Intra-Sector Mobility and Specific Inputs in Tax-Incidence Theory

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Abstract

In a simple three-factor-two-final-good formulation (two factors immobile and sector-specific), a well-known result under competitive and full-employment assumptions is that a partial tax on the mobile factor in either industry hurts that factor everywhere. It can be reversed, however, when the taxed activity uses a sector-specific input produced in the other sector. The model becomes asymmetrical: the same tax often yields different results, depending on where it is levied and the nature and cross-sector linkages of various inputs. Their respective roles in determining tax-incidence are discussed in a series of plausible settings, each 3 x 2, involving primary and produced inputs and intra-sector mobility of some sector-specific factors. Cross-sector linkages of produced inputs, more than any other element, drive the new results which are often similar to those in models with all mobile factors.

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1. Introduction

Factors of production which are specific to a particular locality or region in a given economy are featured prominently in analyses of local public finance issues. Some production inputs have this attribute because of their intrinsic nature. For example, a coal mine cannot be physically relocated to another region. Workers, on the other hand, may not be able to freely move from one locality to another due to licensing requirements, residency restrictions, or linguistic barriers. Some may not wish to move because of a strong preference for a particular province or region. ¹ To use a Canadian example, comparing Ontario and Quebec, two neighbouring provinces, the licenses of many trades (electricians, plumbers) issued in Ontario are not recognized in Quebec; Ontario workers often are required to set up residence in Quebec for some time before they can openly work there; and they must be able to function in French (a majority of Ontario residents are English speakers). Ontario sometimes retaliates in kind, but restrictions are often asymmetrical. There are no such restrictions on capital, however, and it is freely mobile between the two jurisdictions. Analytically, this setting is well represented by the famous 3 x 2 model (Jones 1971), or McLure’s 1971 adaptation of the celebrated Harberger tax model - one mobile factor and two sector-specific or imperfectly mobile factors of production - which is the source of the well-known

¹Whatever the causes, all such scenarios have the same analytical implications in the present context; therefore, the terms “sector-specific,” “immobile,” and “fixed” factors will be used interchangeably. Without loss of generality, it will be assumed that labor, L, is the only mobile primary factor of production and capital, K, is the corresponding sector-specific factor.
tax incidence result that under competitive and full-employment assumptions, if production is carried on with the help of mobile and sector-specific inputs, a tax on the former in either locality will hurt that factor throughout the economy.

In most modern economies, the relationship between two neighbouring provinces is more complex than what the above example or the 3 x 2 model depicts. Continuing with the Quebec-Ontario case, the former has a long-established manufacturing facility for small aircraft which uses parts made in Ontario - a specific input produced in the other locality - and there are avenues for adding values within each province (hence intra-sectoral mobility). Now, small aircraft is internationally traded and Ontario sells aircraft parts all over the world as well. Quebec, thus, may not be able to employ (sector-specific) workers from Ontario directly, but what they produce does move across Canada and abroad. Turning to another industry, Ontario has several automobile factories which use parts made in Quebec. Under NAFTA, which covers Canada, U.S.A., and Mexico, these parts can be purchased from abroad as well. Forces of globalization, thus, are playing an important part in regional development, accentuating and reinforcing linkages within and across national frontiers. Such relatively recent developments must be incorporated into the analytical framework before the incidence of various taxes can be properly examined.

At this stage, it might be helpful to set up a series of inter-linked examples to illustrate the various theoretical constructs which will be deployed later in the paper. Let there be two sectors called “manufacturing” and “agriculture” for easy reference. If labor, the mobile factor, is employed to grow food on fertile farmland and also to manufacture rubber products (using a synthetic raw material) in the other sector, we get the original 3 x 2 model. A wage tax, say, in
manufacturing, then will hurt all workers and benefit owners of farmland. In a “back to nature” move, however, if farmers plant rubber trees and grow organic crops, while manufacturers switch to latex as a raw material, a distinct possibility arises that workers can actually benefit from this wage tax, and the model behaves like a mobile-factor formulation in some respects in spite of sector-specific inputs (Bhatia 2001). Each specification still has a 3 x 2 dimension, also two sector-specific inputs and a mobile factor; in fact, the only discernible change is that one specific input (latex) has replaced another (synthetic raw material), but there is more than meets the eye because several new elements -- a cross-sector production linkage, a value-adding process (gathering and transportation of latex), and intra-sector mobility of a specific factor (farm land) -- come into the analysis.

From the standpoint of taxation theory, intra-sector mobility is arguably the most radical change because, strictly speaking, it is no longer “a model with one immobile factor,” but there are still two sector-specific inputs, which prompts one to ask: Is this what dislodges the text-book result? Do the two final goods still play symmetrical roles? What makes the model behave like a mobile-factor formulation? Such questions are not an idle theoretical curiosity, for stylized examples of the modified framework, often embedded in more complex settings, can be found in many input-output (i-o) tables. For instance, denoting the final goods by Xi (i = 1, 2), the produced specific input (p. s. i.) by X3, and the mobile factor, labor, by L, one may come across coal or oil (X3) produced in the natural-resource sector firing up factory furnaces elsewhere; natural diamonds being used for jewellery, a final good, as well as in industrial production; and, switching sectors, the industrial heartland producing synthetic clothing (X1), and fertilizer (X3) for agriculture where food crops (X2) are grown. There is also a potential policy angle inasmuch as
policy prescriptions based on the text-book model can be quite misleading. That model is commonly identified with “short-run” or first-round effects of a tax change, a la Marshall, which can be very different from the “long run” outcomes generated by the p. s. i. specifications.

The analytical framework draws on a series of alternate specifications involving p.s.i.'s and production linkages. These are set out in Section 2, along with the model assumptions and key equations. Section 3 deals with the solution process, and the “latex” example is discussed in some detail to set out the notation and derive the main results. A different group of production structures is considered in Section 4 where comparisons with mobile-factor models are also made. The tax-incidence literature suggests that partial factor taxes tend to be the most complicated; therefore, to limit the length of this piece, a tax on the mobile input in X₁ alone will be considered in detail. One or two other taxes will be discussed briefly, and the conclusions are summarized in Section 5.

2. Alternative Formulations, Assumptions, and Key Equations

Drawing upon the examples outlined in the Introduction, the three p.s.i. formulations about to be discussed may be referred to as "latex", "single-corn," and "diamond" models. Since the focus of the analysis is on the structure of production, whatever else these labels might connote -- technologies, financial arrangements, different products -- would be of little interest. An intermediate input is involved in all of them, so an appropriately contrasting label for the text-book 3 x 2 set-up may be fgo (final-goods-only). In all cases, there are two final goods, X₁ and X₂, two primary factors -- labor (L) characterized by unrestricted mobility, and K which is sector-specific -- and the p. s. i. specifications, of course, involve a produced input. In the fgo framework, K is replaced by K₁ and K₂, both immobile and fixed in supply.
To formalize the "latex" model, it is assumed that the economy’s only primary specific factor, \( K \), is used in agriculture to produce \( X_2 \) (food) and \( X_3 \) (latex, a p.s.i. needed for manufacturing rubber products, \( X_1 \)). The production functions, then, can be stated as \( X_1 = f_1(L_1, X_3) \) in manufacturing, and \( X_i = f_i(L_i, K_i) \), \( (i = 2, 3) \) in agriculture. If \( a_{ij} \) denotes the amount of the \( i^{th} \) input per unit of \( X_j \), the full employment (F-E) condition for labor can be written as \( a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 = \), and the zero excess demand for \( K \) will be characterized by \( a_{K2}X_2 + a_{K3}X_3 = \). The bars over \( L \) and \( K \) indicate their exogenous endowments.

Formally, all that this formulation does is replace \( K_1 \) by a p.s.i., \( X_3 \), in McLure's production function for \( X_1 \), but \( K \) is not immobile as in the fgo model; it can be reallocated within agriculture, although not directly between \( X_2 \) and \( X_1 \). What is produced with the direct help of \( K \) nonetheless is used only in manufacturing, so \( X_3 \) is a sector-specific input, albeit with a value-added component, and \( X_3 \) does physically move from agriculture to manufacturing. This leads to the key question: Is it the pair of mobility assumptions (\( K \) within agriculture, \( X_3 \) across sectors), rather than the value-adding process or the production linkage, that is the driving force behind the new tax-incidence outcomes?

One way of answering this question is to rule out any role for the mobility of \( K \) by supposing, for example, that there is only one agricultural good, "single corn" for eating as well as further processing in the manufacturing sector. A portion of the agricultural output (\( X_{21} \)) then becomes an intermediate input in \( X_1 \), a one-way i-o setting with two goods instead of three. The production function for \( X_1 \), accordingly, becomes \( f_1(L_1, X_{21}) \), and the \( X_3 \) terms will be dropped from the F-E conditions for \( L \) and \( K \). There is still a production linkage between manufacturing and agriculture, although the only primary factor that physically moves, within or across sectors, is labor. In this
regard, the production structure is simpler than the "latex model," and it will transform itself into the standard fgo formulation if the linkage between the two sectors is ousted and supplanted by a primary factor specific to X₁.

To abstract from intra-sector mobility of K yet again, one may think of a river bed full of diamonds in the rough which can be gathered to serve the needs of final-goods producers everywhere. The production function for gathered diamonds (D) can be written as $D = f(K, L_D)$, and for a final good it will be $X_i = f_i(L_i, D_i), (i = 1,2)$. Equations of this type can depict a wide range of intermediate goods, such as specialized software developers, or an industry producing chips, hard drives, and other paraphernalia, or a separate tertiary sector providing managerial, accounting, and advertising services. Any pure intermediate good, i.e. one that does not have a final demand, can fit this mould if it is produced in a sector all its own. So far as physical mobility goes, the model will have one mobile and one immobile primary factor, the latter getting transformed into a produced input for the two final goods.

A partial factor tax in $X_1$ is the focus of this analysis; therefore, in terms of the analytics of the models, all of the above examples place p. s. i. production in the non-taxed sector, and the taxed activity uses this input. Other formulations nonetheless can be equally plausible: For instance, a taxed sector (corporations) may supply a p. s. i. (tractors) for the other, or each activity may be self-contained so that manufacturing produces the "semi-finished" goods it needs while agriculture uses its own organic compost rather than synthetic fertilizer, and so on. Some of these will be taken up in Section 4 below.

3. The Model Solutions
Besides the assumptions set out above, in keeping with the corresponding tax literature, it is also assumed that the factors of production are owned by consumers whose optimizing decisions (based on identical, homothetic preferences) generate demands for the final goods, and tax revenues are returned to them in a lump-sum fashion. Starting with a no-tax initial equilibrium, for (small) tax levies, the models are solved for changes in the rental-wage ratio which, in turn, determine factor incomes (because of the full-employment assumption) and thus the incidence of a given tax.

The solution process follows Jones (1965) in totally differentiating the F-E conditions for the primary factors and the production functions, invoking competitive results (zero profits, factor rewards equal to marginal value products, etc.), and setting the expressions for proportional changes in the ratio of final outputs to the corresponding changes in demand. The algebra and the presentation will be simplified by setting initial prices to unity and letting asterisks denote proportional changes everywhere. The demand side of the model can be summarized by \((X^* - X) = \sigma_D (p^*_2 - p^*_1)\), as in Atkinson and Stiglitz (1980, Lecture 6), where \(\sigma_D\), defined to be positive, is the elasticity of substitution in demand, and \(p_i\) is the unit price of \(X_i\).

These types of models typically are underdetermined, with one degree of freedom, so the net-of-tax wage rate, \(w\), is chosen as the numeraire. The goal is to solve for \(r^*\), the proportional change in the rental-wage ratio after a "small" wage tax is levied in \(X_1\). Denoting the tax rate by \(t\), \(w(1 + t)\) or \(wt_{L1}\) is the tax-inclusive cost of employing a unit of labor in the taxed industry. In all cases, the production functions are assumed to be linear homogeneous, so \(a_{ij}\) is homogeneous of degree zero in factor prices, and \(a^*_{ij}\) can be related to elasticities of substitution (\(\sigma\)) and factor
shares ($\rho_{ij}$). For instance, $\rho_{L2} = wL_2 / X_2$, and in the "latex" model\(^2\), $aL_1 = \rho_{31}pK3r^*\sigma^1$. Again, $\lambda_{ij}$ can be defined as the proportion of input $j$ ($j = L, K$) used by the $i^{th}$ activity. Thus, $\lambda_{L2} = aL_2X_2 / \sigma^1$, and the individual $\lambda$'s can be arranged in an activity matrix whose determinant, $|\lambda|$, will be a summary indicator of relative factor intensities.

The detailed steps involved in solving the "latex" model, along with some numerical illustrations, have been set out in Bhatia (2001), so we shall focus on the key equations and the $r^*$-solutions. The other formulations also follow the same procedures more or less; therefore, highlighting their important features and the differences among the various $r^*$-expressions would suffice.

### 3.1 The "latex" model

The manufacturing sector uses labor directly (based on $a_{L1}$) and some indirectly, through $X_3$. Its total labor requirement per unit of output, then, becomes $R_L = a_{L1} + a_{31}a_{L3}$. Correspondingly, labor will have a direct factor share, $\rho$ (computed as $wL_1/X_1$ or $wa_{L1}$), and a "total" share, $\theta$ (given by $wR_L$). The two components of each pair, $(a_{Li}, R_{Li})$ and $(\rho_{Li}, \theta_{Li})$, will diverge for any activity that uses a produced input. The F-E conditions can be restated as $R_LX_1 + a_{L2}X_2 = 0$ for labor, and as $a_{K3}a_{31}X_1 + a_{K2}X_2 = 0$ for the other primary factor. Since both final goods are now using $L$ and $K$...

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\(^2\)In this case, $\sigma^1$ is defined as $(a_{L1} - a_{31})/(p_{*3} - w^*)$. The zero-profit condition for $X_3$ states that $\rho_{L3}w^* + \rho_{K3}r^* = p_{*3}$. Competitive firms minimize average cost, and the condition for that is $\rho_{L1}a_{L1}^* + \rho_{31}a_{31}^* = 0$. The solution for $a_{L1}^*$ can be derived by setting $w^* = 0$ (because $w$ is the numeraire), substituting for $p_{*3}$ into the definition of $\sigma^1$, and utilizing the minimum-average-cost condition. See Jones (1965) for more details. A similar procedure is followed for other $a_{ij}^*$'s, and after the tax is levied, $w^*$ is replaced by $w^* + t^*_{L1}$ in the expression for $\sigma^1$. 
K, directly or indirectly, their K/L ratios can be compared, and if X₁ is relatively labor intensive \( (L_1/K_1 > L_2/K_2) \), \( |\lambda| > 0 \). So far as the analytics of the model are concerned, this change is most remarkable because factor intensities of the two final goods simply cannot be compared in the standard 3 x 2 model.

As mentioned earlier, the first part of the solution process is to determine \((X^\dagger - X^\ddagger)\) on the supply side, and that is done by totally differentiating the F-E conditions for the primary factors, plugging in the expressions for \(a_{ij}^*\) and \(R^*L_1\) (derived by the procedures outlined in footnote 2), and solving the resulting equations for \(X^\dagger\) and \(X^\ddagger\). In the vicinity of the initial equilibrium, for "small" changes,

\[ |\lambda|(X^\dagger - X^\ddagger) = \left[ \rho K_3(M_1 - S_1)\sigma^1 + (M_2 - S_2)\sigma^2 + (M_3 - S_3)\sigma^3 \right] r^* \tag{1} \]

where \(M_i's\) and \(S_i's\) are sums or products of \(\lambda's\), \(\rho's\), and \(\theta's\) (all positive fractions). A local stability condition described by Atkinson and Stiglitz (1980) stipulates that equation (1) will be negative³.

In the post-tax situation, a term, \((S_1 - M_1)\sigma^1 t^*L_1\) will be added to the right-hand side of equation (1).

The demand side, after substituting for \(p^*_i\) and setting \(w^* = 0\) (because \(w\) is the numeraire), can be summarized as:

\[ (X^\dagger - X^\ddagger) = \sigma_D[(\theta K_2 - \theta K_1)r^* - \rho L_1 t^*L_1] \tag{2} \]

³A fall in the wage-rental ratio \((r^* > 0)\) should lead to a decline in the relative price of the labor-intensive good as well as its output. The "supply curve" of \(X_1/ X_2\), in other words, is upward sloping. For instance, if \(X_1\) is relatively labor intensive, \(|\lambda| > 0\), so \((X_1^* - X_2^*)\) must be negative.
And equations (1) and (2) yield:

\[ r^* = \left[ \rho_{L1}|\lambda| \sigma_D + (S_1 - M_1)\sigma^1 \right] t_{L1}/D_1 \] (3)

The sign of \( r^* \) determines the direction in which the rental-wage ratio moves when the tax on \( L_1 \) is imposed. Now, \( \rho_{L1} \) is the factor share of \( L_1 \), the labor directly employed in \( X_1 \), whose earnings are the base for this tax; there is a role for \( \sigma_D \) because the tax will affect relative output prices (demand or output effect); and \( \sigma^1 \) comes in because cost-minimizing firms would substitute away from the taxed factor (input-substitution effect). Both \( (S_1 - M_1) \) and \( D_1 \) turn out to be positive\(^4\), and we get Result 1:

\textit{Result 1.: If the taxed industry is relatively labor intensive, labor throughout the economy will suffer from a partial tax upon itself. It can benefit from that tax if the taxed industry is relatively K-intensive.}

In equation (3), \( \sigma_D \) has been defined to be positive; therefore, if \( |\lambda| > 0, r^* > 0, \) and \( |\lambda| < 0 \) will be a necessary condition for \( r^* \) to be negative, i.e., for the wage-rental ratio to rise, but \( r^* \) can be positive even if the taxed industry is relatively K-intensive \((|\lambda| < 0)\), and a sufficient condition for such an outcome will be \( \sigma^1 > \sigma_D \rho_{L1}|\lambda| / (S_1 - M_1) \). In a nutshell, the tax leads to a higher unit cost of production in \( X_1 \), so its output is lowered, which brings in its wake a reduced demand for both labor and \( X_3 \). Moreover, firms producing \( X_1 \) will tend to substitute \( X_3 \) for labor, and its primary consequence would be to exacerbate labor's position. All things considered, based on total factor usage, if \( X_1 \) is relatively labor intensive, there will be an excess supply of labor, and the

\(^4\)For details of the proof, see Bhatia (2001), Section 3.1. Briefly, it invokes the Atkinson-Stiglitz local stability condition mentioned above in connection with equation (1), and it is also assumed that \( X_1 \) and \( X_2 \) have the same rank whether the K/L ratio or factor shares (\( \rho \)'s and \( \theta \)'s) are compared.
rental-wage ratio will rise. An exception can occur only if $X_1$ is relatively $K$-intensive.

Incidentally, the above result will hold even when a partial tax on labor ($t_{L2}$) is levied

$$t^* = \left[ -\rho L_2^2|\lambda|\sigma_D + (S_2 - M_2)^2 \right] t_{L2} / D_1.$$  The term $(S_2 - M_2)$ is positive, analogous to $(S_1 - M_1)$ in equation (3), the wage-rental ratio therefore can go up or down, and labor may benefit from this tax (for that, $|\lambda| > 0$ and $\sigma^2 = 0$ will be a sufficient condition). Therefore, whether the tax is imposed on $X_1$ or $X_2$ does not matter, as in the text-book $3 \times 2$ model; its symmetry property, although not its key result, is thus preserved in the “latex” specification.

Tax-incidence outcomes like Result 1 are typical of $2 \times 2$ models that feature full mobility for all factors. These would be regarded as “long run” models in the Harberger tax literature (one example is Rosen et al. (1999, pp.443-444)), and an expression very similar to (3) - a factor-intensity term and another involving the elasticity of substitution in the taxed industry appearing in the numerator - can be found in Mieszkowski (1967) for a mobile-factors-only model. The really interesting aspect of Result 1, however, is the possibility that labor might benefit from this tax, which cannot happen in the $3 \times 2$ fgo model. As noted earlier, the two new elements contributing to this possibility are the intra-sector mobility of $K$ between $X_2$ and $X_3$, and the p. s. i. with a cross-sector production linkage. They do not simultaneously appear in any of the other formulations presented above in Section 2 which, therefore, would help in disentangling their respective contributions.

3.2 The "single corn" formulation
This specification is useful in the present context because it does away with the effect of K-mobility within agriculture, and the goal is to see if the original, McLure model result is restored. The production activities can still be ranked in terms of factor intensities, but unlike the “latex” model, one good does double duty and satisfies both intermediate and final demand. Even though there are now two goods instead of three, the solution for $r^*$ does get a little more complicated. For starts, only a portion of $X_2 (x_2)$ now meets final demand; therefore, the F-E conditions, the factor shares, and $\lambda_{ij}$'s have to be restated in terms of $x_2$, and some of the $a_{ij}^*$ terms (notably, $a_{L2}^*$ and $a_{K2}^*$) become more complicated. The F-E condition for labor is

$$\lambda_{L1}X_1^* + \lambda_{L2}x_2^* = -\lambda_{L1}R_{L1} - \lambda_{L2}a_{L2}^*,$$

and $\lambda_{K1}X_1^* + \lambda_{K2}x_2^* = -(a_{K2}^* + \lambda_{K1}a_{L2}^*)$ for $K$. The solution process by and large follows the steps outlined above for the "latex" model, and the $r^*$-expression is:

$$r^* = [\rho_{L1}\lambda_{L1}\sigma_D + A_1 \sigma_1^t]t_{L1}/D_2$$

where $A_1 = \rho_{L1}[\lambda_{K1}(\lambda_{L1} - \lambda_{L2}) + \lambda_{L2}\theta_{K1}(\lambda_{K1} - \lambda_{K2})/\theta_{L1}]$. The denominator, $D_2$, is positive, for reasons analogous to those advanced above for $D_1$, so the sign of $r^*$ depends on the numerator, essentially on a $\sigma_D$- and a $\sigma_1$-term, as in (3). $A_1$ is also positive, like $(S_1 - M_1)$ in (3), and for similar reasons. Result 1 is thus confirmed even when $K$ is physically immobile.

The really noteworthy aspect of the "single corn" specification nonetheless is that since a portion of $X_2$ is being used as an intermediate input, so long as any labor is also directly engaged in the production of $X_1$, it will have a higher L/K ratio than $X_2$; i.e. $|\lambda| > 0$, which rules out the possibility of a negative $r^*$. The tax, therefore, will hurt labor, as in the text-book model, and ease of input substitution in the taxed industry ($\sigma_1 > 0$) will only make matters worse in this regard. Without further elaboration, the following result can be stated:
Result 2.: If a portion of the output of sector 2, where the immobile input is located, is used as an intermediate input by $X_1$, the mobile factor (labor) cannot benefit from a partial tax upon itself in $X_1$.

A similar tax in the other sector ($t_{1,2}$), however, tells a different story. Using the same notation and procedures as before, the solution for $r^*$ turns out to be:

$$r^* = \left[ -\rho L_1|\lambda|\sigma D - A_1 \sigma^1 + A_2 \sigma^2 \right] t_{1,2}/D_2$$

Given that, as noted above, $|\lambda|$, $D_2$, $A_1$, and $A_2$ are all positive, $r^*$ can be positive or negative. For “small” values of $\sigma^2$, $r^*$ would be negative, indicating that labor would benefit from a partial tax upon itself; in fact, $\sigma^2 = 0$ (fixed proportions in the production of $X_2$) will be sufficient for this outcome.\(^5\)

In the “single corn” formulation, thus, the two taxes do not behave symmetrically, unlike the standard 3 x 2 fgo specification or the “latex model.” Workers, regardless of their place of employment, are invariably hurt by $t_{1,1}$, whereas the other tax is more likely to lead to an increase in the net-of-tax wage rate. In both cases, however, a “large” elasticity of substitution in the taxed sector will be detrimental to the workers’ interests.

### 3.3 The "diamond" model

This specification also has an immobile primary input, but it is assigned to a separate sector which produces an intermediate input for the two final-good activities. In the textbook model,

\(^5\)This result is directly comparable to Atkinson and Stiglitz’s second conclusion (1980, p.175). Restated in terms of $t_{1,2}$ in the present framework, it shows that this partial tax, when levied in a relatively K-intensive sector ($X_2$ in the “single corn” formulation) will benefit labor if the substitution elasticity in the taxed sector is zero.
“immobility” implies “sector-specificity,” and vice versa. Here, K (the diamond mine, the river bed) never physically moves, although it is indirectly "used" in producing both final goods, through X3. Again, following the steps described earlier, the solution for $r^*$ is:

$$r^* = \left( \rho_{L1} |\lambda| \sigma_D + A_1 \sigma_1 \right) t^*_{L1}/D_3$$

which is the same as (4) except for slight changes in the denominator stemming from the intermediate usage in $X_2$ - terms involving $\rho_{32}$, $a_{32}$, etc. - which do not affect the sign of $r^*$ because they appear in the denominator of (6), and $D_3$ turns out to be positive. More than these similarities and differences, however, the feature worth highlighting in this formulation is that even though the $r^*$ expression is virtually the same as in the "single corn" specification, there is no a priori reason to expect that $|\lambda|$ will have a particular sign. Result 1 is therefore confirmed in a formulation with production linkages and no physical mobility for the primary specific factor. In contrast with Result 2, $r^*$ can be negative because there is nothing to prevent the taxed industry from being relatively K-intensive.

Returning to the questions posed in the Introduction, it seems that, at least in some cases, cross-sector production linkages, rather than intra-sector mobility of the specific input, is responsible for the possibility that labor might benefit from a partial tax upon itself, as the "diamond" model demonstrates. The “single corn” formulation nonetheless shows that not all production linkages are created equal, and their differences are reflected in the results presented above. More generally, the location of the specific factor and what it produces may prove to be more important than assumptions about factor mobility in some settings. What if latex gets processed in the agricultural sector itself, or only $X_2$ uses diamonds? The formulations in the next section will shed more light on such questions.
4. Some Other Production Structures

Among the large number of other possible formulations, some already touched upon, it is useful to concentrate on three which specially contrast with the specifications in Section 3: (i) self-contained sectors so that both the specific factor and what it produces remain in the same segment of the economy (e.g., a synthetic raw material produced in the manufacturing sector replaces X3); (ii) one-way linkage only among non-taxed activities (latex processed in the agricultural sector, for instance); and (iii) a primary specific factor located in the taxed sector and a one-way linkage with the other activity (for example, with t_{L1}, iron ore smelted and used for producing a final good, X1, as well as farm tools). Examples of this sort are just as plausible and plentiful as the ones considered thus far, and they provide a useful extension of the analysis in Section 3. Detailed derivations of changes in the rental-wage ratio nonetheless are not needed to address the questions being considered here.

The first two groups have intra-sector mobility for the specific factor, but in the absence of cross-sector connections, there are no L/K ratios to compare between X1 and X2, although there will be interactions on the demand side and in the labor market, as in the 3 x 2 fgo set-up. The essential features of the third category can be examined by taxing L2 rather than L1 in the "latex" model, and that tax was briefly discussed in Section 3.1 (the sign of r*, again, determined by the relative factor intensity of the taxed good, |λ|, and its elasticity of input substitution, σ^2). Of course, in this case |λ| > 0 indicates that the taxed activity is relatively K-intensive, so the second part of Result 1 will apply.

These formulations, along with those discussed in Section 3 earlier, show that so far as
similarities with mobile-factor models are concerned, say, with the 2 x 2 Harberger model, cross-section connections are of paramount importance. They allow $X_1$ and $X_2$ to be ranked in terms of factor intensities, and the end result is that the sign for $r^*$ depends on more or less the same factors in the two types of models, but there is no stipulation that the exact numerical value of $r^*$ would be the same in the two cases. The 2 x 2 mobile-factor fgo formulation, like the standard 3 x 2 model, also treats the two final goods symmetrically. The symmetry breaks down totally in the “single corn” formulation. Even in the “latex model” a tax in one final-good industry ordinarily would not have the same incidence outcome as a corresponding tax in the other, although the underlying logic and the solution process are identical. If $|\lambda| > 0$, $t_{L_{1,1}}$ is a tax on labor directly employed in the relatively labor intensive industry, so $t_{L_{1,2}}$ will be the corresponding tax in the K-intensive activity; therefore, the retal-wage ratio could move in opposite directions in the two cases. Such complications do not arise in the 3 x 2 fgo model, and the well-known result cited in the Introduction indicates that either tax will tend to hurt labor.

5. Summary and Conclusions

This research has been motivated by the observation that a simple modification of the standard 3 x 2 sector-specific model (three primary factors, two final goods) can lead to a reversal of one of its celebrated results -- that the mobile factor is always hurt by a partial factor tax upon itself (typically, a wage tax in one of the final-good industries) -- and the analysis for the most part has focussed on why and how that outcome may be affected.

The analytical framework relies on a number of production structures, each 3 x 2 in its physical dimension and also incorporating a produced input and some cross-sector production
linkages. These two elements enable us to compare the factor intensities (based on direct and intermediate usage) of the two final goods, which cannot be done in the standard model. It seems that this feature, rather than intra-sector mobility of the sector-specific factor, drives the new results, although in a "single corn" economy (one-way linkage, the taxed industry using a portion of the other sector's output as an intermediate input), the 3 x 2 fgo result holds when the tax is levied in the industry that uses the p.s.i. but not when it is switched to the other sector. Without production linkages, the final-good industries cannot be ranked in any of the production settings considered above, with or without intra-sector mobility of some inputs. Cross-sector production connections also appear to be responsible for similarities with models in which all factors of production are fully mobile.

These findings have an appeal beyond the concerns of tax-incidence theory. To mention just one application, policy makers must carefully weigh the distributional consequences of the tax changes they propose. Returning to a theme touched upon in the Introduction, the text-book model is generally identified with the short run, and “immobility” implies “sector-specificity.” It works well when immobility is being caused by institutional restrictions (licensing requirements), the intrinsic nature of certain inputs (a mine cannot be moved), or locational preferences. But in most modern economies, such immobile factors of production can and do produce intermediate inputs which may be unique to a given activity or used more broadly. The analysis in this paper suggests that production linkages will dominate the effects of factor immobility in many situations, and tax-incidence outcomes one might expect from the standard model would not always hold. Such considerations will also affect questions of tax substitution. In the McLure model, for instance, a selective output tax mimics the incidence of a partial factor tax. Although this issue has
not been taken up in this analysis, it would not be surprising to find that these two types of taxes lead to different incidence outcomes in some p.s.i. formulations.

References


