2015 SISMID Spatial Short Course Ohio Exercise

1. We will carry out disease mapping for Ohio lung cancer mortality data from 1988 that is on the class website along with various R functions for producing maps for counties within Ohio.

(a) Provide a map of the SMRs, with expected numbers adjusted for gender, race and age.
(b) Provide a map of the estimated standard errors of the SMRs,

\[ \text{s.e.}(\text{SMR}_i) = \frac{\hat{\theta}_i^{1/2}}{E_i^{1/2}}. \]

2. In this question we will carry out disease mapping for Ohio lung cancer mortality data from 1988, following on from the previous question using the model

\[ Y_i | \delta_i, \alpha \sim_{iid} \text{Poisson}(e^\alpha E_i \delta_i) \]
\[ \delta_i, a \sim_{iid} \text{Gamma}(a, a) \]

(a) Use the eBayes function in the SpatialEpi package to obtain empirical Bayes estimates of \( \text{RR}_i = e^\alpha \delta_i \). Map these estimates and compare with the SMR map.
(b) Use the EBpostdens function (in the SpatialEpi package), which plots the gamma densities

\[ \text{Gamma}\left(\hat{\alpha} + y_i, \frac{\hat{\alpha} + E_i \exp(\hat{\alpha})}{\exp(\hat{\alpha})}\right), \]

for \( i = 1, \ldots, 4 \).
(c) Using the function EBpostthresh, calculate the posterior probabilities that \( \text{RR}_i \) exceeds the threshold 1.2, and map these quantities.

3. In this question we will carry out disease mapping for Ohio lung cancer mortality data from 1988, following on from the previous part, but now turning to Poisson-Lognormal models:

\[ Y_i | \alpha, V_i \sim_{iid} \text{Poisson}(E_i e^\alpha e^{V_i}) \]
\[ V_i | \sigma^2 \sim_{iid} \text{N}(0, \sigma^2) \]

(a) Using the inla function in R fit this model using the default prior for \( \alpha \) and a Gamma(1, 0.026) prior \( \sigma^2 \). Report the posterior medians and a 95% interval for \( \alpha \) and for \( \sigma^2 \).
(b) Extract the relative risk estimates and provide a map of these. Compare these estimates with those obtained from the Poisson-Gamma model considered in question 2.

4. In this question we will carry out Bayesian disease mapping for Ohio lung cancer mortality data from 1988 using spatial models within \texttt{inla}.

We analyze the Ohio data using the ICAR conditional model discussed in class. For this we assume:

\[
\sigma_u^{-2} \sim \text{Ga}(1, 0.026) \\
\sigma_v^{-2} \sim \text{Ga}(1, 0.026)
\]

with the default prior on $\alpha$.

(a) On the class website you will find a file containing the adjacency information, \texttt{ohio.graph}. Fit the ICAR model (including spatial and non-spatial random effects) and report the posterior medians and a 95\% interval for $\alpha$, $\sigma_v$ and $\sigma_u$.

(b) Produce maps of the posterior medians of the non-spatial and spatial random effects $\hat{V}_i$ and $\hat{U}_i$. Comment on the magnitude of the residual relative risks and on the relative contributions of the non-spatial and spatial components.

(c) Produce a map of the estimate of the smoothed relative risks $\exp(\alpha + V_i + U_i)$. 