How sensitive is climate sensitivity?

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Estimates of climate sensitivity are typically characterized by highly asymmetric probability density functions (pdfs). The reasons are well known, but the situation leaves open an uncomfortably large possibility that climate sensitivity might exceed 4.5°C. In the contexts of (1) global-mean observations of the Earth’s energy budget and (2) a global-mean feedback analysis, we explore what changes in the pdfs of the observations or feedbacks used to estimate climate sensitivity would be needed to remove the asymmetry, or to substantially reduce it, and demonstrate that such changes would be implausibly large. The nonlinearity of climate feedbacks is calculated from a range of studies and is shown also to have very little impact on the asymmetry. The intrinsic relationship between uncertainties in the observed climate forcing and the climate’s radiative response to that forcing (i.e., the feedbacks) is emphasized. We also demonstrate that because the pdf of climate forcing is approximately symmetric, there is a strong expectation that the pdf of climate feedbacks should be symmetric as well.


1. Introduction

Climate sensitivity (≡ $T_{2x}$), the equilibrium response of global-mean, annual-mean, near-surface air temperature to a doubling of carbon dioxide above preindustrial concentrations, is a conceptually convenient metric for comparing different methods of estimating climate change. However, both the observations from which $T_{2x}$ is estimated and the climate simulations from which $T_{2x}$ is derived are uncertain, so that we cannot establish a single value but only its probability density function (pdf), $h_{T_{2x}}$. Both observations and simulations yield highly skewed pdfs, with finite probabilities of large sensitivities [e.g., Knutti and Hegerl, 2008].

Because the large asymmetry of $h_{T_{2x}}$ has been questioned [e.g., Hannart et al., 2009; Zaliapin and Ghil, 2010; Solomon et al., 2011, section 3.2], it is appropriate to revisit the underlying assumptions on which its derivation rests. First, $h_{T_{2x}}$ must be consistent with observations, so we analyze what modifications of those observations would lead to a significantly more symmetric pdf. Secondly, we examine the effect of relaxing assumptions underlying the simple model of Roe and Baker [2007, hereafter RB07], who derived an asymmetric $h_{T_{2x}}$ from the pdf of the total feedback factor $f$.

2. Estimates of Climate Sensitivity From Observations

A linearization of Earth’s annual-mean, global-mean energy budget is $H = R - \lambda T$, where $H$ is ocean storage, $R$ is radiative forcing, and $\lambda T$ is the climate response in terms of the global-mean, annual-mean, near-surface air temperature change, $T$, and the climate sensitivity parameter, $\lambda$ [e.g., Gregory et al., 2002]. Let $R_{2x}$ be the forcing due to a doubling of CO₂ over pre-industrial values ($\approx 3.7 \text{ Wm}^{-2}$). Computation of $h_{T_{2x}}$ can be made purely from observations of the modern state via the relationship:

$$T_{2x} = \frac{T_{obs}R_{2x}}{R_{obs} - H_{obs}}$$  \hspace{1cm} (1)

since $H$ is zero in equilibrium. Simplifying notation, let $F_{obs} = R_{obs} - H_{obs}$. Pdfs of these quantities are related by:

$$h_{T_{2x}} = \int_{0}^{\infty} h_{F_{obs}} \cdot h_{T_{obs}} \left( \frac{T_{2x}F_{obs}}{R_{2x}} \right) F_{obs} \cdot dF_{obs}$$ \hspace{1cm} (2)

where $h_{T_{obs}}$ and $h_{T_{obs}}$ are the pdfs of the observations. Both are found to be nearly normal distributions [e.g., Forster et al., 2007, Figure 2.20], given by

$$h_{T_{obs}} = \frac{1}{\sigma_{T} \sqrt{2\pi}} \exp \left[ -\frac{(T_{obs} - \bar{T}_{obs})^{2}}{2 \sigma_{T}^{2}} \right]$$

$$\equiv \phi(T_{obs}, \bar{T}_{obs}, \sigma_{T})$$ \hspace{1cm} (3)

and $h_{F_{obs}} = \phi(F_{obs}, \bar{F}_{obs}, \sigma_{F})$. Various estimates of $F_{obs}$ and $T_{obs}$ have been made. We use values from Armour and Roe [2011, hereafter AR11] of $\bar{F}_{obs} = \pm 0.90 \pm 0.55 \text{ Wm}^{-2}$, and $\bar{T}_{obs} = \pm 0.76 \pm 0.11^\circ\text{C}$, which are the same as Forster et al. [2007] and Trenberth et al. [2007], but updated with new ocean storage observations [$H_{obs} = \pm 0.74 \pm 0.08 \text{ Wm}^{-2}$, Lyman et al., 2010; Purkey and Johnson, 2010] (see auxiliary material). We assume independent errors.

Thus, from equation (2) and the aforementioned uncertainties, the skewed nature of $h_{T_{2x}}$ estimated from global-mean observations (Figure 1) is an inevitable result of the fractional uncertainty in $F_{obs}$ being much larger than the fractional uncertainty in $T_{obs}$ [e.g., Gregory et al., 2002]: the skewed tail towards high climate sensitivity is because the observations allow for the possibility that the relatively well constrained observed warming might have occurred with little or no net climate forcing. Allen et al. [2006] present several other estimates for various time periods: in all cases,

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observations and reconstructions are more constrained for temperature than forcing.

3. Can Observation-Based \( h_{F,2} \) Be Unskewed?

[6] How different would the aforementioned assumptions have to be in order to significantly reduce the asymmetry of \( h_{F} \)? As a metric for the symmetry of the sensitivity pdfs, we define

\[
S = \frac{T_{95} - T_{50}}{T_{50} - T_{05}}
\]

where \( T_x \) is the \( x \)th quantile. \( S \) is the interquantile skewness [e.g., Hinkley, 1975], mapped onto the range 0 to \( \infty \). The more common moment skewness (i.e., \( E((x - \mu)/\sigma)^3 \)), is infinite for the form of equations (2) and (3). The use of 90% bounds is prevalent in other studies of \( T_{2x} \). A symmetric distribution has \( S = 1 \), whereas for \( h_{F,2} \) based on observations, \( S = 6.0 \). We now focus on \( h_{F,ob} \) because it matters much more than \( h_{F,obs} \). Let \( h_{F,ob} \) now be represented by the so-called ‘skew normal’ distribution:

\[
h_{F,ob} = \phi(F_{obs}, T_{obs}, \sigma_F) \times \left( 1 + Erf(\alpha_F (F_{obs} - T_{obs})/(\sqrt{2}\sigma_F)) \right)
\]

\[
= \Psi_{ob}(F_{obs}, T_{obs}, \sigma_F, \alpha_F).
\]

For \( \alpha_F = 0 \) this is the normal distribution given by equation (3); for \( \alpha_F \neq 0 \) the skewness of \( h_{F,ob} \) has the same sign as that of \( \alpha_F \).

[7] The parameters necessary to achieve \( S = 1 \) are given in Table 1, and the corresponding pdfs are shown in Figures 2a and 2c. It is obvious that to remove the skewness completely would require a drastically different \( h_{F,ob} \) (e.g., a 100-fold reduction in \( \sigma_F \)). We thus conclude that, without exceeding large reductions in forcing uncertainty, or compelling arguments why \( h_{F,ob} \) has to be highly asymmetric, some skewness is inevitable in \( h_{F,2} \). For the rest of the paper, we ask whether that skewness might perhaps be, if not completely removed (i.e., \( S = 1 \)), then moderated substantially, and pick \( S = 2 \) as our measure. Table 1 shows this requires an approximate halving of \( \sigma_F \), a five-fold increase in \( F_{obs} \), or an \( \alpha_F \approx 2.0 \). The accompanying distributions are shown in Figures 2b and 2d. Table 1 gives guidance to the search for lower \( S \) by means of new observations.

4. Estimates of Climate Sensitivity From Models

[8] Climate sensitivity may also be estimated by diagnosing feedbacks within climate models. Let \( f = \) the linear sum of individual climate feedbacks, \( f = \sum f_i \). Then there is a one-to-one correspondence between values of this total feedback factor, \( f \), and \( T_{2x} \) [e.g., Roe, 2009]. Thus the pdf of \( T_{2x} \) can be calculated from \( h_f \), the pdf of \( f \). To derive estimates of \( h_{F,erial} \), RB07 further assumed: 1) \( h_f \) is Gaussian:

\[
h_f = \phi(f, \bar{f}, \sigma_f), \quad (6)
\]

and 2) feedbacks are independent of temperature, which led to the relationship between sensitivity \( T_{2x} \) and \( f \):

\[
T_{2x}(f) = \frac{T_0}{1 - f}
\]

where \( \lambda_0 = 0.3 \), \( T_0 = \lambda_0 R_{2x} \approx 1.2^\circ C \). Assumptions (6) and (7) yield an asymmetric \( h_{F,2x} \). For current best estimates \( \sigma_f = 0.13, \bar{f} = 0.65 \) the resulting pdf has \( S = 4.0 \).

[9] Given these assumptions, the skewed nature of \( h_{F,2} \) is an inevitable result of the asymmetric amplification by the feedback response on the high side of the mode of \( h_f \). This amplification serves to underscore the magnitude of the challenge of refining model-based estimates of the high side of \( h_{F,2} \). It requires a high degree of confidence in the shape of the high side of \( h_f \) and, moreover, how that shape changes with mean climate state.

[10] In previous work [RB07; Roe and Baker, 2011, hereafter RB11], we have shown that a model based on equations (6) and (7) is supported by its ability to reproduce the multi-thousand member ensemble results of climateprediction.net; by observational studies that find an approximately Gaussian distribution to the total feedback factor [e.g., Allen et al., 2006]; and by the fact that for a system of many feedbacks, the Central Limit Theorem

<table>
<thead>
<tr>
<th>( f )</th>
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<th>( \bar{f} )</th>
<th>( S )</th>
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<td>.55</td>
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</table>

*The first line are the standard combination of parameters for \( h_{F,2} \) in equation (5), and subsequent lines show the changes in parameters necessary to obtain the given value of the asymmetry parameter, \( S \). In each case only a single parameter has been altered (shown underlined).
would suggest that the distribution of $h_f$ would converge on a Gaussian.

Despite these successes of the model, assumptions (6) and (7) have been questioned. Hannart et al. [2009, hereafter HDN09] take issue with the RB07 result that it is hard to reduce the likelihood that $T_{2\times}$ is higher than the IPCC ‘likely range’ (i.e., $>4.5^\circ$C) by reducing uncertainty in climate parameters, or equivalently in observations [Allen et al., 2006]. They point out that equation (6) allows the possibility that $f \geq 1$, which they argue is an indictment of the model. The applicability of equation (7) has also been questioned by HDN09, and Zaliapin and Ghil [2010]. It is therefore appropriate to examine the effect of relaxing assumptions (6) and (7) on the symmetry parameter $S$.

### 4.1. Can Model-Based $h_{T_{2\times}}$ Be Unskewed?

We consider the following set of analyses, taken one at a time:

- Vary $f$, $\sigma_f$, keeping relationships (6) and (7). We extend the arguments of RB07 here.

- Let the pdf of feedbacks be asymmetric: $h_f = \Psi_{\text{as}}(f, \sigma_f, \alpha_f)$: in order to decrease the asymmetry in $h_{T_{2\times}}$, $\alpha_f$ must be negative.

- Let the feedbacks be nonlinear: $f(T) = f_0 - 2a\lambda_0 T$, where $f_0$ is independent of temperature, and the constant $a$ must be positive to reduce the asymmetry of $h_{T_{2\times}}$.

In our opinion, Table 2 shows that it is virtually impossible to achieve $S \approx 1$ by any single parameter change in the RB07 model: it requires either a $10^4$-fold reduction in $\sigma_f$, or a highly skewed $h_f$ with $\alpha_f \leq -5$. The lowest value of $S$ achievable for non-negative $f$ is 1.2. Table 2 also shows single parameter variations in the model that result in $S \approx 2.0$. The corresponding $h_f$s and $h_{T_{2\times}}$s are shown in Figure 3, as well as RB07’s model for comparison.

### 4.2. Nonlinear Feedbacks

Allowing for nonlinearities (see RB11, and auxiliary material), equation (7) is replaced by

$$T_{2\times} = \frac{-(1 - f_0) + \sqrt{(1 - f_0)^2 + 4a\lambda_0^2 R_{2\times}}}{2a\lambda_0}.$$  

---

**Figure 2.** The effect of (top) altered pdfs of radiative forcing observations on (bottom) the asymmetry of $h_{T_{2\times}}$. The thick grey curve shows current uncertainties (AR11, $\alpha_F = 0$, $S = 6.0$) for comparison. (a and c) $S \approx 1$. (b and d) $S = 2$. The pdfs are normalized between 0 and $\infty$. 

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Table 2. Variation of Feedback Model Parameters and the Impact on $S$

<table>
<thead>
<tr>
<th>$\sigma_f$</th>
<th>$\tilde{f}$</th>
<th>$\alpha_f$</th>
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<tr>
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<td>0.65</td>
<td>0.</td>
<td>1.2</td>
<td>1.0</td>
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</table>

The auxiliary material derives the value of $a$ from a large number of published studies. We find $a \leq 0.06$, from which $S \geq 2.8$. To achieve $S = 1$ requires $a$ to be 20 times greater (Table 2). Figure 3b shows the $h_T$ implied by equation (8) after adjusting $f_0$ so all curves pass through $f = 0.65$, $T_{2x} = 3.5^\circ C$, the best linear estimate for today’s climate (see auxiliary material). For $a = 0.11$, $S = 2$ and the high sensitivity tail ($T_{2x} \geq 8^\circ C$) is eliminated, while at lower values of $a$, $h_T$ is virtually identical to the linear model.

5. Why Are Observation-Based and Model-Based Estimates of $h_T$ So Similar?

[18] A striking feature of Figure 1 is that observation-based and model-based estimates of climate sensitivity are very similar. If they differed wildly, it might perhaps imply that there was important unused information, or that there were troubling biases among different methods. Another reason for their similarity is also worth emphasizing. From equation (1) and the fact $\lambda = \lambda_0/(1 - \Sigma_i f_i)$, we can write

$$\lambda_0 R - \lambda_0 H = \frac{\lambda_0 T}{\lambda} = T - \Sigma_i f_i(T). \quad (9)$$

$\lambda_0$ is known, $H$ and $T$ are well constrained in the current climate (i.e., Section 2), and estimating $\lambda$ is the goal. Term (i) on left-hand side of equation (9) reflects the principal source of uncertainty in global-mean energetics (the radiative forcing of aerosols), and term (ii) on the right-hand side reflects the uncertainty in climate feedbacks. Equation (9) shows that these two approaches are equivalent to each other. Therefore, because $R_\delta$ is broad (relative to $H$ and $H_T$) and nearly symmetric [e.g., Forster et al., 2007] $h_T$ should be too. Moreover, to the extent that ensembles of models (1) adequately simulate $T$ and $H$ and (2) faithfully represent the observed forcing uncertainties, the modeled $h_T$ must behave similarly. As noted previously [e.g., Knutti, 2008], the AR4 ensemble undersamples observed $h_T$, implying an undersampling of $h_T$ and consequently of $h_T$ (Figure 1).

6. Discussion

[19] We have used the frameworks of global-mean energy budget observations and global-mean feedback analysis to explore how asymmetry in $h_T$ might be reduced. While we have only varied the parameters one at a time, we’ve shown that the asymmetry cannot be eliminated by any realistic change to the parameters of either the observed uncertainty distribution or RB07’s model (see auxiliary material for multiple parameter changes). We have also shown that estimates of $h_T$ based on global energetics and estimates based on feedbacks are intrinsically linked. Therefore HDN09, for example, overreach in asserting that the analysis of RB07 is “a mathematical artifact with no connection whatsoever to climate”. Moreover it is critical for future climate projections to appreciate that uncertainties in forcing are not independent of uncertainties in $\lambda$, though this is sometimes overlooked [e.g., Ramanathan and Feng, 2008; Hare and Meinhausen, 2006].

[20] Our results add to a body of work demonstrating that, if further substantial progress is to be made on constraining the fat tail of $h_T$, it will likely only come from combining multiple estimates of $h_T$, or going beyond the global mean to spatio-temporal comparisons of models and observations. Bayesian approaches that combine multiple estimates can in principle lead to narrower and less skewed distributions [Annan and Hargreaves, 2006], though there are some formidable challenges to objectively establishing the inde-
dependence and relative quality of the different estimates [e.g., Lemoine, 2010; Henriksson et al., 2010]. Hegerl and Knutti [2008] review the many studies that constrain $h_{T_2}$ by minimizing disagreement between observations and models. Such estimates of $h_{T_2}$, as well as those based on model-diagnosed feedbacks are typically somewhat narrower than that permitted by modern observations (Figure 1), and are narrower still when including apparent correlations among feedbacks [Huybers, 2010]. Confidence in these model-based estimates depends on whether models fully sample the observed uncertainties, and whether they adequately represent the relationship between other aspects of the climate system and the global-scale energetics with sufficient skill [e.g., Knutti et al., 2010]. Finally, a practical measure of the acceptance of any of these narrower estimates is whether they become formally used as constraints to narrow uncertainties in current climate forcing (AR11).

[21] Ominous consequences have been thought to follow from the skewness of $h_{T_2}$ [e.g., Weitzman, 2009]. The argument has been made that we should focus our efforts on decreasing the probabilities of high $T_2$, by making more accurate observations. Our results provide clear targets in terms of improved observations or more certainty among models. However, this focus is to some extent misplaced. Firstly, because, as shown by RB07 and the present analysis, it would take large decreases in observed or modeled uncertainties to have much of an impact. Moreover, a reduction of uncertainty in $F_{obs}$ or $f$ moves the mode of $h_{T_2}$ to higher values (e.g., Figures 1 and 2). So, as noted by RB07, while the probabilities become more focussed, in other words the range—however measured—gets less, the cumulative likelihood beyond 4.5°C remains stubbornly persistent. Secondly, and more fundamentally, $T_2$ is only a metric of a hypothetical global mean temperature rise that might occur thousands of years into the future. Very high temperature responses, if they develop, are associated with the very longest time scales [e.g., Baker and Roe, 2009]. On the other hand, in this century we face the very real threat of climate changes that will have very damaging impacts on life and society [e.g., Intergovernmental Panel on Climate Change, 2007]. While understanding the basic relationship between radiative forcing, climate feedbacks and climate sensitivity is important, arguments about the details of the pdf shape are not.

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