1. Explain the concept of opportunity cost of capital and how it applies to PV analysis. [2 pts]

The opportunity cost of capital is the best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted. The idea is that when you invest your money (capital), you bear an opportunity cost that is equal to the next best opportunity for investing your money on similar terms (risk and time). Thus, the appropriate discount rate in PV analysis should be your opportunity cost of capital.

2. Inflation is expected to be 2.5% per year. You are considering an investment offering a return of 6% APR, compounded quarterly. You want to have a total real annual return of at least 3.5%. Should you make the investment? [2 pts]

First, you need to convert the APR into an effective annual rate so that you can apply the inflation rate, which is already an annual rate (it is per year):

\[ r = \frac{APR}{m} = \frac{.06}{4} = .015 \quad EAR = (1.015)^4 - 1 = .06136 \]

Then you can convert the nominal EAR into a real EAR:

\[ 1 + real = \frac{1 + Nom}{1 + Infl} = \frac{1.06136}{1.025} = 1.0355 \], so the real EAR is 3.55%, which is more than your minimum, so you should take the investment.

3. You would like to endow a scholarship today. You want the first scholarship to be awarded 10 years from today and you would like the amount to grow by 3% each year forever. The endowment’s discount rate is 7% and you would like the first scholarship award to be $10,000. How much do you have to donate TODAY? [2pts]

This is a deferred growing perpetuity. Break it into 2 parts: first compute the value of the growing perpetuity and then bring it back to the present.

\[
\begin{array}{cccccc}
0 & 1 & \ldots & 9 & 10 & 11 \ldots \\
0 & 0 & \ & 0 & 10,000 & \text{Grows by 3%} \\
\end{array}
\]

The value of a growing perpetuity one period before it starts is

\[ CF_1 = \frac{10,000}{r - g} = 250,000. \text{ This is the value in YEAR 9, one period before the cash flows start.} \]

You need to discount this value back to the present:

\[ \frac{250,000}{(1.07)^9} = 135,983.44 \]

The formula for the present value (PV) is:

\[
PV = \frac{CF}{r} \left[ \frac{1}{1 + r)^n} - 1 \right] \quad PV = \frac{FV}{(1 + r)^n} \quad PV = \frac{CF}{r} \left[ 1 - \frac{1}{(1 + r)^n} \right] \quad FV = PV \left(1 + \frac{Nom}{1 + Infl} \right) = 1 + Real \quad PV = \frac{CF}{r - g} \quad PV = \frac{CF}{r}
\]