1. Explain how to decide which cash flows to consider in a capital budgeting decision? [6]

The best way is to use the “with vs. without” rule. You ask yourself is this cash flow different with the project vs. without the project. If you can’t answer “yes,” then you should not consider the cash flow. Remember, we are looking only for incremental changes—how will the project change the firm’s cash flows?

2. You are choosing between two loans. One is pegged to inflation and charges a real interest rate of 6% APR, compounded annually. The other charges a nominal rate of 9.8% APR, compounded semi-annually. If inflation is 0.3% per month, which has the lower rate? [5]

You need to get these both on equal footing. The easiest is to convert them both into nominal EAR’s. Since the real rate is annual, you need to convert the inflation rate to annual as well. It is given as 0.003% per month, so its annual equivalent is \((1+(.003))^{12}\cdot1=0.0366\). Thus, the annual nominal rate to go with the 6% real rate is \((1.06)(1.0366)-1=0.0988\)

The other rate is already a nominal rate, but needs to be converted into an EAR:
\((1+(.098/2))^{2\cdot1}=1.1004\)

Thus, the real rate is lower.

3. My sons like to play with Star Wars action figures. Each figure costs $10. When I was their age (33 years ago), I played with the original Star Wars action figures that cost $3 each. If the average rate of inflation has been 3.34% per year (annual compounding), has the price of action figures increased by more or less than the rate of inflation? [5]

$3 (1.0334)^{33} = \$8.87$, so the price of Star Wars action figures has increased faster than the rate of inflation.
4. Your company is considering a 3-year contract to produce organic chocolates in Seattle. The equipment needed would cost $50,000 and would be depreciated on a 5-year straight line basis to zero. You think you might be able to sell the equipment at the end of the contract for $24,000. You will also use some existing equipment with a book value of zero and a market value of $10,000. Your company’s vice president of operations would supervise production along with her other duties. Her salary is $120,000 per year. The cost of producing the chocolate would be $200,000 per year. The revenues are expected to be $225,000 per year. New working capital for production will be $20,000 immediately. Project working capital will stay level at $20,000 in year 1, decrease to $10,000 in year 2 and then drop to 0 at the end of the project. Your discount rate is 12% and your tax rate is 35%. Forecast all incremental free cash flows and compute the project’s NPV. [20]

The existing equipment is an opportunity cost. After-tax, you are giving up:
$10,000 - (10,000 - 0)(.35) = $6500 by using it instead of selling it.
When you sell the new equipment, it will be partially depreciated (it will have 3 years of depreciation at $10,000 per year), leaving it with a book value of $20,000. If you sell it for $24,000, you will have to pay taxes on the $4000 gain, so your net will be:
$24,000 – ($24000-20000)(.35) = $22,600.
The vice president is existing overhead. The increase in working capital represents a commitment of $20,000 in funds at the beginning of the project that flows back out as $10000 in year 2 and another $10000 in year 3.

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<td></td>
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<tr>
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<td>200000</td>
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<td>10000</td>
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<td>FCF</td>
<td>-76500</td>
<td>19750</td>
<td>29750</td>
<td>52350</td>
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\[ NPV = -76500 + \frac{19750}{1.12} + \frac{29750}{1.12^2} + \frac{52350}{1.12^3} = 2112.14 \]
5. Let’s say that starting one month from today, you save $500 per month for 40 years and then retire. Starting one month after you retire, you make monthly withdrawals for 20 years. If your interest rate is 0.7% per month, what is the amount you could withdraw each month so that you would not run out until the last withdrawal? [8]

\[
P V = \frac{500}{0.007} \left[1 - \frac{1}{1.007^{480}}\right] = 68,918.27
\]

\[
F V = 68,918.27 \left(1.007\right)^{480} = 1,961,011.88
\]

Then compute the monthly withdraw based on that amount

\[
1,961,011.88 = CF \left[\frac{1}{0.007} - \frac{1}{0.007 \left(1.007\right)^{240}}\right] = 16,894.21
\]

If inflation is 0.3% per month, what is the real value of your first withdrawal? [2]

The nominal cash flow is 16894.21, it comes 481 months from now (you save for 480 months and make your first withdrawal one month later). So, to convert this nominal cash flow into a real cash flow, you need to take out 481 months of inflation of 0.3% per month:

\[
\frac{16894.21}{\left(1.003\right)^{481}} = 3999.35, \text{ so in terms of today's buying power, you will have about $4000 per month in retirement.}
\]

Now let’s assume that you increase your savings each month by the amount of inflation, which is 0.3% per month. If everything else stays the same and you want to make withdrawals in retirement that have a constant real value, what will the real value of your withdrawals be? [5]

This was intended to be hard—a stretch problem. The key to solving this problem is to realize that if you do everything in real terms, these are just regular, flat annuities (the cash flows increase in nominal terms, but remain constant in real terms). First, you must compute the monthly real rate. You have a monthly nominal and monthly inflation rate, so you can just do the division: 1.007/1.003 -1 = 0.003988. Then just use this rate throughout. One final issue is that the first cash flow at the end of this month won’t be $500 in real terms because one month of inflation will have gone by, so it will be $500/1.003 (the question was a bit ambiguous, so I don’t care whether you used $500, or $500/1.003):

\[
P V = \frac{500}{0.003988} \left[1 - \frac{1}{\left(1.003988\right)^{480}}\right] = 106,498.92
\]

\[
F V = 106,498.98 \left(1.003988\right)^{480} = 719,508.27
\]

Your answer would be slightly higher if you kept all of the significant digits for the interest rate in your calculator.

Computing the real monthly withdrawal based on the real starting value and real rate:

\[
719508.27 = CF \left[\frac{1}{0.003988} - \frac{1}{0.003988 \left(1.003988\right)^{240}}\right] = 4,663.63
\]

Again, you could have a slightly higher answer without rounding at any point.

This is the constant real value in terms of today’s purchasing power of your monthly withdrawal. The actual nominal value of your first withdrawal will be 19700.31
6. Explain the difference between a sunk cost and an opportunity cost in capital budgeting. [6]

A sunk cost is a cost that you have already incurred and will not be able to recover, regardless of whether you take the project or not. An opportunity cost refers to any value or cash flow that you give-up by taking the project. Sometimes this refers to something that you already own, such as a piece of land. Even though you have already incurred the cost of the land (like a sunk cost), it is still (at least partially) recoverable in that if you don’t take the project, you could sell the land or do something else with it.

7. Assume the following term structure of interest rates:

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<th>1.5 years</th>
<th>2 years</th>
<th>2.5 years</th>
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<tbody>
<tr>
<td>Interest</td>
<td>1.0%</td>
<td>1.1%</td>
<td>1.4%</td>
<td>1.8%</td>
<td>2.4%</td>
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All rates are quoted as semi-annually compounded APR’s.

What is the price of a 4% bond (semi-annual coupons) that matures in exactly 1.5 years? [6]

\[ PV = \frac{20}{\left(1 + \frac{.01}{2}\right)} + \frac{20}{\left(1 + \frac{.01}{2}\right)^2} + \frac{1020}{\left(1 + \frac{.014}{2}\right)^3} = 1038.56 \]

What is the price of a 1.5-year $1000 zero-coupon bond (a $1000 STRIP)? [4]

\[ PV = \frac{1000}{\left(1 + \frac{.014}{2}\right)^3} = 979.29 \]

Why does one sell at a premium and the other at a discount? [4]

The bond sells at a premium because its coupon rate is greater than the market rates, so if it sold at or below par, it would offer too high a return relative to what else is being offered in the market. Its price must rise until at the higher price, its higher cash flows offer a fair market return. The STRIP sells at a discount because it offers no coupons, so it must sell at a discount to par, otherwise you would have no return or a negative return (if it sold at a premium)!

What is the current yield of the STRIP? What is the current yield of the bond? [4]

The current yield of the STRIP is zero because it pays no income.
The current yield of the bond is \(\frac{40}{1038.56} = 0.0385 \) or 3.85%

Which would have a higher yield-to-maturity: the STRIP or the bond? Why? [5]

The STRIP must have a higher YTM. The STRIP’s YTM is simply 1.4% since there is only one spot rate relevant to pricing the STRIP. The bond’s YTM has to be a weighted average of 1, 1.1 and 1.4%. Even though most of the weight will be on 1.4%, it will necessarily be less than 1.4%.
[Problem 7 continued—keeping the same term structure of interest rates]

You own a 2-year BBB-rated zero-coupon corporate bond with a $1000 par value. Its price is currently $920. What is its credit spread? [4]

First, solve for its YTM, then compare it to the YTM of a 2-year STRIP from the previous page (1.8%)

\[ r = \left( \frac{1000}{920} \right)^{\frac{1}{2}} - 1 = 0.021064, \quad APR = 0.021064 \times 2 = 0.042128, \quad \text{credit spread} = 0.042128 - 0.018 = 0.024128 \]

So, the credit spread is 2.41%, or 241 basis points.

If the credit spread on BBB bonds changes to 260 basis points, what will the change in price of the bond be? [4]

Add the new credit spread to the STRIP’s YTM and recompute the price:

\[ APR = 0.018 + 0.026 = 0.044, \quad r = \frac{0.044}{2} = 0.022, \quad \text{so} \quad \text{Price} = \frac{1000}{(1.022)^2} = 916.63 \]

So the price change will be $916.64 - $920 = -$3.37

8. You are leasing a $20,000 car with a $10,000 residual value. The salesperson says they are giving you a great financing rate of 3.6% APR (compounded monthly) and that your payments will be $360 per month for a 3-year car lease. Is the salesperson telling the truth about your financing rate? [Assume the 1st payment is in one month] [8]

\[ PV = \frac{10000}{1.003^{36}} = 8977.73 \quad \text{so} \quad 20000 - 8977.73 = 11,022.27 \]

\[ 11,022.27 = CF \left[ \frac{1}{.003} - \frac{1}{.003(1.003)^{36}} \right] \]

\[ CF = 323.46 \]

So, the salesperson is not telling the truth—otherwise your payments would be $323.46! (Note, you could also answer this by computing the PV of the lease with $360 payments and showing that it is more than the price of the car.)

If not, is the true interest rate they are charging you higher or lower than 3.6%? Explain. [4]

The true rate is higher. A higher true rate would decrease the PV of the residual value, increasing the amount you have to finance. At the same time, a higher rate will also require a higher payment in order for the PV of the payments to equal the value of the car you are driving off the lot.

Some people missed the point of the question and simply stated that the EAR is higher than the APR. Given your calculation in the first part of the problem, even the APR is higher than 3.6%.