AMATH 383 Homework Assignment #1
(Due in class: Jan 14)

1. The first 4 numbers in Fibonacci sequence $F_n$ with $n = 1, 2, \ldots$ are 1, 2, 3, 5. Find the values for $a, b, c, d$ in

$$G_n = an + bn^2 + cn^3 + dn^4,$$

such that the first four $G_n$ fit the first four $F_n$. What is $F_5$? and what is $G_5$? Compare $F_n$ and $G_n$ for $0 \leq n \leq 100$ in a graph.

2. Exercise 2 of Chapter 1.

3. The populations of two interacting biological organisms $X$ and $Y$, generation after generation, follow the pattern

$$x_{n+1} = 6x_n - 4y_n, \quad y_{n+1} = 2x_n,$$

in which $x_n$ and $y_n$ are the population sizes of the $n$th generation of $X$ and $Y$, respectively. If $x_0 = y_0 = 1$, find out the populations $x_n$ and $y_n$ for all $n \geq 0$.

Extra credit: There is a very easy way to solve this problem. However, a more systematic way to solve this problem is by assuming a solution in the form of

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = c_1 \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \lambda_1^n + c_2 \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} \lambda_2^n$$

and determine the $\lambda$’s, $a$’s, $b$’s, and $c$’s. Consult Boyce and DiPrima’s *Elementary Differential Equations* for solving a system of linear, constant coefficient, homogeneous first-order ODEs.