Time: MW 4:00-5:20; Place: LOW 101

CourseWebsite: http://faculty.washington.edu/hqian/amath572/

by Crispin Gardiner, Springer (2009)


Reference Book 2: Probability Theory: The Logic of Science


Reference Book 4: Numerical Solution of Stochastic Differential Equations

Syllabus

1. Review on Probability and Random Variables:

The first week, I shall give an overview of why stochastic dynamics is important. Then in the second and third weeks, we shall review the materials on the theory of probability, and also have a brief historical account. Required reading: Ch. 1, Ch. 2; optional reading: a paper by Montroll (on the course web). I shall give lectures on related material, but not follow the book exactly. This is a review: In principle you should know this material a priori.

New science, even social sciences, needs new mathematics. The stochastic mathematical view of the world is fundamentally different from that of Newtonian mathematics. It is not an alternative; It is a more complete, more sophisticated, and more realistic perspective. The deterministic theory is only a special extreme case.

Some people, such as Bernoulli (1700-1782), Laplace (1749-1827), and E. T. Jaynes (1922-1998), actually articulate an even stronger view that the theory of probability is the language of science.

Statistical physics as an example: kinetic of gases and fluid dynamics; But in physics the stochastic effect is only considered as a perturbation to the deterministic theory. It is a part of the Nature due to atomic theory of the matters, thermal fluctuations, or incomplete information. It was Darwin who put the stochasticity, called “variation” into a much more positive light. In Darwin’s view, the stochasticity is a basis of living process. It is the constructive origin of life. According to a deterministic world view, things can only converge; but according to Darwinian view of the world, there are diversities.

Probability review. (1) Distribution and dynamics, (2) Joint and conditional probabilities, (3) Mean values and probability density, (4) Characteristic function and correlation functions, (5) Binomial, Poisson and Gaussian distributions, (6) Central limit theorem.

2. Markov Processes (Chapter 3)

What is a stochastic process? Markov process and the Chapman-Kolmogorov equation, discrete space and time;

Continuous-time Markov processes and its representation;

Diffusion processes — Fokker-Planck equation;
Deterministic processes — Liouville’s equation;
Kolmogorov forward and backward equations.

4. **Stationary Processes and Correlation Function (Section 3.7)**
   - Stationary and homogeneous Markov processes;
   - Autocorrelation function for Markov processes;
   - Reversibility and entropy production;
   - Time reversibility, symmetry, detailed balance, and potential;
   - Relative entropy and entropy production (see reading material).

5. **Fokker-Planck Equation (FPE) (Chapter 5)**
   - FPE in 1D, FPE in 2D;
   - Reversibility and circulation;
   - FPE in several dimensions;
   - First passage time problem.

6. **Master Equations and Jump Processes (Chapter 11)**
   - Random walk and birth-death processes;
   - Approximation of Master equations by FPEs;
   - Mean first passage times;
   - Birth-death systems with many variables and the chemical master equation;
   - Poisson representation.

7. **Brownian Motion, Diffusion Processes, and Stochastic Differential Equations (Chapter 4)**
   - Einstein’s Brownian motion theory;
   - Diffusion equation;
   - Langevin equation;
   - Stochastic integration and stochastic differential equations.

8. **Approximation Methods for Diffusion Processes**
   - Small noise perturbation (Chapter 7);
   - Elimination of fast variables (Chapter 8);
   - Beyond the white noise limit (Chapter 9);
   - Kramers’ problem and barrier crossing (Chapter 14).

8. **Lévy Process and Financial Applications (Chapter 10)**
   - Central limit theorem and the origin of Lévy process;
   - The Paretoian processes;
   - Stochastic description of financial markets and the theory of Black-Scholes;
   - Fractional Brownian motion and Gaussian processes.