C_{H'} \), \( C_{F'} \), \( C'_{H'} \), and \( C'_{F'} \) are given by

\[
C_{H'} = \gamma \left( \frac{P_{H}}{P} \right)^{-1} C; \quad C_{F'} = (1-\gamma) \left( \frac{P_{F}}{P} \right)^{-1} C \quad [10]
\]

\[
C'_{H'} = (1-\gamma) \left( \frac{P'_{H}}{P'} \right)^{-1} C'; \quad C'_{F'} = \gamma \left( \frac{P'_{F}}{P'} \right)^{-1} C' \quad [11]
\]

Combining [8] and [10], and [9] and [11] gives respectively

\[
C_{H'}(v) = \gamma \left( \frac{P_{H}(v)}{P_{H}} \right)^{-\theta} \left( \frac{P_{H}(v)}{P} \right)^{-1} C; \quad C_{F'}(v) = (1-\gamma) \left( \frac{P_{F}(v)}{P_{F}} \right)^{-\theta} \left( \frac{P_{F}(v)}{P} \right)^{-1} C \quad [12]
\]
The world consumption for each variety produced in home and foreign countries is defined as follows.

\[ C^*_H(v) = (1 - \gamma) \left[ \left( \frac{P^*_H(v)}{P^*_H} - \gamma \frac{P^*_H}{P^*_H} \right) + \left( \gamma \frac{P^*_H}{P^*_H} \right) \right] C^* \hspace{1cm} C^*_F(v) = \gamma \left[ \left( \gamma \frac{P^*_F}{P^*_F} \right) - \left( \frac{P^*_F}{P^*_F} \right) \right] C^* \]  

[13]

where \( C^*_H(v) \) and \( C^*_F(v) \) represent total world consumption each variety produced in home and foreign countries respectively. Plugging [12] and [13] into [14] gives

\[ C^W_H(v) = \gamma \left[ \left( \frac{P^*_H(v)}{P^*_H} \right) - \left( \frac{P^*_H}{P^*_H} \right) \right] C + (1 - \gamma) \left[ \left( \gamma \frac{P^*_H}{P^*_H} \right) - \left( \frac{P^*_H}{P^*_H} \right) \right] C^* \]  

[15]

\[ C^W_F(v) = (1 - \gamma) \left[ \left( \frac{P^*_F(v)}{P^*_F} \right) - \left( \frac{P^*_F}{P^*_F} \right) \right] C + \gamma \left[ \left( \gamma \frac{P^*_F}{P^*_F} \right) - \left( \frac{P^*_F}{P^*_F} \right) \right] C^* \]  

[16]

The goods market for each variety produced in home and foreign countries clears when the demand equals the supply. Taking into account the population of two countries and evaluating it at the symmetric equilibrium, where, \( P^*_H(v) = P^*_H \) and \( P^*_F(v) = P^*_F \), we obtain the world market clearing condition for each variety produced in home and foreign countries as follows.

\[ C^W_H(v) = \left\{ \gamma \left( \frac{P^*_H}{P^*_H} \right) - \left( \frac{P^*_H}{P^*_H} \right) \right\} C + \left( 1 - \gamma \right) \left\{ \left( \gamma \frac{P^*_H}{P^*_H} \right) - \left( \frac{P^*_H}{P^*_H} \right) \right\} C^* = Y(v) \]  

[17]

\[ C^W_F(v) = \left\{ (1 - \gamma) \left( \frac{P^*_F}{P^*_F} \right) - \left( \frac{P^*_F}{P^*_F} \right) \right\} C + \left( \gamma \left( \gamma \frac{P^*_F}{P^*_F} \right) - \left( \frac{P^*_F}{P^*_F} \right) \right\} C^* = Y^*(v) \]  

[18]

**Asset Market Equilibrium**

Asset markets are assumed to be incomplete either because Arrow-Debreu type state-contingent futures contracts or real bonds ensuring one unit of composite consumption in the
next period are not available, or because the number of risky assets to span idiosyncratic output shocks is limited. Because including risky assets that are not enough to span idiosyncratic shocks into the model would only complicate the analysis without any benefit, we simply exclude them from the model.

Only two nominal bonds denominated in home and foreign currencies are available to the households across countries. Bonds denominated in home currency pay one unit of home currency in the next period, while foreign currency denominated bonds pay one unit of foreign currency. The current price of the futures contract in home currency is the discounted present value of one unit of home currency by the current domestic nominal interest rate, \( Q_t = 1/(1 + i_t) \). The current price of futures contract in foreign currency is the current spot exchange rate times the discounted present value of one unit of foreign currency by the current foreign nominal interest rate, \( S_t Q^*_t = S_t / (1 + i^*_t) \).

The market clearing conditions in the world asset markets for home and foreign currency denominated bonds are respectively,

\[
B_{H,t}^* + B_{H,t} = 0, \quad B_{F,t}^* + B_{F,t} = 0 \quad \forall t \tag{19}
\]

where \( B_{H,t} \) and \( B_{F,t} \) are home and foreign currency denominated bonds.

**The Budget Constraint**

Given intra-temporal consumption choices, the budget constraint of the representative household in the home country is as follows.

\[
P_t C_t + M_t + Q_t B_{H,t} + S_t Q^*_t B_{F,t} = M_{t-1} + P_{H,t}(v) Y_t(v) + B_{H,t-1} + S_t B_{F,t-1} + T_t \tag{20}
\]

where \( Q_t \) and \( Q^*_t \) are the prices of home and foreign currency denominated bonds. \( T_t \) is the
monetary transfer from the government to each citizen. Households in each country are assumed to hold only domestic currency, not foreign currency.

The government budget constraint is as follows. The change in domestic money supply is given directly to each household by the government as transfer payments. There are no government expenditures over time in the model.

\[
M_t = M_{t-1} + T_t \tag{21}
\]

**First Order Conditions for the Representative Households in Home and Foreign**

The problem of the representative households in each country is to choose rules for holding nominal money balances, \( M \), home-currency denominated bonds, \( B_{hi} \), and foreign-currency denominated bonds, \( B_{fi} \), by maximizing his lifetime expected utility [1] given his lifetime budget constraint that are constructed as the present discounted sum of the combination of the equations [20] and [21]. The initial values of \( M \), \( B_{hi} \), and \( B_{fi} \) are given.

First order conditions for the representative home household are as follows.

\[
Q_i = \beta \frac{E_i(P_{t+1}^{-1}C_{t+1}^-)}{P_t^-C_t^-} \tag{22}
\]

\[
Q_{i}^* = \beta \frac{E_i(S_{t+1}^{-1}P_{t+1}^{-1}C_{t+1}^-)}{S_t^-P_t^-C_t^-} \tag{23}
\]

\[
\left( \frac{M_i}{P_t} \right)^{\epsilon} = \frac{XC_t^\rho}{1 - Q_i} \tag{24}
\]

First order conditions for the representative foreign household are as follows.

\[
Q_i = \beta \frac{E_i(S_{t+1}^{-1}P_{t+1}^{-1}C_{t+1}^-)}{S_t^-P_t^-C_t^-} \tag{25}
\]

\[
Q_{i}^* = \beta \frac{E_i(P_{t+1}^{-1}C_{t+1}^-)}{P_t^-C_t^-} \tag{26}
\]
\[ \left( \frac{M^p}{P^*} \right)^\varepsilon = \frac{XC^p}{1 - Q} \]  

**Short Run Inflexible Prices in Goods Markets**

At time \( t - 1 \), the representative consumers along with producers in home and foreign countries determine their monopoly prices, \( P_{H,t}(v) \) and \( P_{F,t}^*(v) \) one period ahead by maximizing their expected lifetime utility [1] given their expected life-time budget constraint at the symmetric equilibrium where \( P_{H,t}(v) = P_{H,t} \) and \( P_{F,t}^*(v) = P_{F,t} \), based on the information available at time \( t - 1 \). Their predetermined prices for time \( t \) would not change unless their participation constraints that guarantee them at least zero profits are violated.

\[
P_{H,t}(v) = P_{H,t} = \left( \frac{\theta}{\theta - 1} \right) \frac{E[\eta(Y_t(v))]}{E[P_tC^p]} \tag{28}
\]

\[
P_{F,t}^*(v) = P_{F,t}^* = \left( \frac{\theta}{\theta - 1} \right) \frac{E[\eta^*(Y_t^*(v))]}{E[P_t^*C^{p*}]} \tag{29}
\]

**3. Real Exchange Rates and ex post Country-specific Output and Money Shocks**

Suppose that the logs of output and money supply in both home and foreign countries follow a random walk.

\[
\log \eta = \log \eta_{t-1} + \xi_t; \quad \log \eta^* = \log \eta^*_{t-1} + \xi^*_t \tag{30}
\]

\[
\log M_t = \log M_{t-1} + \mu_t; \quad \log M^*_t = \log M^*_{t-1} + \mu^*_t \tag{31}
\]

where \( \xi_t \sim N(0, \sigma^2_{\xi}) \), and \( \mu_t \sim N(0, \sigma^2_{\mu}) \) for every time \( t \) are defined as an adverse output shock and a money shock. Distributions from which output and monetary shocks are redrawn are assumed to be time-invariant and lognormal.
Log-linearizing the money demand equations of the home and foreign countries, \([24]\) and \([27]\) at a non-stochastic steady state where \(Q = Q' = \overline{Q}\) (or, equivalently \(i = i' = \overline{i}\)) gives

\[
\varepsilon \{m_i - p_i\} = \log \chi - \log(1 - Q_i) + \rho \varepsilon_i
\]  
\[32\]

\[
\varepsilon \{m_i' - p_i'\} = \log \chi' - \log(1 - Q_i') + \rho \varepsilon_i'
\]  
\[33\]

Adding two equations \([32]\) and \([33]\) under the assumption that at the initial equilibrium,

\[
\log M_{t-1} = \log M'_{t-1}, \quad \chi = \chi', \quad \text{and} \quad Q_i = Q_i'
\]
gives the following expression.

\[
\rho \{\varepsilon_i + \varepsilon_i'\} = \varepsilon\{\mu_i + \mu_i'\} - \varepsilon\{p_i + p_i'\}
\]
\[34\]

where

\[
\rho(\varepsilon_i - \varepsilon_i') = (2\gamma - 1)(\varepsilon_i + p_{F,i} - p_{H,i})
\]
\[35\]

\[
p_i = \gamma p_{H,i} + (1 - \gamma)s_i + (1 - \gamma)p_{F,i}
\]
\[36\]

\[
p_i' = (1 - \gamma)p_{H,i} - (1 - \gamma)s_i + \gamma p_{F,i}
\]
\[37\]

Plugging \([36]\) and \([37]\) into \([34]\) and \([35]\), and solving for \(\varepsilon_i\) and \(\varepsilon_i'\) gives

\[
\varepsilon_i = \frac{(2\gamma - 1)}{2\rho} \{\varepsilon_i - (p_{H,i} - p_{F,i})\} + \left(\frac{\varepsilon}{2\rho}\right)\{\mu_i + \mu_i'\} - \left(\frac{\varepsilon}{2\rho}\right)\{p_{H,i} + p_{F,i}\}
\]
\[38\]

\[
\varepsilon_i' = -\frac{(2\gamma - 1)}{2\rho} \{\varepsilon_i - (p_{H,i} - p_{F,i})\} + \left(\frac{\varepsilon}{2\rho}\right)\{\mu_i + \mu_i'\} - \left(\frac{\varepsilon}{2\rho}\right)\{p_{H,i} + p_{F,i}\}
\]
\[39\]

Subtracting the equation \([39]\) from \([38]\) under the assumption that at the initial equilibrium,

\[
\log M_{t-1} = \log M'_{t-1} = 0, \quad \chi = \chi', \quad \text{and} \quad Q_i = Q_i'
\]
gives the following expression.

\[
\rho \{\varepsilon_i - \varepsilon_i'\} = \varepsilon\{\mu_i - \mu_i'\} - \varepsilon\{p_i - p_i'\}
\]
\[40\]

Plugging \([38]\) and \([39]\) into \([40]\) gives the following expression of the ex post terms of trade as a function of ex post relative money shocks and the current period prices of home and foreign goods.
Taking a log of \([28]\) and \([29]\) in the current period gives

\[
p_{W,F} = \log \left( \frac{\theta}{\theta - 1} \right) + E \log \eta_i + E p_i + \rho E \epsilon_i \quad [42]
\]

\[
p_{W,F}^* = \log \left( \frac{\theta}{\theta - 1} \right) + E \log \eta_i^* + E p_i^* + \rho E \epsilon_i^* \quad [43]
\]

Subtracting \([43]\) from \([42]\) under the assumption that \(\log \eta_{i-1} = \log \eta_{i-1}^*\), and combining it with the equations \([35]\), \([36]\) and \([37]\) gives the ex ante terms of trade.

\[
p_{W,F} - p_{W,F}^* - E \xi_i = (\xi_i - \xi_i^*) \quad [44]
\]

where \(E \xi_i = -\left( \frac{(e - 1)(2\gamma - 1)}{(2\gamma - 1) + 2e(1 - \gamma)} \right) (p_{W,F} - p_{W,F}^*) \quad [45]\)

The expected exchange rate in \([45]\) is derived by taking the expectation of the equation \([41]\) under the assumption that future period monetary surprises are not expected by the households.

Combining \([44]\) and \([45]\) gives

\[
p_{W,F} - p_{W,F}^* = \left( \frac{(2\gamma - 1) + 2e(1 - \gamma)}{e} \right) (\xi_i - \xi_i^*) \quad [46]
\]

Plugging \([46]\) into \([41]\) gives the ex post terms of trade as a function of ex post relative money supply and relative output shocks.

\[
s_t + p_{W,F}^* - p_{W,F} = \left( \frac{e}{(2\gamma - 1) + 2e(1 - \gamma)} \right) (\mu_i - \mu_i^*) - (\xi_i - \xi_i^*) \quad [47]
\]
The real exchange rate is defined as the relative cost of living across countries. Combining it with the home and foreign price indices gives

\[ \frac{S_t P_t^*}{P_t} = \left( \frac{S_t P_{t,F}^*}{P_{t,H}} \right)^{\frac{\gamma-1}{\sigma}} \]  \[48\]

Log-linearizing it gives

\[ s_t + p_t^* - p_t = (2\gamma - 1)(s_t + p_{t,F}^* - p_{t,H}) \]  \[49\]

Plugging [47] into [49] gives the following expression for the real exchange rate as a function of ex post relative money and output shocks.

\[ s_t + p_t^* - p_t = (2\gamma - 1) \left( \frac{\varepsilon}{(2\gamma - 1) + 2\varepsilon(1 - \gamma)} \right) \left( \mu_t - \mu_t^* \right) - \left( \xi_t - \xi_t^* \right) \]  \[50\]

From [50], note that real exchange rate changes can be caused by the change in the degree of trade openness given ex post relative money and output shocks. When \( \gamma > \frac{1}{2} \), the adverse home output shock appreciates the real exchange rate while a monetary expansion depreciates it. As \( \gamma \) goes to \( \frac{1}{2} \), ex post money and output shocks would affect the real exchange rate by less. This result is consistent with Hau [2000, 2002]'s who showed that the more open the economies are, the less are the fluctuations of real exchange rates with respect to relative output shocks. When \( \gamma = \frac{1}{2} \), ex post output and money shocks would not have an influence on the real exchange rate at all.

4. Trade Balance as a function of Real Exchange Rates

From our model economy, trade balance can be expressed as follows.
Trade Balance = Exports - Imports = $S_y P_{iH}^* C_{iH}^* - P_{iF} C_{iF}$ \[51\]

From the demand equations for representative home and foreign products \[12\] and \[13\], exports and imports can be expressed as follows.

$$Exports = S_y P_{iH}^* C_{iH}^* = (1 - \gamma) S_y P_{iF}^* C_{iF}^*, \quad Imports = P_{iF} C_{iF} = (1 - \gamma) P_{iF} C_{iF}$$

Home country exports \((1 - \gamma)\) portion of foreign demand to foreign country and imports \((1 - \gamma)\) portion of home demand from foreign country.

Substituting these expressions into \[51\] gives

$$Trade Balance = S_y P_{iH}^* C_{iH}^* - P_{iF} C_{iF} = (1 - \gamma) [S_y P_{iF}^* C_{iF}^* - P_{iF} C_{iF}]$$ \[52\]

Trade balance equals \((1 - \gamma)\) portion of the difference between home and foreign demand.

Using the demand functions for representative home and foreign products, \(C_{iH}^*\) and \(C_{iF}^*\), evaluating them at the initial symmetric equilibrium where \(P_{iH}(\nu) = P_{iH}^*\) and \(P_{iF}(\nu) = P_{iF}^*\), and combining the consumption risk sharing condition yields the following expression for trade balance as a fraction of GDP.

$$\frac{Trade Balance}{GDP} = 1 - \frac{1}{\gamma + (1 - \gamma) \left( \frac{SP^*}{P} \right)^{\frac{\rho - 1}{\rho}}}$$ \[53\]

where \(P = (P_{iH})^\gamma (SP^*)^{1-\gamma} = Home\ Consumption\ Price\ Index\)

\(P^* = (P_{iF} / S)^{1-\gamma} (P_{iF}^*)^\gamma = Foreign\ Consumption\ Price\ Index\)

Trade balance as a fraction of GDP in the equation \[53\] is a function of the real exchange rate. If the degree of relative risk aversion of households is greater than unity, \(\rho > 1\), the increase in
the relative cost of living of home households worsens trade balance to smooth consumption over time. The greater the degree of relative risk aversion, the more do households try to smooth consumption. As a result, the fluctuations in trade balance also become greater. If households have a log utility function, that is, \( \rho = 1 \), households do not care about the changes in their cost of living and consumption. In this case, trade would be always balanced because households would not like to trade goods, services, and financial assets for their intertemporal consumption risk sharing.

Combining [53] with [48] gives the following expression for trade balance as a function of the terms of trade.\(^4\) The increase in the price of exports worsens the terms of trade while the decrease in the price of exports improves it.

\[
\frac{\text{Trade Balance}}{\text{GDP}} = 1 - \left[ \gamma + (1 - \gamma) \left( \frac{SP^*_F}{P_H} \right)^{(2\gamma - 1) \left( \frac{\rho - 1}{\rho} \right)} \right]^{\gamma}
\]

where \( \frac{SP^*_F}{P_H} = \text{The Terms of Trade} \)

In our model, we assume that agents’ preferences across countries are identically asymmetric for computational convenience, that is, the share of imports in the consumption basket is the same across countries. Therefore, when the share of imports in the consumption basket is a half (\( \gamma = \frac{1}{2} \)), consumption baskets across countries become identical. When agents across countries have identical preferences, the change in the terms of trade does not have an influence on the real exchange rate and the trade balance. According to equation [40], trade balance will be zero in the case of \( \gamma = \frac{1}{2} \) or \( \rho = 1 \). If \( \rho > 1 \) and \( \gamma > \frac{1}{2} \), the worsening of the terms of trade

\(^4\) The terms of trade is defined as the relative price of imports to exports.
appreciates the real exchange rate and worsens trade balance.


7. Concluding Remarks

In our paper, we construct an open economy macroeconomic model that shows that the changes in the relative prices of home-produced goods resulted from country-specific shocks influence relative consumption and trade balance through the changes in real exchange rate in the setting of intertemporal utility maximization. If the consumption baskets across countries are home-biased, an increase in the relative prices of home produced goods would raise home country’s cost of living more than foreign country’s, and then reduce home relative consumption. Intertemporal utility maximizing agents with the degree of relative risk aversion greater than unity would like to smooth their relative consumption over time. By reducing exports and increasing imports, home agents are able to increase their consumption, which leads to trade deficit in the current period. The magnitude of real exchange rate fluctuations, the degree of consumption synchronization and the magnitude of trade balance deviations critically depend on the parameters of households’ CES utility function such as the degree of relative risk aversion and the share of home-produced goods in the households’ consumption baskets.

References


Appendix 1: Consumption Risk Sharing Condition
From the first order conditions for bond holdings, equating [22] and [25], and [23] and [26] respectively gives the following two equations.

\[ Q_t = \beta \frac{E_t(p_t^{-1}C_t^{-\rho})}{p_t^{-1}C_t^{-\rho}} = \beta \frac{E_t(S_t^{-1}p_t^{-1}C_t^{-\rho})}{S_t^{-1}p_t^{-1}C_t^{-\rho}} \]  \[ \text{[A1]} \]

\[ Q_t^* = \beta \frac{E_t(S_t^{-1}p_t^{-1}C_t^{-\rho})}{S_t^{-1}p_t^{-1}C_t^{-\rho}} = \beta \frac{E_t(p_t^{-1}C_t^{-\rho})}{p_t^{-1}C_t^{-\rho}} \]  \[ \text{[A2]} \]

Equating [A1] and [A2] under the assumption that both countries are initially in the symmetric equilibrium where \( p_t^{-1}C_t^{-\rho} = (p_t')^{-1}(C_t')^{-\rho} \), gives the following expression.

\[ \Rightarrow \left( \frac{C_t}{C_t'} \right)^{\rho} = \frac{S_t p_t^*}{p_t'} \quad \forall t \]  \[ \text{[A3]} \]

The equation [A3] says that the presence of asset markets for nominal one-year bonds in different currency denomination ensures that the ratio of the marginal utility of consumption in home and foreign countries equals the real exchange rate. It implies that relative consumption between countries varies with respect to the changes in the real exchange rate. This condition is called ‘consumption risk sharing condition.’

Substituting the equation [48] into the equation [A3] transforms the consumption risk sharing condition [A3] into the following expression.

\[ \frac{C_t}{C_t'} = \left( \frac{S_t p_t^*}{p_t'} \right)^{\frac{1}{1+\gamma}} \quad \forall t \]  \[ \text{[A4]} \]

From the equation [A4], we see that relative consumption relies not only on the change in the terms of trade but also on the parameters of the CRRA utility function of the share of the home-produced goods in the consumption basket, and the degree of relative risk aversion.