Assume throughout that inflation is 3% APR compounded monthly

1. Current interest rates are very low. If you can get an interest rate of 1.5% APR, compounded quarterly, how long would it take for a $1000 deposit to grow to $1500? [6]

\[ n = \frac{\ln \left( \frac{FV}{PV} \right)}{\ln (1 + r)} = \frac{\ln \left( \frac{1500}{1000} \right)}{\ln \left( 1 + \frac{0.015}{4} \right)} = 108.33 \text{ qtrs}, \text{ or about 27 years} \]

2. What is the relation between bond prices and interest rates and what is the economic intuition explaining this relation? [6]

Bonds are just a fixed package of cash flows. When you decide how much you are willing to pay for a given package of cash flows, you compare it to other uses for your money. Current interest rates represent those other uses. If interest rates increase, the bond’s fixed package of cash flows looks relatively less attractive. If interest rates decrease, the bond’s cash flows look relatively more attractive compared with your other opportunities (the interest rates). That’s why there is an inverse relation between bonds and interest rates.

3. After a wildly successful career in finance, you decide to thank the guy who showed you the true path to happiness, Prof. Harford. You thought about just buying him a Maserati, but inexplicably decided to endow a scholarship in his name instead. You want the first scholarship payment to be $10,000 immediately. Payments will be made annually and you want them to grow to keep up with inflation. The nominal financing rate appropriate for funding this scholarship is 7% APR, compounded annually. How much will it cost you to establish the “I’d rather just have the Maserati” scholarship? [8]

Growing to keep up with inflation means that the growth rate is equal to the annual inflation rate. You are given inflation as 3% APR, compounded monthly, meaning that it is 0.25% per month. You can convert this to an effective annual rate as \((1.0025)^{12} - 1 = 0.0304\)

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>10000(1.0304)</td>
<td>10000(1.0304)^2</td>
<td>…</td>
</tr>
</tbody>
</table>

Note that the first cash flow is CF0 and we want it always to be worth $10,000, so in a year, CF1 will be 3.04% more and the formula takes CF1. That’s why it is 10000(1.0304).

\[ PV = 10000 + \frac{10000(1.0304)}{.07 - .0304} = 270,202 \] So the Maserati would have been cheaper, too!!
Assume throughout that inflation is 3% APR compounded monthly.

4. Jen dreams of retiring in Hawaii. She is starting a savings plan by putting $500 into a retirement account at the end of the month. She will continue to deposit $500 every month until retirement. She plans to retire in 40 years (480 months). One month after she retires, she will start making withdrawals from her retirement account. Her retirement account is expected to earn 12% per year, nominal APR, compounded monthly.

a. How much money will Jen have when she retires? [6]

This is a future value of annuity question. First get the present value of the 480 deposits of $500 and then compute the FV. The appropriate rate is .12/12 = .01 per month.

\[
PV = \frac{500}{.01} \left( 1 - \frac{1}{(1+.01)^{480}} \right) = 49,578.58
\]

\[
FV = 49,578.58 \times (1 + .01)^{480} = 5,882,386.26
\]

b. What will the real value of her retirement account be when she retires? [4]

Converting from a nominal cash flow to a real cash flow simply requires you to take-out the inflation. In this case, the inflation rate is 3% APR, compounded monthly, so the inflation rate is .03/12 = .0025 per month.

\[
\frac{5,882,386.26}{(1.0025)^{480}} = 1,774,395.87
\]

If Jen plans to spend 25 years (300 months) in retirement, what monthly withdrawal (in real terms) can she make so that she just runs out of money after 25 years? [6]

If you take the real amount you computed in part (b) and use it as the PV of a real annuity (meaning that you use a real discount rate), you can compute a real monthly payment:

The real monthly rate is \[
\frac{1 + \text{Nom}}{1 + \text{Infl}} - 1 = \frac{1 + .01}{1 + .0025} - 1 = 0.00748
\]

\[
CF = \frac{1,774,395.87}{0.00748} - \frac{1}{0.00748(1.00748)^{300}} = 14,863.40
\]

d. Now assume Jen plans to increase the nominal savings amount each month to keep up with inflation. Her first deposit will still be $500, but she will increase her deposit each month so that the real value of her deposit stays constant (but the nominal amount increases). When she retires, what will the real value of her retirement account be under this scenario? [6]

This is a growing annuity in nominal terms, but in real terms, it is just staying constant because it is only growing at exactly the rate of inflation. That means we can treat it as a
Assume throughout that inflation is 3% APR compounded monthly.

regular annuity where the discount rate is real and the cash flows are constant in real terms. The first cash flow is made one month from the start, so its real value is actually \( \frac{500}{1.0025} = 498.75 \), which is the real cash flow we’ll use in the annuity formula.

\[
PV = \frac{498.75}{0.00748} \left[ 1 - \frac{1}{(1.00748)^{480}} \right] = 64,803.52
\]

\[
FV = 64,803.52 \left( \frac{1.00748}{480} \right) = 2,319,289.70
\]

5. Your company, PBR, Inc. is considering the launch of a new product. Designing the new product cost $500,000 and it estimates that it will sell 800,000 units per year for $3 per unit and variable non-labor costs will be $1 per unit. PBR will close down production after year 3. New equipment costing $1 million will be required. The equipment will be depreciated to zero on a straight-line basis over 10 years. PBR thinks it can sell the equipment in year 3 for $400,000. PBR’s current level of working capital is $300,000. The new product will require the working capital to increase to a level of $380,000 immediately, then to $400,000 in year 1, in year 2 the level will be $350,000, and finally in year 3 the level will return to $300,000. PBR’s tax rate is 35%. The discount rate for this project is 10%. Do the capital budgeting analysis for this project and calculate its NPV. [20]

Design already happened and is sunk (irrelevant). Depreciation will be $100,000 per year (straight line for 10 years gives us $1,000,000/10). Figure the cash flow from selling the equipment at the end (when its book value will be $700,000): 400-(400-700)(.35)=505. Selling it at a loss has created a deduction of $300,000, reducing taxes by $105,000. Thus, the total cash flow effect of the sale is 400+105=505.

Also, note that we are only concerned about changes in working capital.

<table>
<thead>
<tr>
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<th>Time : 0</th>
<th>1</th>
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<tbody>
<tr>
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<td>400</td>
<td>350</td>
<td>300</td>
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<tr>
<td>Change</td>
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<td>-20</td>
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<td></td>
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<tr>
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<tr>
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<tr>
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<tr>
<td>+CapEx</td>
<td>-1000</td>
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<td>505</td>
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</tr>
<tr>
<td>+CF from Chg NWC</td>
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<td>50</td>
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</tr>
<tr>
<td>=FCF</td>
<td>-1080</td>
<td>1055</td>
<td>1125</td>
<td>1630</td>
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</tr>
</tbody>
</table>

\[
NPV = -1080 + \frac{1055}{1.10} + \frac{1125}{(1.10)^2} + \frac{1630}{(1.10)^3} = 2033.486
\]

It has an NPV of $2033.486, or $2,033,486.
Assume throughout that inflation is 3% APR compounded monthly.

6. After negotiating with a car salesman, you get him down to the point where the purchase price for the car will be $19,000. You will lease the car for 36 months with an option to purchase or return the car at the end of the lease. The contracted value of the car at the end of the lease is $10,000. The interest rate underlying the lease is 6% APR, compounded monthly. You will make 36 monthly lease payments with the first one due in one month.

   a. What will your lease payments be? [10]

   Ok, we think about a lease as just a loan with a balloon payment at the end. The balloon payment comes from you giving the car back (so it is like a payment of $10,000). Thus, the cash flows look like this:

<table>
<thead>
<tr>
<th>0</th>
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<th>48</th>
</tr>
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<tbody>
<tr>
<td>PMT</td>
<td>PMT</td>
<td>...</td>
<td>PMT+10,000</td>
<td></td>
</tr>
</tbody>
</table>

   The PV of the 10,000 at the end just reduces the effective amount of the loan (which equals the PV of your payments).

   \[
   Loan = 19,000 - \frac{10000}{\left(1+\frac{.06}{12}\right)^{36}} = 10,643.55
   \]

   \[
   10,643.55 = PMT \left[ \frac{1}{.06/12} - \frac{1}{\left(1 + \frac{.06}{12}\right)^{36}} \right], \text{ so } PMT = 323.80
   \]

   b. If you total (wreck) the car after 12 months, how much will you owe the leasing company? [4]

   You will still owe 24 months of payments and the residual value of the car. That PV of that total at the end of month 12 is:

   \[
   323.80 \left[ \frac{1}{0.06/12} - \frac{1}{\left(1 + \frac{.06}{12}\right)^{24}} \right] + \frac{10,000}{\left(1 + \frac{.06}{12}\right)^{24}} = 16,177.71
   \]

7. A 4-year $1000 par zero coupon bond has a YTM of 8% APR, compounded semi-annually. You think there is a 20% chance that the bond will default and pay-off nothing. What is your expected return from buying the bond? Express your answer as a semi-annually compounded APR? [6]

   First, compute the price of the bond, we know that if you discount the cash flows from the bond at the YTM, you must arrive at the price. This bond has only one cash flow ($1000 at the end). Given the price, you can compute your expected return from buying the bond. You buy it at the price and expect to get $800 (your expected CF is .2(0)+.8(1000)), which
Assume throughout that inflation is 3% APR compounded monthly you can treat as your FV and solve for r. The r will be a 6-month r and then you convert to APR:

\[ PV = \frac{1000}{\left(1 + \frac{.08}{2}\right)^r} = 730.69 \]

Expected CF = .2(0)+.8(1000) = 800

\[ r = \left(\frac{800}{730.69}\right)^{\frac{1}{2}} - 1 = 0.0114 \]

Expected return as an APR = 2 x 0.0114 = 0.0228

8. Suppose you own a 4-year 5% coupon bond and the yield curve is its normal shape. If long-run rates decrease such that the yield curve switches to slope downward, is it likely that bond’s yield to maturity would increase or decrease? Explain. [6]

It is likely that the YTM of the bond will decrease. Thinking about the YTM as a weighted average of the spot rates associated with the bond with the weights approximately proportional to the size of the cash flows, the most influential spot rate will be the 4-year one (where the big, principal repayment cash flow occurs). If the yield curve goes from sloping up to sloping down, that rate will go down and so will the YTM.

9. The price of a 1-year 1% coupon $1000 par Treasury bond with semi-annual payments is $998.03 (the next coupon is due in 6 months). The one-year spot rate is 1.2% APR, compounded semi-annually. What must the price of a 6-month STRIP be? [6]

\[ \text{Price} = \text{PV} = PV(\text{5 in 6 months}) + PV(\text{1005 in one year}) \]

\[ $998.03 = PV(\text{5 in 6 months}) + \frac{1005}{\left(1 + \frac{.012}{2}\right)^r} \]

\[ $998.03 - $993.05 = PV(\text{5 in 6 months}) \]

PV(5 in 6 months) = $4.98, so the STRIP price is $100 \times \frac{4.98}{5} = $99.65 or 99.60 with rounding

10. What does a present value tell us? In other words, what does it mean to say that the present value of $110 received in one year at a 10% discount rate is $100? [6]

It tells us what amount of money we would need to have today in order to be indifferent between having the money today and waiting for the future cash flow(s). In the example, we would need to have $100 today to be indifferent between $100 now and waiting for $110 in one year. That is because at a 10% opportunity cost of capital, we could take the $100 and invest it at 10% to produce the $110 cash flow for ourselves.