1. Hermitian Operators in Quantum Mechanics

a) Show that the momentum operator \( \hat{p} = -i\hbar d/dx \) is Hermitian.

b) Show that the Hamiltonian operator \( \hat{H}(p, x, t) = \hat{p}^2/2m + V(x, t) \) is Hermitian.

c) Show that the eigenvalues of any Hermitian operator are real.

2. Momentum space wave functions –

a) Determine the momentum space wave function \( \Phi_0(p, t) \) (see Griffiths, Eq. [3.54]) for a particle in the ground state of a \( \delta \)-well potential of strength \( \alpha \).

b) Determine the momentum space wave functions \( \Phi_0(p, t) \) and \( \Phi_1(p, t) \) for a particle in the ground and 1st excited states of a dimensionless Harmonic oscillator. Hint: for \( \Phi_1(p, t) \) use the raising operator \( a^+ = (Q - iP)/\sqrt{2} \) with \( Q = +(i\hbar)\partial/\partial P \).

3. Commutators

a) Show for any operators \( A, B \) and \( C \) that \([AB, C] = A[B, C] + [A, C]B\)

b) Use part a) and the fundamental commutator \([\hat{p}, x]\) to evaluate \([\hat{p}^n, x]\).

4. Dynamics

a) Use Griffiths Eq. [3.71] to evaluate \( d\langle x \rangle/dt \) and compare with Ehrenfest’s theorem.

b) Use Griffiths Eq. [3.71] to evaluate \( d\langle \hat{p} \rangle/dt \) and compare with Ehrenfest’s theorem.