Finance 423, Banking, Assignments, Professor Hess, Mackenzie 214, Telephone 206 543-4579. Email hess@u.washington.edu. Office hours, T and Th afternoons and by appointment.


Grading:

There are four tests. I use the following formula to determine your G.P.A. on each test.

\[
Test \ GPA = \frac{4 \times (Your \ score)}{(Highest \ score \ among \ all \ students)}.
\]

Your course G.P.A. is an average of your G.P.A.s on each test plus an adjustment for each student based on his or her skill in answering questions I pose in class.

CLASS-BY-CLASS ASSIGNMENTS

<table>
<thead>
<tr>
<th>Date</th>
<th>Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep 28 Th</td>
<td>No class. I am presenting a research paper at a banking conference at Wharton. Class starts on Oct. 3</td>
</tr>
<tr>
<td>Oct 12 Th</td>
<td>Test on the theory of financial intermediation</td>
</tr>
<tr>
<td>Oct 17 T</td>
<td>Read textbook Ch. 7. Risks of financial intermediation, and Ch. 8. Repricing and maturity models of interest rate risk.</td>
</tr>
<tr>
<td>Oct 19 Th</td>
<td>Read textbook Ch. 9. The duration model Read “Interest Rate Risk” in Course Pack, pages 37-56.</td>
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<tr>
<td>Date</td>
<td>Day</td>
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<tr>
<td>Oct 24 T</td>
<td>Read textbook Ch. 9 Appendix “Convexity and other complications”</td>
</tr>
<tr>
<td>Oct 26 Th</td>
<td>Read textbook Ch. 10. Market risk: The Value at Risk Model, and <em>Course Pack</em> page 57-62.</td>
</tr>
<tr>
<td>Oct 31 T</td>
<td>Finish Ch. 10 and review for test 2.</td>
</tr>
<tr>
<td>Nov 2 Th</td>
<td><strong>Test on interest rate risk</strong></td>
</tr>
<tr>
<td>Nov 7 T</td>
<td>Read textbook Ch. 11. <em>Course Pack</em> pages 68-71.</td>
</tr>
<tr>
<td>Nov 2 Th</td>
<td>Test on interest rate risk</td>
</tr>
<tr>
<td>Nov 7 T</td>
<td>Read textbook Ch. 11. <em>Course Pack</em> pages 68-71.</td>
</tr>
<tr>
<td>Test 3 preparation: Find the spreadsheet that is on p. 67 of the <em>Course Pack</em>. Set up a spreadsheet that duplicates the spreadsheet using formulas to calculate all data that are not input data. I recommend that you work together in teams of your choosing. See reading in <em>Course Pack</em>, “Using Net Present Value Analysis in Bank Lending,” pages 63-67.</td>
<td></td>
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<tr>
<td>Nov 9 Th</td>
<td>No class</td>
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<tr>
<td>Nov 14 T</td>
<td>Read textbook Ch. 11 RAROC</td>
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<tr>
<td>Nov 16 Th</td>
<td>Read textbook Ch. 11. Credit risk continued through end of chapter. Option model.</td>
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<tr>
<td>Nov 21 T</td>
<td><strong>Test on credit risk</strong></td>
</tr>
<tr>
<td>Nov 28 T</td>
<td>Read textbook Ch. 24. Futures and forwards</td>
</tr>
<tr>
<td>Nov 30 Th</td>
<td>Futures and forwards continued.</td>
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<tr>
<td>Dec 5 T</td>
<td>Finish futures and forwards</td>
</tr>
<tr>
<td>Mar 7 T</td>
<td>Continue swaps</td>
</tr>
<tr>
<td><strong>Dec 11 M</strong></td>
<td><strong>Test on hedging.</strong> 10:30-12:20 p.m.</td>
</tr>
</tbody>
</table>
WHAT ALAN HESS THINKS EVERY FINANCE STUDENT SHOULD KNOW.

1) Discounted cash flow analysis
   a) Expected incremental cash flows
      i) Cash flows, not earnings
      ii) Real options
   b) WACC for each cash flow
      i) CAPM
         (1) Risk free interest rate
             (a) Expected real rate
             (b) Inflation premium
         (2) Diversification and beta
         (3) Equity premium and risk aversion
      ii) Cost of debt
           (1) Tax advantage of debt
           (2) Debt risk premiums
      iii) MM theorem and the WACC
   c) Term structure of interest rates
      i) Efficient market theory
      ii) Term premiums
   d) Investment decision making rules
      i) Net present value rule
      ii) Rate of return rule

2) Consequences of information and transaction costs
   a) Principal vs. agent conflicts
   b) Effects of capital structure
      i) Incentives
      ii) Signals
   c) Role of intermediaries and the bid to ask spread

3) Risk management
   a) Option pricing theory
      i) Binomial approach
      ii) Black-Scholes model
   b) Forward contracts
Commonly used words in banking whose meanings you should know.

- Adverse selection
- Agency costs
- Asset transformer
- Asymmetric information
- Bid-ask spread
- Borrowers' surplus
- Broker
- Capital charge
- CAPM
- Correlation
- Covenants
- DCF
- Dealer
- Delegated monitor
- Denomination intermediation
- Diminishing marginal returns
- Diversification
- Economic profits
- Externalities
- Financial intermediary
- Fixed costs
- Hedging
- Information costs
- Liquidity
- Loan loss provisions
- Maturity intermediation
- MM
- Monitoring
- Moral hazard
- mrr
- Net interest income
- NPV
- Operating costs
- Payments system
- Perfect market assumptions
- Price discovery
- Primary securities
- Project valuation
- Rate of time preference
- Savers' surplus
- Scale economies
- Scope economies
- Search costs
- Secondary securities
- Specialization economies
- Systematic risk
- Transaction costs
- Unsystematic risk
- Variable costs
1.0 Introduction

Financial markets and intermediaries exist to reduce the information and transaction costs of transferring money from savers to borrowers and back to savers at future dates. These transfers have two main benefits: First, they provide for the external financing of investment, which if the investments have positive net present values, increases the nation's wealth. Second, they allow each household to adjust the timing and risk exposure of its saving to maximize the expected utility of its lifetime consumption. Investors' and savers' surpluses measure the benefits of external investment financing and saving. Intermediation costs increase the interest rate paid by borrowers, decrease the interest rate received by savers, and reduce investors' and savers' surpluses. An operationally efficient financial market or intermediary is one that minimizes information and transaction costs thereby maximizing investors' and savers' surpluses.

2.0 A Model of an Intermediated Financial Market

In a perfect financial market the interest rate paid by the borrower equals the interest rate received by the lender, the amounts borrowed and lent are as large as they simultaneously can be, and trading benefits are maximized. Intermediation costs cause the interest rate paid by the borrower to exceed the interest rate received by the lender by the unit cost of supplying intermediation services. If financial intermediaries can reduce the spread between the borrowing rate and the lending rate, they can increase the quantity of saving and investment and savers' and investors' surpluses.

2.1 A perfect financial market

In a perfect financial market: (1) transaction costs are zero. All participants have free and equal access to the market and can trade at zero marginal cost. (2) information costs are zero. Information about the risks and returns to financial assets are freely and widely available to all participants. No participant has control over prices. All participants are price takers. (3) No distorting taxes exist.

The bold line in Figure 1 labeled "I perfect market" represents external financing of investment in a perfect financial market. Its height for any quantity of investment is the marginal rate of return on the investment. As the amount of financing and investment increase, the marginal rate of return declines due to the law of diminishing marginal returns. The diminishing marginal rate of return is shown as the downward slope of the investment schedule.

The bold line labeled "S perfect market" represents lending by savers in a perfect financial market. The height of the saving schedule is the rate of time preference. This is
the rate of return savers require to save instead of consume. As saving increases and consumption decreases, people require ever-higher rates of return to save more.

Equality between perfect market saving and investment sets the perfect market interest rate \( r_{pm} \), and the perfect market quantities of saving and investment \( q_{pm} \). For investments less than \( q_{pm} \), the marginal rate of return is greater than the market interest rate. These investments have positive net present values. For all investments greater than \( q_{pm} \), the marginal rate of return is less than the market interest rate. These investments have negative net present values. The marginal investment, \( q_{pm} \), for which the marginal rate of return equals the market interest rate has a zero net present value.

The area of the triangle that lies above the market interest rate \( r_{pm} \) and below the investment schedule \( I_{pm} \) is the wealth created by external financing of positive net present value investments. This area is called investors' surplus. It represents the accumulated difference between what investors would pay to borrow, the marginal rates of return, and what they had to pay, the market interest rate.

The area of the triangle that lies above the saving schedule and below the market interest rate is called savers' surplus. It represents the aggregate gains to savers from lending. These gains are the differences between the interest rate they receive and the interest rates they required based on their rates of time preference.

The sum of these two triangles, the area above the saving schedule and below the investment schedule, is the total trading surplus from being able to trade in a competitive market. Investors gain because they can finance their projects at an interest rate that is less than their projects' marginal rates of return, and savers gain because the interest rate they receive more than compensates them for foregoing current consumption.

### 2.2 A financial market with trading costs but no intermediary.

Borrowers and lenders face two types of trading costs. Information costs include the costs of finding trading partners, estimating the payoffs to investments, estimating the risk-adjusted, expected rate of return for the investment, and monitoring the performance of borrowers to ensure that borrowers use the money in the ways they promised when they solicited the loan. Transaction costs are the costs of effecting a transaction once a decision to trade has been made. They include the costs of agreeing on a transaction size and maturity, writing contracts to document the transaction, transferring money from a lender to a borrower and back to the lender, and managing the risks of the saving portfolio.

In the presence of trading costs, the net marginal rate of return to the investor is the marginal rate of return in a perfect market less the marginal cost of trading. The downward sloping, dashed line labeled "I imperfect market, no banks" is the net marginal rate of return when there are trading costs but no intermediaries. The vertical distance, denoted \( \gamma_{\text{imb}} \), between this line and the perfect market investment schedule is the unit
cost of borrowing without intermediaries. It is the cost of homemade, that is, non-specialized, intermediation.

In the presence of trading costs, the lender requires compensation for foregoing consumption and for incurring trading costs. The saving line labeled "S imperfect market, no banks" is the gross rate of return required by lenders when they trade with trading costs and without banks. It consists of the rate of time preference given by the perfect market saving schedule plus $\gamma_{s, nb}$ which is the unit cost of saving without banks.

In an imperfect, non-intermediated market, lending and borrowing are equal at the quantity $q_{nb}$. The rate of time preference at this quantity of saving is $r_{s, nb}$. Savers must receive the interest rate $r_{s, nb}$ after paying trading costs to be induced to save the amount $q_{nb}$. The marginal rate of return at the quantity of investment of $q_{nb}$ is $r_{i, nb}$. Savers lend the amount $q_{nb}$ to borrowers. Borrowers pay the interest rate $r_{i, nb}$ which covers the unit costs of trading $\gamma_{i, nb} + \gamma_{s, nb}$ and the required return to savers of $r_{s, nb}$. The spread between the borrowing rate and the lending rate is the cost of trading.

$$r_{i, nb} - r_{s, nb} = \gamma_{i, nb} + \gamma_{s, nb}$$  \hspace{1cm} (1)

Trading costs divide the perfect market-trading surplus into four pieces. First, at the borrowing rate $r_{i, nb}$ fewer investments have positive net present values. Investors' surplus is reduced to the area of the triangle that lies above the interest rate $r_{i, nb}$ and below the perfect market investment schedule. The vertical distance $\gamma_{i, nb}$ is the cost of effecting the loan and $r_{i, nb}$ is the cost of the borrowed money.

Second, at the lending rate $r_{s, nb}$ the reward to saving is lower. Savers' surplus is the area above the perfect market saving schedule and below the interest rate $r_{s, nb}$.

Third, the area of the rectangle with height $r_{i, nb} - r_{s, nb}$ and width $q_{nb}$ is the total cost of trading.

Fourth, the area of the triangle that lies above the perfect market saving schedule and below the perfect market investment schedule and between $q_{nb}$ and $q_{pm}$ is the lost surplus due to trading costs. It is the sum of the projects that no longer have positive net present values, and the benefits that are lost to savers who have high rates of time preference.

2.3 A financial market with trading costs and banks.

If a bank or other financial intermediary can profitably reduce trading costs, it can pay a higher interest rate to savers and charge a lower interest rate to borrowers. These attract savers and investors and increase its value as it adds value.
If banks' incurs unit information costs of $\gamma_{ib}$ and unit transaction costs of $\gamma_{sb}$, they can offer savers the interest rate $r_{sb}$ and charge borrowers the rate $r_{ib}$. Savers decide how much to save by comparing the interest rate they receive net of all trading costs to their rate of time preference. The increase in the interest rate from $r_{snb}$ to $r_{sb}$ induces them to increase their saving up to the quantity $q_b$. To loan the amount $q_b$, banks post a borrowing rate of $r_{ib}$. This rate consists of the banks' cost of money $r_{sb}$ plus the banks' cost of trading which is $\gamma_{sb} + \gamma_{ib}$. Borrowers compare their borrowing rate to the marginal rates of return on their investments and borrow the amount $q_b$. Thus, borrowing equals lending even though the borrowing rate is greater than the lending rate.

Investors' surplus is the area of the triangle above $r_{ib}$ and below the marginal rate of return schedule. Savers' surplus is the area of the triangle below $r_{sb}$ and above the rate of time preference schedule. If banks are low cost suppliers of trading services, both investors' and savers' surpluses increase as compared to the case with trading costs and no banks. The banks' spread is $r_{ib} - r_{sb}$. The product of this spread times the amount of trading $q_b$ is the total cost of trading services supplied by banks. The remaining lost trading surplus is the triangle above the rate of time preference schedule, below the marginal rate of return schedule, and between $q_b$ and $q_{pm}$. Efficient banks minimize this triangle.

From the investors' and savers' views, who are the demanders of financial services, the spread between the banks' lending and borrowing rates is the sum of unit information and transaction costs.

$$r_{ib} - r_{sb} = \gamma_{ib} + \gamma_{sb}. \quad (2)$$

From the banks' views, the suppliers of financial services, the spread is unit operating costs which is the sum of unit labor and capital costs, plus expected loan losses per dollar of loans, plus any possible profit. Let $w$ be the wage rate paid by the bank, $N$ the number of bank employees, $k_b$ the bank's weighted average cost of capital, and $K_b$ the amount of debt and equity capital used to finance the bank. The debt excludes deposits. $LL$ stands for loan losses, and $\pi$ is profits. For the bank to continue to exist its net revenues must at least cover its costs. This requires

$$\gamma_{ib} + \gamma_{sb} \geq w \frac{N}{q_b} + k_b \frac{K_b}{q_b} + \frac{E(LL)}{q_b} + \frac{\pi}{q_b}. \quad (3)$$

The bank's bid/ask spread of $\gamma_{ib} + \gamma_{sb}$ must cover the sum of its average variable costs, $wN / q_b$, and average fixed costs, $k_bK_b / q_b$, expected loan losses as a percent of loans, $E(LL) / q_b$, and unit profits $\pi / q_b$.

Banks exist because they reduce trading costs through economies of scale, scope, and specialization. These economies work either by increasing the average product of
labor, $q_b / N$, or the average product of capital, $q_b / K_b$, or by reducing expected loan losses. Economies of scale exist when banks can spread fixed costs over a large volume or value of transactions, or when they can reduce risks by pooling the risks of many different loans. Banks have made large investments in computing and communications equipment. Most U.S. banks are large enough that they have exhausted possible economies of scale. While banks may not differ from each other in their economies of scale, they have large economies of scale compared to individuals. Economies of scope exist when the joint production of two goods or services is less costly than separate production. Banks appear to have economies of scope in many of the financial services they provide that deal with information. They can reuse information gathered in providing one service to provide other services without having to search anew for the information. Economies of specialization are due to learning by doing, training and education. Often, a well-trained, experienced person can do a task much faster and more accurately than an inexperienced, untrained person. Even though the experienced person may earn more, he or she may be sufficiently more productive to be the lowest cost producer of the service.

3.0 Banks' Supply of Intermediation Services

There are two dimensions to the services that banks provide to savers and investors. One dimension distinguishes between the origination and portfolio management activities. The other distinguishes between information and transaction services (see Table 1). To understand these services we analyze the steps prudent investors and savers take before and during the process of transferring money from savers to borrowers and back to savers.

3.1 Information services. Information about the payoffs to investment projects, the skills and effort needed to implement the projects, costs of capital, and the identities of savers and investors is costly to produce and distribute. We first discuss the role of financial intermediaries in reducing information costs.

Search for counterparties. Borrowers do not know who or where the lenders are, and lenders do not know which enterprises seek financing. Banks can eliminate direct search between savers and investors by providing either the broker function - they conduct the search and bring the parties together but do not take a position in the transaction, or the dealer function - they bring borrower and lender together and take a position in the transaction. U.S. commercial banks provide the dealer function for reasons discussed below in connection with independent valuation, price discovery, and monitoring.

The cost to a bank of supplying dealer services is the sum of a broker's costs of finding borrowers and savers, plus the costs of originating and managing a portfolio of assets and liabilities. Brokerage costs include a fixed component for the physical facilities used as the central meeting place for conducting business, and a variable component for the costs of loan officers and deposit takers. Banks spread the fixed costs over large numbers of transactions. They reduce variable costs by becoming knowledgeable about who the savers and investors are. Gains in knowledge increase the marginal and average
products of labor. This reduces average variable costs and spreads, increases volume and reduces average fixed costs.

Project valuation. Lenders recognize that borrowers have incentives to overstate their ability to repay and understate the risk of their investment. To overcome adverse selection, either the lender or a third party provides an independent valuation of the borrower's ability to repay. We use the net present value model to identify the elements of investment valuation. The components of the model are the investment, \( I \), the expected future cash flows \( E(C_t) \) for each of the \( T \) periods in the life of the project, and the set of discount rates \( k_t \) for each cash flow. The net present value equation is

\[
NPV = -I + \sum_{t=1}^{T} \frac{E(C_t)}{(1 + k_t)^t}. \tag{4}
\]

The valuator projects the future cash flows from the borrower's pro forma financial statements. At a minimum, a cash flow forecast depends on projected revenues, projected costs, and taxes. Projected revenues are the product of the total size of the output market times the enterprise's projected market share times the unit price of the product. Costs consist of variable costs, as recorded in cost of goods sold, and fixed costs, as recorded in overhead or administrative costs. There may be other allowable deductions from net revenues before the tax liability is computed. A simple cash flow statement includes the following items.

- Projected market size
- \( \times \) Projected market share
- \( \times \) Price per unit
  = Projected revenues
  - Cost of goods sold
  = Gross margin
  - Overhead expenses
  - Allowable non cash expenses
  = Taxable income
  - Taxes
  + Allowable non cash expenses
  = Projected cash flow

The bank must either make these projections itself or buy them from an agent who has an incentive to provide accurate projections. Banks have an advantage in independent valuation if a borrower is more willing to give the bank confidential information that may affect its market share because the bank keeps the information private. The bank signals to savers that it views the projections as accurate by taking an at risk position in the investment. Taking this position is what makes the bank a dealer and not a broker. If the bank did not take this position, savers would have less confidence in the bank and be unwilling to deposit their savings with it. Banks have an economy of specialization in project valuation as they gain information on market sizes, potential market shares, and project's costs of good sold and overhead costs. Banks use the information they produced in previous valuations as inputs to current and future valuations.
Project valuation shows in Figure 1 as the investment schedule. Every point on the investment schedule represents a project.

Price discovery. The cost of capital, the set of interest rates used to discount future cash flows to their present value, depends on expected real interest rates, expected rates of inflation, the risk of repayment, and the liquidity and maturity of the financial instrument issued to finance the investment. Efficient markets provide interest rates that correctly discount cash flows and provide accurate estimates of value. They induce the optimal aggregate quantity of saving and investment and the optimal distribution of investment across projects. Inefficient and fragmented markets do neither. They can produce interest rates that do not reflect aggregate amounts of saving and investment and do not correctly distinguish between the relative risks of investments. This can lead to over-investment in overvalued projects, underinvestment in undervalued projects, and non-optimal total amounts of saving and investment.

Banks that operate in inefficient financial markets must produce their own estimates of discount rates. Let \( r_f \) be the real risk free interest rate, \( E(r_{m}) \) the expected real rate of return on a well diversified portfolio of assets, \( \beta_j \) a measure of the contribution of investment \( j \) to the risk of a well diversified portfolio, \( E(\pi) \) the expected rate of inflation, and \( \gamma_j \) the marginal cost of supplying intermediation services to the \( jth \) borrower. A commonly used estimate of the cost of capital in western financial markets is the augmented capital asset pricing model given by

\[
k_j = r_f + \beta_j [E(r_m) - r_f] + E(\pi) + \gamma_j.
\]  

(5)

A bank that handles a large volume of loans and deposits can gain an advantage in price discovery in an inefficient market because it sees a sizable portion of the market supply and demand schedules for credit. Thus, it has information on what market clearing yields should be. Banks may have an information economy between project valuation and price discovery. Through project valuation banks estimate cash flows and gain evidence on the costs of capital for which projects have positive net present values. They become knowledgeable about the range of marginal rates of return across projects, and the amount of positive net present value investments at each internal rate of return.

In Figure 1, price discovery refers to the intersection of the investment and saving schedules. It gives the one interest rate at which saving equals investment.

Monitoring. After a borrower has received financing, he has an incentive to use the money in a way that benefits him, possibly to the detriment of the lender. To counter this moral hazard the loan contract specifies many actions that the borrower must perform to maintain the credit quality of the loan. Once the lender has delivered the money to the borrower, the lender must monitor the borrower's actions to reduce the chance that the borrower takes actions that transfer wealth from the lender to the borrower. Banks may have an economy in monitoring if the borrower keeps its payroll, accounts receivable,
and accounts payable accounts with the bank. This provides it with a low cost source of information how the borrower is managing the enterprise. Banks also have economies of specialization in monitoring since they learn what data, for example, financial ratios, best describes borrower behavior and the best way to analyze these data. If the cost of signaling is less than the cost of monitoring, banks have an economy of specialization since they incur monitoring cost just once and signal their continuing confidence in the project to depositors by maintaining their at-risk position in the project.

3.2 Transaction services. Financial contracts must be structured so they are mutually acceptable to both borrowers and lenders. In indirect financing, intermediaries make one set of contracts with borrowers and another set with different terms with lenders. These contracts provide the following services.

- **Denomination intermediation.** Investment projects usually require a larger amount of credit than one saver can supply. A bank makes few large loans and breaks the financing into small denominations so pieces of it can be sold to each of many limited wealth savers. There may be an economy between search and denomination intermediation. Since banks are the market maker, both borrowers and savers deal with them and they have the necessary large numbers of small savings accounts to aggregate into large loans.

- **Maturity intermediation.** Borrowers usually have a continuing demand for credit since they are going concerns, while lenders want to have their money returned so they can spend it on consumption goods in the future. Financial intermediaries must be able to supply loans with maturities that best serve borrowers, and saving instruments with the maturities that best serve savers. Banks are able to fund long-term, but not necessarily fixed-rate, loans with short-term deposits because the withdrawal behavior of their pool of deposits is more predictable than the behavior of any single depositors. This is because absent an economy-wide disturbance, individual depositors deposit and withdraw money at times and in amounts that are unique to each depositor. Additions to and withdrawals from deposit accounts tend to offset each other leaving smaller and more predictable changes in aggregate deposits.

- **Diversification.** A portfolio of loans, bonds, and stocks issued by a wide variety of enterprises in many different industries and regions, has less risk than a portfolio concentrated in one region or industry. Banks have economies of specialization in diversification both through learning by doing and by spreading fixed costs over many loans and over large size loans. Portfolio management includes estimating efficient amounts of each of the assets to be held in the portfolio from the vector of expected rates of return and their variance/covariance matrix. Banks gain expertise in generating estimated returns and their risk, and in using these data to construct efficient frontiers.

Banks may have economies between their search, denomination intermediation, and diversification activities. Being the center of search exposes the bank to a pool of loans, which begets diversification, and to a pool of depositors each of whom buys a small share of the diversified portfolio of loans. Banks have a conflict between diversification and project valuation. To gain expertise in project valuation banks evaluate loans in the same
region or in the same industry. But this concentrates their loan portfolio in a few assets and reduces its risk diversification. Loan sales among banks allow them to keep the economies from search and diversification. Both these economies serve to reduce the bid/ask spread.

**Hedging.** Individuals and enterprises differ in their desires and abilities to hold and manage risk. Costs of capital are lowest in financial markets where the risk that remains after diversification is transferred to those most willing and able to bear it. Risk management using forward contracts, futures, options, swaps and securitization require well-developed secondary markets and expertise in measuring risk exposures. In inefficient financial markets where these derivative instruments are not supplied, banks provide hedging services by offering loans with the durations that best serve borrowers and deposits with the durations that best serve savers. This transfers their customers' interest rate risk to themselves. Banks manage their resulting exposure to unexpected changes in the level or term structure of interest rates by holding sufficient capital to absorb losses during periods when the yield curve moves up sharply or is inverted. This additional capital is costly and must be part of the bank's spread.

**Liquidity.** In well functioning secondary financial markets, trading occurs in response to news about the prospects of an enterprise or the economy, to changes in peoples' plans, and to rebalancing to maintain well diversified portfolios. Liquidity is the ability to conduct a trade in a short period of time at a price close to the currently quoted price. Liquidity depends on the price elasticity of demand. In intermediated markets banks are low cost suppliers of liquidity to savers and borrowers because of economies of scope and specialization. Scope economies may exist between liquidity and the bank's search/valuation/monitoring/denomination/maturity activities. The bank's search activities give it a large base of diversified borrowers and savers whose aggregate transactions are more predictable than their individual transactions. In addition through their valuation and monitoring activities banks can learn to anticipate the liquidity demands of their borrowers. This leaves deposit flows as the less predictable component of the bank's net cash flow. However, the maturity of the bank's deposits gives it some clues to depositors' likely liquidity demands. In addition because banks have many deposits of small denominations, it gains evidence through time on the flows into and out of its deposits. This economy of specialization increases the bank's ability to predict its deposit flows.

**Payments system.** Both primary and secondary financial transactions entail frequent payments of large sums of money. Borrowers and lenders must be confident that the money they part with will reach the intended counterparty. Banks have large volumes of transactions that reduce their average fixed costs of supplying a payment system. Borrowers and depositors have confidence in the payment system because the bank has an at risk position in the system.
Figure 1. Effects of Trading Costs on a Financial Market

Quantities of Investment and Saving
Table 1. Services Supplied by Financial Intermediaries

<table>
<thead>
<tr>
<th>Stage of financial intermediation</th>
<th>Reduce Information Costs</th>
<th>Reduce Transaction Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Originate financial assets</td>
<td>Reduce search costs of bringing borrowers and savers together</td>
<td>Denomination intermediation</td>
</tr>
<tr>
<td></td>
<td>Independent valuation to reduce adverse selection</td>
<td>Maturity intermediation</td>
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<tr>
<td></td>
<td>Price discovery</td>
<td>Provide payments system</td>
</tr>
<tr>
<td>Own financial assets</td>
<td>Monitoring to reduce moral hazard</td>
<td>Diversify risk</td>
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<td></td>
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<td>Hedge risk</td>
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<td>Liquidity including providing payments system</td>
</tr>
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</table>

Summary.
Banks exist to reduce the information and transaction costs associated with credit transactions.
Information and transaction costs are the labor, capital and overhead costs of providing the ten financial services, which are listed in the preceding table.
Banks reduce costs via economies of scale, specialization, and scope.
Economies → average costs ↓ → spread ↓ → borrowing rate ↓ → borrowers’ surplus ↑, and spread ↓ → deposit rate ↑ → savers surplus ↑.
OVERVIEW

Our approach to analyzing past performance is to compare the bank’s actual return to the return that investors expected to receive when they invested in the bank. Investors are the last in line to get paid by the bank. The bank uses its revenue to pay its depositors, workers and overhead expenses, and to buy or rent capital equipment and facilities. Whatever is left over, the bank pays to its investors either in dividends or retained earnings. By examining payments to shareholders we are sure that we have not omitted any party with a higher-priority claim on the bank’s revenues.

The bank’s actual return is its net cash flow that is available for distribution to its shareholders. Investors’ expected return is the bank’s cost of equity capital, which is the rate of return investors could have received if they had invested in other assets with the same risk as the bank, times the bank’s net worth. The bank’s net worth is the historical value, measured as the book value, of investors’ investment in the bank. Net worth is the sum of paid-in-capital plus retained earnings.

Economic profit is the difference between what investors actually receive at the end of the performance period minus what they expected to receive when they invested in the bank. It is given by the formula.

\[ \pi_t = (ROE_t - k_t) \cdot \bar{NW}_t \]

\( \pi_t \) is economic profit for period \( t \), observed at the end of the period

\( ROE_t \) is the bank’s return on equity for period \( t \), observed at the end of the period

\( k_t \) is the bank’s equity cost of capital at the start of period \( t \)

\( \bar{NW}_t \) is the book value of the bank’s average net worth over the period.

COST OF CAPITAL

The bank’s cost of capital is \(^2\)

---

\(^1\) This worksheet is based on Dennis Uyemura, Charles Kantor, and Justin Pettit, “EVA\(^R\) for Banks: Value Creation, Risk Management, and Profitability Measurement,” *Journal of Applied Corporate Finance*, Summer 1996, 9:2, pages 94-113.
\[ k_t = R_{F,t} + \beta_{m,t} \lambda_{m,t} + \beta_{i,t} \lambda_{i,t} \]

- \( R_{F,t} \) is the risk-free interest rate for period \( t \) known at the start of period \( t \)
- \( \beta_{m,t} \) is the bank’s systematic exposure to market risk. We have to estimate this using data for the bank.
- \( \lambda_{m,t} \) is the market risk premium for systematic risk. It is sometimes called the equity premium. We can approximate it as the difference between the average annual return on the S&P 500 minus the average annual return on long-term U.S. treasury bonds.
- \( \beta_{i,t} \) is the bank’s exposure to interest rate risk. We have to estimate this using data for the bank.
- \( \lambda_{i,t} \) is the market risk premium for interest rate risk. We can approximate this as the difference between the yield on a U.S. Treasury bill with one year to maturity minus the yield on a U.S. Treasury bill with 6 months to maturity.

---

DUPONT DECOMPOSITION OF RETURN ON EQUITY

We can use basic models of a bank’s balance sheet and income statement to develop an understanding of the factors that affect a bank’s return on equity. The simplest balance sheet has assets financed by deposits and net worth.

<table>
<thead>
<tr>
<th>Basic Elements of a Bank’s Balance Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Basic Elements of a Bank’s Income Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Revenue (IR)</td>
</tr>
<tr>
<td>- Interest Expense (IE)</td>
</tr>
<tr>
<td>= Net Interest Income (NII)</td>
</tr>
<tr>
<td>+ Non-interest Revenue (NIR)</td>
</tr>
<tr>
<td>- Non-interest Expense (NIE)</td>
</tr>
<tr>
<td>- Loan Loss Provisions (LLP)</td>
</tr>
<tr>
<td>- Taxes</td>
</tr>
<tr>
<td>= Net Income (NI)</td>
</tr>
</tbody>
</table>

A standard measure of the ROE is the ratio of the bank’s net income, NI, for a period of time divided by its average net worth, $\bar{NW}$, over the period.

$$ROE = \frac{NI}{\bar{NW}}.$$  

The ROE links the income statement, which reports performance for a period of time, to the balance sheet, which shows the bank’s assets, liabilities and net worth at a point in time.

The ROE can be decomposed into three terms

$$ROE = \frac{NI}{R} \cdot \frac{R}{A} \cdot \frac{A}{\bar{NW}}.$$
The first term is the profit margin, $NI/R$, the ratio of net income to revenue. A bank’s two sources of revenue are interest revenue, $IR$, and non-interest revenue, $NIR$, which is what we often call fee income. From the income statement we can see that the bank’s net income differs from its revenue by all the costs the bank incurs. These are interest expenses, $IE$, and non-interest expenses, $NIE$, which include employee costs, overhead costs, loan losses, and taxes. Banks that are good at controlling their costs have high profit margins. Banks that are poor at controlling their costs have low profit margins. The profit margin is a measure of the skill of the bank in controlling its costs relative to its revenue. Note that the profit margin contains only income statement items.

The second term of the ROE is asset utilization, $R/\bar{A}$, the ratio of revenue to average assets. The bank’s assets generate its interest revenue and some of its non-interest revenue. If the bank chooses to make risky loans it will be able to charge relatively high interest rates and its asset utilization will be high. If it chooses to make relatively low-risk loans it will have low interest revenue and low asset utilization. Thus, asset utilization is a measure of the risk of the bank’s assets. Note that asset utilization links the balance sheet to the income statement.

The third term of the ROE is a leverage ratio, $\bar{A}/\bar{NW}$, the ratio of average assets to average net worth. The bank funds its assets with net worth and deposits. The greater its leverage the more assets it can have for any amount of net worth. The leverage ratio contains terms from the balance sheet.

Using the income statement we can rewrite the bank’s ROE as

$$ROE = \frac{IR - IE + NIR - NIE}{IR + NIR} \frac{IR + NIR}{A} \frac{\bar{A}}{\bar{NW}}.$$ 

THE EXPANDED ECONOMIC PROFIT EQUATION

We substitute the expanded ROE equation and the expanded cost of capital equation into the economic profit equation to get the equation that we use to evaluate a bank’s past financial performance. It is

$$\pi = \left( \frac{IR - IE + NIR - NIE}{IR + NIR} \frac{IR + NIR}{A} \frac{\bar{A}}{\bar{NW}} - r_f - \beta_m \lambda_m - \beta_i \lambda_i \right) \bar{NW}.$$
## APPLICATION OF THE PROCEDURE TO WAMU

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>REAL ESTATE LOANS</td>
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<td>16,330,000</td>
<td>17,620,000</td>
<td>18,690,000</td>
<td>18,192,418</td>
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<td>COMMERCIAL LOANS</td>
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<td>1,197,000</td>
<td>946,000</td>
<td>878,000</td>
<td>657,447</td>
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<td>-7.69</td>
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<tr>
<td>INDIVIDUAL LOANS</td>
<td>587,000</td>
<td>732,000</td>
<td>881,000</td>
<td>987,000</td>
<td>1,047,412</td>
<td>-3.45</td>
<td>-9.81</td>
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<tr>
<td>AGRICULTURAL LOANS</td>
<td>278,000</td>
<td>247,000</td>
<td>366,000</td>
<td>369,000</td>
<td>330,595</td>
<td>-11.46</td>
<td>-19.88</td>
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</tbody>
</table>

## BALANCE SHEET - ASSETS, LIABILITIES AND CAPITAL ($000)

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>CHARTER # NA</td>
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<tr>
<td>BALANCE SHEET - ASSETS, LIABILITIES AND CAPITAL ($000)</td>
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<tr>
<td>CHARTER # NA</td>
<td>COUNTY: KING</td>
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</tr>
</tbody>
</table>
**Washington Mutual Inc.** is a financial services company serving small- to mid-sized businesses. The Company accepts deposits from the general public, originates, purchases, services and sells home loans, makes consumer loans and commercial real estate loans (primarily loans secured by multi-family properties) and is engaged in certain commercial banking activities, such as providing credit facilities and cash management and deposit services. Washington Mutual originates, purchases from correspondents, sells and services loans to higher-risk borrowers through its specialty mortgage finance program. The Company also markets annuities and other insurance products, offers full-service securities brokerage services and acts as the investment advisor to, and the distributor of, mutual funds. Washington Mutual has a concentration of operations in California. The Company has three major operating segments: banking and financial services, home loans and insurance services and specialty finance.
Washington Mutual’s excess return, the difference between the return to its equity investors less the return they expected to receive, has been positive but variable for the years 2000 - 2003. In 2003 Washington Mutual earned an excess return that is 21 percent higher than the return that investors could have received on alternative investments of equal risk.

Washington Mutual’s excess return increased by 10 percent from 2001 to 2002 primarily because its return on equity increased 9 percent. Abetting this increase is a 1 percent decrease in its cost of capital, which occurred because the risk-free rate decreased and Washington Mutual’s beta decreased.

Washington Mutual’s return on equity increased primarily because its margin increased 9 percent. Its margin increased primarily because its noninterest revenue ratio increased 10 percent. Its net interest income ratio decreased 4 percent and its efficiency ratio decreased 2 percent.

Washington Mutual’s market risk, as measured by its beta, has declined from 1.17 in 1999 to 0.3 in 2004. This reduced the risk premium that investors require to own its shares. This decline in its beta mirrors the decline in its leverage. Because it has positive earnings not all of which it pays out as dividends, Washington Mutual’s net worth continues to increase. This net worth increases coupled with the decrease in its assets caused Washington Mutual’s leverage to decrease. Washington Mutual was a less-risky bank in 2003 than in 2000.
Estimate WAMU’s cost of capital

- $R_{F,t}$: one year constant maturity Treasury rate from FRED on the last trading day of the previous year.
- $\lambda_{m,t}$: difference between average annual return on the S&P500 index and the long-term government bond rate. The average annual return on the S&P 500 is 12.2% for 1926-2002 and the long-term U.S. government bond rate is 5.9%. Thus the average equity premium is 6.3%.

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<tbody>
<tr>
<td>IR</td>
<td>1414</td>
<td>1611</td>
<td>2310</td>
<td>2534</td>
<td>2349</td>
</tr>
<tr>
<td>E</td>
<td>447</td>
<td>563</td>
<td>1201</td>
<td>1666</td>
<td>1398</td>
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<tr>
<td>NIR</td>
<td>561</td>
<td>356</td>
<td>473</td>
<td>329</td>
<td>405</td>
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<tr>
<td>OE</td>
<td>1041</td>
<td>1081</td>
<td>1036</td>
<td>951</td>
<td>998</td>
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<tr>
<td>NIAT</td>
<td>487</td>
<td>323</td>
<td>546</td>
<td>246</td>
<td>358</td>
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<td>Assets</td>
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<td>26723</td>
<td>31639</td>
<td>34715</td>
<td>35036</td>
</tr>
<tr>
<td>NW</td>
<td>1834</td>
<td>2242</td>
<td>2043</td>
<td>2023</td>
<td>1806</td>
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<tr>
<td>Beta</td>
<td>0.3</td>
<td>0.362</td>
<td>0.61</td>
<td>0.78</td>
<td>1.17</td>
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<td>ROE</td>
<td>0.24</td>
<td>0.15</td>
<td>0.27</td>
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<tr>
<td>Rf</td>
<td>0.0132</td>
<td>0.0217</td>
<td>0.0532</td>
<td>0.0598</td>
<td>0.0453</td>
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<tr>
<td>Equity premium</td>
<td>0.063</td>
<td>0.063</td>
<td>0.063</td>
<td>0.063</td>
<td>0.063</td>
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<tr>
<td>Calculations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>0.03</td>
<td>0.04</td>
<td>0.09</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>ROE-k</td>
<td>0.21</td>
<td>0.11</td>
<td>0.18</td>
<td>0.02</td>
<td></td>
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<tr>
<td>Profit</td>
<td>422</td>
<td>228</td>
<td>360</td>
<td>37</td>
<td></td>
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<tr>
<td>Margin</td>
<td>0.25</td>
<td>0.16</td>
<td>0.20</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>Utilization</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Leverage</td>
<td>13.75</td>
<td>13.62</td>
<td>16.32</td>
<td>18.22</td>
<td></td>
</tr>
<tr>
<td>Revenue</td>
<td>1975</td>
<td>1967</td>
<td>2783</td>
<td>2863</td>
<td>2754</td>
</tr>
<tr>
<td>NII/R</td>
<td>0.49</td>
<td>0.53</td>
<td>0.40</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>NIR/R</td>
<td>0.28</td>
<td>0.18</td>
<td>0.17</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>OE/R</td>
<td>0.53</td>
<td>0.55</td>
<td>0.37</td>
<td>0.33</td>
<td>0.36</td>
</tr>
</tbody>
</table>

WAMU 2002 to 2003

The excess return increased 0.10 because ROE increased 0.09 and the cost of capital decreased 0.01.

The cost of capital decreased because the risk free rate decreased about 1%.

ROE increased because the margin increased 0.09.

The margin increased because NIR/R increased 0.10. NII/R decreased 0.04 and OE/R decreased 0.02.

DIMENSIONAL FUND ADVISORS INC.
Jan 1926 - Dec 2002

<table>
<thead>
<tr>
<th>Long Term</th>
<th>Govt Bonds</th>
<th>S&amp;P 500</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized Return</td>
<td>5.47</td>
<td>10.21</td>
<td></td>
</tr>
<tr>
<td>Annual Standard Deviation</td>
<td>9.41</td>
<td>20.49</td>
<td></td>
</tr>
<tr>
<td>Average Annual Return</td>
<td>5.86</td>
<td>12.2</td>
<td></td>
</tr>
</tbody>
</table>
Campbell and Kracaw’s Theory of Financial Intermediation

The economic profit equation for a bank.

\[ \Pi = \left( \frac{IR - IE - OH - LLP - EC - TX}{NW} - k \right)^{NW} \]

\( \Pi \): economic profit. The difference between the realized return on invested capital and the return investors require to own the bank’s stock.

\( IR \): interest revenue
\( IE \): interest expense
\( OH \): overhead expenses
\( LLP \): loan loss provisions
\( EC \): employee costs
\( TX \): taxes
\( NW \): net worth, which is paid-in capital
\( k \): investors’ required rate of return. This is the rate of return they require to willingly hold the bank’s stock.

1) Valuation is one of the ten financial services a bank offers to borrowers and savers. Valuation means determining the present value of the project the borrower is financing with the bank loan. The bank hires skilled personnel (graduates of F423) and buys data and computers to value projects. These costs are components of the bank’s employee costs and overhead costs. As costs increase, ceteris paribus, the bank’s VA decreases. The return to the bank is a reduction in loan loss provisions as valuation hopefully reduces the number of loans the bank makes to borrowers who will default. The challenge to the bank is to have loan loss provisions decrease more than operating costs increase.

2) Campbell and Kracaw use game theory to develop the conditions under which it is likely that the bank’s loan losses decrease more than their operating costs increase. At the end of today’s class you should be able to list and explain the conditions that increase the chance that the bank provides value-adding valuation services.
Situation 1

Assumptions:

A1. There are \( N_A \) firms of type A with true value \( V_A \), and \( N_B \) firms of type B with true value \( V_B \), \( V_A > V_B \).

A2. Investors only know the average value, \( \overline{V} \), of all firms. \[ \overline{V} = \frac{N_A \times V_A + N_B \times V_B}{N_A + N_B} \]

A3. There are no gains from diversifying across A and B.

A4. Market trading discloses bids of all traders.

Implications

1. What is the market value of an A-type firm? \( \overline{V} \)

2. Are they over or under valued?
   The market undervalues them, \( \overline{V} < V_A \).

3. What is the market value of a B-type firm? \( \overline{V} \)

4. Are they over or under valued?
   The market overvalues them, \( \overline{V} > V_B \).

5. What do owners of the A firms do?
   Announce that they are type A firms.

6. What do owners of the B firms do?
   Announce that they are type A firms.

7. Is investment efficiently allocated between A and B?
   No, investors cannot tell the type of any firm. They invest too much in B-type firms and too little in A-type firms. The market fails to allocate investment optimally because the B-type firms are dishonest.

8. Is there a way an intermediary can create value?
   Yes, if they can identify which firms are type A and which firms are type B. This is the financial service known as project valuation.
Situation 2

Assumptions:

A1. There are \( N_A \) firms of type A with true value \( V_A \), \( N_B \) firms of type B with true value \( V_B \), \( V_A > V_B \).

A2. Investors only know the average of all firms.

A3. There are no gains from diversifying across A and B.

A4. Market trading discloses bids of all traders.

A5. Investors can obtain information on the true values of all firms at a cost of \( C_i \) for saver I.

Implications

1. What is the return from investing in information?

   Is it \( W_i \frac{V_A - \bar{V}}{V} - C_i \) or is it \( W_i \frac{V_A - V_A}{V_A} - C_i \) where \( W_i \) is the wealth of investor \( i \)?

2. Will anyone be willing to produce information?

   No. The return from producing information is negative because of assumption 4, which is the assumption of perfect price discovery. Information in this situation is a public good. It is non-exclusive, once it is produced it is freely available to all, and it is non-rival, once it is produced one person using it does not preclude another person from using it.

3. Is investment efficiently allocated between A and B?

   No. Because investors are unable to identify which firms are of each type there is too much investment in the B-type firms and too little investment in the A-type firms. Investors cannot trust the B-type firms to identify themselves, and no investor will incur the costs of producing a public good.

4. Is there a way an intermediary can create value?

   Yes, if it can identify the type of each firm.
Situation 3

Assumptions:

A1. There are \( N_A \) firms of type A with true value \( V_A \), \( N_B \) firms of type B with true value \( V_B \), \( V_A > V_B \).

A2. Investors only know the average of all firms.

A3. There are no gains from diversifying across A and B.

A4. Market trading discloses bids of all traders.

A5. Investors can obtain information on the true values of all firms at a cost of \( C_i \) for saver I.

A6. Firms offer side-payments to investors to induce them to invest in information.

A7. There is a lowest-cost, monopoly information producer for whom \( C_i < C_j \).

Implications

1. What is the net return to producing information?

   The maximum amount that the A-type firms will collectively pay to have information produced about their true values is \( N_A \cdot (V_A - \bar{V}) \). The cost of producing information is \( C_i \). Therefore, the net return to producing information is \( N_A \cdot (V_A - \bar{V}) - C_i \).

2. What is the net return to not producing information?

   The B-type firms will pay the monopolist information producer up to \( N_B \cdot (\bar{V} - V_B) \) if he promises not to produce information. He has the same revenue but lower costs for a higher net return if he does not produce information.

3. Will information be produced?

   No, as is seen by comparing the net returns from producing and not producing information.

4. Is investment efficiently allocated between A and B?

   No, because investors cannot identify the type of each firm.

5. Is there a way an intermediary can create value?

   Yes, if it can identify the true type of each firm.
Situation 4

Assumptions:

A1. There are $N_A$ firms of type A with true value $V_A$, $N_B$ firms of type B with true value $V_B$, $V_A > V_B$.

A2. Investors only know the average of all firms.

A3. There are no gains from diversifying across A and B.

A4. Market trading discloses bids of all traders.

A5. Investors can obtain information on the true values of all firms at a cost of $C_i$ for saver I.

A6. Firms offer side-payments to investors to induce them to invest in information.

A7'. There are competitive, honest information producers, $C_i = C_j$.

Implications

1. Will information be produced?

   Yes. The A-type firms randomly select one of the competitive information producers and offer to pay him to produce information. The B-type firms pay him a bit more than $N_A \cdot (V_A - \bar{V}) - C_i$ not to produce information. The potential information producer chooses not to produce information. The A-type firms then randomly select a second competitive information producer and offer to pay him to produce information. The B-type firms have already spent most of their money on the first information producer and cannot make a viable counteroffer. Thus, the second chosen information producer produces information on the true value of every firm.

2. Is investment efficiently allocated between A and B?

   Yes, because investors know the true value of each firm.

3. Is there a way an intermediary can create value?

   The second information producer is the intermediary. He produces project valuation. He has added value by providing information that correctly allocates investment to the two types of firms. The market requires competitive, honest, low-cost information producers.
Situation 5

Assumptions:

A1. There are \( N_A \) firms of type A with true value \( V_A \), \( N_B \) firms of type B with true value \( V_B \), \( V_A > V_B \).

A2. Investors only know the average of all firms.

A3. There are no gains from diversifying across A and B.

A4. Market trading discloses bids of all traders.

A5. Investors can obtain information on the true values of all firms at a cost of \( C_i \) for saver I.

A6. Firms offer side-payments to investors to induce them to invest in information.

A7”. There are competitive, dishonest information producers.

Implications

1. Is information produced?

   No. After the same sequence of offer and counter-offer as in the previous case, the second information producer takes the money from the A-type firms, ostensibly to produce information, but instead randomly selects firms and declares that they are type A firms. In a rational expectations economy, the A-type firms would have anticipated this action and not have made an offer to anyone to produce information.

2. Is investment efficiently allocated between A and B?

   No, because information is not produced.

3. Is there a way an intermediary can create value?

   Yes, if it is a low cost, honest information producer.
Situation 6

Assumptions:

A1. There are \( N_A \) firms of type A with true value \( V_A \), \( N_B \) firms of type B with true value \( V_B \), \( V_A > V_B \).

A2. Investors only know the average of all firms.

A3. There are no gains from diversifying across A and B.

A4. Market trading discloses bids of all traders.

A5. Investors can obtain information on the true values of all firms at a cost of \( C_I \) for saver I.

A6. Firms offer side-payments to investors to induce them to invest in information.

A7*. There are competitive, dishonest information producers.

A8. Information producers must invest in their information.

Implications

1. Will information be produced?

   The \( A \)-type firms require that the information producer invest his own money in the firms that he declares to be type A. The information producer, who we assume is potentially dishonest, compares the net return from being honest and dishonest.

   The net return from being honest is \( N_A \* (V_A - \overline{V}) - C_I \).

   The net return from being dishonest is \( N_A \* (V_A - \overline{V}) + W_i \* \frac{\overline{V} - V_A}{V_A} \).

   The information producer will be honest if \( C_I < W_i \* \frac{V_A - \overline{V}}{V_A} \).

2. Is investment efficiently allocated between A and B?

   It is if the cost of producing information is low compared to the loss from investing in overvalued firms.

3. Is there a way an intermediary can create value?

   Yes, if it has low costs of producing information, it increases the incentive to be honest instead of dishonest.
Situation 7

Assumptions:

A1. There are \( N_A \) firms of type A with true value \( V_A \), \( N_B \) firms of type B with true value \( V_B \), \( V_A > V_B \).

A2. Investors only know the average of all firms.

A3. There are no gains from diversifying across A and B.

A4. Market trading discloses bids of all traders.

A5. Investors can obtain information on the true values of all firms at a cost of \( C_i \) for saver I.

A6. Firms offer side-payments to investors to induce them to invest in information.

A7”. There are competitive, dishonest information producers.

A8. Information producers must invest in their information.

A9. Investors pool their wealth to form an intermediary that produces information and buys securities.

Implications

1. Will information be produced?

   *It is more likely to be produced. The cost comparison between honesty and dishonesty is now* \( C_i < \sum_{i=1}^{t} W_i \cdot \frac{V_A - V}{V_A} \). *As the amount of their own money the information producers invest in the firms they identify as type A increases, \( \sum_{i=1}^{t} W_i \), so does their cost of being dishonest.*

2. Is investment efficiently allocated between A and B?

   *It will be if the cost of dishonest exceeds the cost of honesty.*

3. Is there a way an intermediary can create value?

   *Yes, if it has low costs of producing information and a high at-risk position in the firms it is valuing. If the intermediary is a bank, it compares its additional operating costs to its additional loan losses.*
Study questions.

1. What is investors’ surplus?
2. How is investors’ surplus related to the interest rates banks charge on loans?
3. How is investors’ surplus related to the rates of return on investment projects?
4. Using the symbols $r_{i,b}$ for the interest rate a bank charges on loans, $ror_j$ for the rate of return on the $j^{th}$ investment project, and $A_j$ the amount of money a bank lends to the $j^{th}$ borrower, write a formula for investors’ surplus.
5. Which of the ten financial services affects investors’ surplus? Explain.
6. What is savers’ surplus?
7. How is savers’ surplus related to the interest rates banks pay on deposits?
8. Write a formula for savers’ surplus.
9. Using the symbols $EC$, $OH$, $LLP$, $Tx$, $k_e$, $A$, and $NW$, whose meanings you are supposed to know from lectures and the Course Notes, write a formula for a bank’s bid/ask spread.
11. Which of the ten financial services are asset transformation services? Explain.
12. Which of the ten financial services are broker services? Explain.
13. Which might be higher to the bank, the cost of making loans or the cost of taking deposits? Explain.
14. Which of the components of a bank’s bid/ask spread can the bank reduce via economies of specialization?
15. Which components of a bank’s bid/ask spread can the bank reduce via economies of scope?
16. Which of the components of a bank’s bid/ask spread can the bank reduce via economies of scale?
17. What is a bank’s efficiency ratio?
18. How is a bank’s bid/ask spread related to its efficiency ratio?
19. How is investors’ surplus related to a bank’s efficiency ratio?
20. How do the three economies affect a bank’s efficiency ratio?
21. How do the three economies affect a bank’s bid/ask spread?
22. How do the three economies affect investors’ surplus?
23. How might a bank’s profit margin be related to how well it controls its expenses?
24. How might a bank’s asset utilization ratio be related to risk and return?
25. Why might banks’ asset utilization ratios be similar?
26. How might a bank’s leverage ratio be related to risk and return?
27. In the presence of mispriced federal deposit insurance, why might banks’ leverage ratios be similar?
28. If a bank increases its leverage ratio, what happens to its ROE?
29. If a bank increases its leverage ratio, what might happen to its cost of equity?
30. If a bank increases its leverage ratio, what might happen to the difference between its ROE and its cost of equity? How if at all is this related to the MM hypothesis?
31. How might a bank obtain economies of scale?
32. How might a bank obtain economies of scope?
33. How might a bank obtain economies of specialization?
34. Do economies of scale, scope and specialization come free to the bank? If not, how should a value-maximizing bank decide on the optimal amount to invest to obtain the economies?

35. In the Campbell/Kracaw analysis, how does the cost of honesty relate to the bank’s financial statements?

36. Briefly explain how the three cost economies can affect a bank’s cost of being honest.

37. If honesty is costly, who pays the bank to be honest?

38. It has been suggested that small companies that borrow from banks get lower rates from other lenders, such as its trade creditors. How might this be?

39. How does the cost of dishonesty in the Campbell/Kracaw analysis relate to a bank’s financial statements?

40. How might we measure whether a bank has a high or low cost of dishonesty?

41. Give and explain a formula that shows how to decide whether a bank is honest or dishonest.

42. If all borrowers were honest, would we still have banks? Explain.
Diversification example.

Asset 1 has five uniformly distributed payoffs ranging from –2 to +2 in value. Its expected payoff is 0 and the standard deviation of its payoff is 1.58. Asset 2 has the same potential payoffs but they occur in different states of the economy from those on Asset 1. Asset 2 has the same expected payoff and standard deviation of payoff as does Asset 1. If an investor buys equal amounts of assets 1 and 2, his portfolio payoffs range from –1 to 1.5. The expected payoff on the portfolio is zero, but it has lower risk than the individual assets. This is an example of how diversification reduces risk. The amount of risk reduction depends on the correlation between the payoffs to the assets in the portfolio. For assets 1 and 2 the correlation is –0.3.

Diversification also works if the correlation between payoffs is positive. Asset 3 has five uniformly distributed payoffs ranging from –2 to +2. But, payoffs occur in different states of the economy from those on Asset 1. Buying equal amounts of Asset 1 and Asset 3 gives a portfolio whose payoff standard deviation is 1.37. This is less than the standard deviation of the individual assets, but not as great a reduction as for Portfolio 1,2. The reason the risk reduction is less for Portfolio 1,3 is because the payoffs have a higher correlation 0.5, as compared to –0.3 for Assets 1 and 2.

### Asset Payoffs

<table>
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<th>State of the Economy</th>
<th>Asset 1</th>
<th>Asset 2</th>
<th>Portfolio 1,2</th>
<th>Asset 3</th>
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<td>0</td>
<td>-1</td>
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</table>

| Average              | 0       | 0       | 0             | 0       | 0             |
| Std. Dev.            | 1.5811  | 1.5811  | 0.9354        | 1.5811  | 1.3693        |
| Variance             | 2.5     | 2.5     | 0.875         | 2.5     | 1.875         |
| Correlation          | Asset 1 | Asset 2 | Asset 1       | Asset 3 |
| Asset 1              | 1       | 1       | 1             |         |
| Asset 2              | -0.3    | 1       | 0.5           | 1       |

\[
\text{Variance} = 0.5^2 \text{V}_1 + 0.5^2 \text{V}_2 + 2 \times 0.5 \times 0.5 \times \text{cov}_{1,2}
\]

<table>
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<th>Covariance</th>
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<th>Asset 2</th>
<th>Asset 1</th>
<th>Asset 3</th>
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1. **Interest rate risk** is the risk of changes in the value of an asset due to unexpected changes in interest rates. An unexpected change in interest rates can change the market value of an asset, and it can change the future value of an asset.

The value equation says that the value of an asset is the present value of the future cash flow that the asset produces or has a claim to. The Gordon growth model is a simplified version of the value equation that is useful for our purposes. It assumes that the cash flow stream grows at a constant rate.

\[
V_0 \approx \frac{CF_1}{k - g} = \frac{(IR - IE + NIR - EC - OH - LLP - TX)}{k - g}.
\]

*\(V_0\): value at time 0
*\(CF_1\): cash flow for the period ending at time 1
*\(k\): cost of capital
*\(g\): growth rate of net income

\[
IR = \sum_{i=1}^{n} r_i A_i.
\]

*\(IR\): interest revenue
*\(r_i\): interest rate on asset \(i\)
*\(A_i\): book value of asset \(i\)

\[
IE = \sum_{j=1}^{m} r_j L_j.
\]
Interest rate risk, ©Alan C. Hess, May 1991

IE: interest expense

$r_j$: interest rate on liability $j$

$L_j$: book value of liability $j$

\[ k = r_f + \beta [E(r_m) - r_f] \]. Capital asset pricing model.

$k$: cost of capital (assumes 100% equity financing)

$r_f$: risk free interest rate

$\text{Beta}$: nondiversifiable risk of equity

$E(r_m)$: expected rate of return on the market portfolio

An unexpected change in interest rates affects a bank’s value in four ways.

1. $\Delta r_f \rightarrow \Delta IR \rightarrow \Delta V$ (+). **Reinvestment risk.** If interest rates rise, the bank receives higher interest revenues on its variable rate assets immediately, and higher interest revenue on its fixed rate assets when the bank reinvests its interest and principal receipts.

2. $\Delta r_f \rightarrow \Delta IE \rightarrow \Delta V$ (-). **Refinancing risk.** If interest rates rise, the bank makes higher interest payments on its variable rate liabilities immediately, and it receives pays higher interest on its fixed rate liabilities when they mature and are refinanced.

3. $\Delta r_f \rightarrow \Delta k \rightarrow \Delta V$ (-). **Market or price risk.** If interest rates rise, the bank’s cost of capital increases and its value decreases.

4. $\Delta r_f < 0 \rightarrow \Delta IR \rightarrow \Delta V$ (-). **Prepayment risk.** If interest rates decrease, mortgage borrowers can refinance at the lower interest rate. This reduces the bank’s interest revenue and value.

The **risk profile** diagram shows the relationship between unexpected changes in interest rates and unexpected changes in the value of an asset. The risk profile for a bond is the bond’s price-yield curve centered at the bond’s current yield. Note that a bond with a constant coupon has fixed cash payments consisting of coupons and principal. The price-yield curve is a plot of the price of the bond at each of several yields surrounding the
The price-yield curve is downward sloping and convex. This follows from the present value formula used to price the bond.

\[ V = \sum_{t=1}^{T} C_t (1 + r)^{-t} \]. The value of an asset is the present value of its future cash flows. In the case of a bond, the cash flows are the coupon and principal payments.

The following chart plots the prices of a bond with an 8% coupon, paid semiannually, and 20 years to maturity at various yields.
2. **Duration** is the percent change in the value of an asset due to a percent change in its discount factor. We use duration to approximate the slope of an asset's risk profile.

**DERIVE DURATION.**

Take the derivative of the value with respect to a change in yield.

\[
\frac{\partial V}{\partial r} = -\sum_{t=1}^{T} C_t (1 + r)^{-t-1}.
\]

Rewrite this as an elasticity to try to get it into a more recognizable form.

\[
\frac{\partial V (1 + r)}{\partial r V} = -\frac{\sum_{t=1}^{T} C_t (1 + r)^{-t}}{\sum_{t=1}^{T} C_t (1 + r)^{-t}}.
\]

Define duration as minus the interest elasticity of value.

\[
\frac{\partial V (1 + r)}{\partial r V} \equiv -D.
\]

**INTERPRET DURATION.**

Rearrange the terms on the right-hand-side of the duration formula.

\[
D = -\frac{\partial V (1 + r)}{\partial r V} = \sum_{t=1}^{T} \left[ \frac{C_t (1 + r)^{-t}}{V} \right] t.
\]

The numerator of the bracketed term is the present value of the \( t \)th cash flow. The denominator is the value of all cash flows. Thus, the term in brackets is the fraction of the value of the asset contributed by the \( t \)th cash flow. This is multiplied by the maturity of the \( t \)th cash flow. Thus, duration is the present value weighed average of the maturity of each cash flow.
Duration of a 20-yr Treasury bond with a coupon of 8 and 7/8 % and a yield of 5.66%.

Duration of a 15 year bond with a coupon of 8% and a yield of 5.62%.

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<th>Cash flow factor</th>
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Sum: 139.38
Duration: 11.21
Risk sensitivity: -1478.30
Price change for 0.1% yield change: -14783.02
Convexity

1. What are the pricing consequences of the convex relationship between price and yield?

2. How do we measure the convexity correction to duration?

3. Is convexity important?

**Price versus Yield**

![Graph showing the relationship between price and yield, with a formula for price and duration approximation.](image)
Duration measures the price risk of an asset. It is the elasticity of value with respect to a small change in the discount rate. The approximation is inaccurate for large changes in the discount rate. How inaccurate is duration for large changes in the discount rate? Here are some examples.

Example 1. 4-year maturity, $100 level cash flow, priced to yield 10%.

<table>
<thead>
<tr>
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<th>Cash Flow</th>
<th>Present Value @ 10%</th>
<th>Maturity times Present Value</th>
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</tr>
<tr>
<td>4</td>
<td>100</td>
<td>68.30</td>
<td>273.21</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>316.99</td>
<td>754.79</td>
</tr>
<tr>
<td>Duration</td>
<td></td>
<td></td>
<td>2.38</td>
</tr>
</tbody>
</table>

If yield falls from 10% to 9%, the exact value of the cash flow at 9% is $323.97. The value change for a 1% yield change is $6.98.

The duration approximation to the value change is

\[
\Delta V = -2.38 \times \frac{-0.01}{1.10} \times 316.99 = \$6.86
\]

The duration approximation error is $0.12. The percent approximation error is

\[
\frac{0.12}{316.99} \times 100 = 0.04\% \text{ or 4 basis points.}
\]

Example 2. A 4-year maturity, zero-coupon, $1000 bond priced to yield 10%. Its value @ 10% = $683.01. Its value @ 9% = $708.43. Its value change for a yield change of -1% is $25.42.

Duration approximation to value change. Duration is 4 years.
\[ \Delta V = -4 \cdot \frac{-0.01}{1.10} \cdot 683.01 = 24.84 \]

Duration approximation error is $0.58. Percent approximation error is
\[ \frac{0.58}{683.01} \cdot 100 = 0.085\% \text{ or 8.5 basis points.} \]

Example 3. 4-year maturity, 10% coupon bond, priced to yield 10%.

\[ D = \frac{316.99}{1000} \cdot 2.38\text{yrs} + \frac{683.01}{1000} \cdot 4\text{yrs} = 3.49\text{yrs} \]

Value @ 10\% equals $1000. Value @ 9\% equals $1032.40. Value change is $32.40.

Duration approximation to value change is
\[ \Delta V = -3.49 \cdot \frac{-0.01}{1.10} \cdot 1000 = 31.73 \]

Duration error is $0.67. Percent approximation error is 0.067\% or 6.7 basis points.
The duration approximation underestimates the value change for a decrease in the yield, and the approximate value change for the bond does not equal the sum of the approximate value changes for its components. These are because duration is a linear approximation to the nonlinear value to yield relationship.

A second order Taylor series expansion of value provides a closer approximation to a change in value caused by a change in yield.

\[
V(r_1) = V(r_0) + V'(r_0) \cdot (r_0 - r_1) + \frac{V''(r_0) \cdot (r_0 - r_1)^2}{2}.
\]

The value at the new interest rate \( r_1 \) is the value at the original interest rate \( r_0 \), plus the derivative of value with respect to yield evaluated at the original yield times the difference between the two yields, plus one-half of the second derivative of value evaluated at the original yield times the squared difference between the yields.

This can be rewritten as the sum of Duration plus the Convexity Correction.

\[
\Delta V = V'(r_0) \cdot \Delta r + \frac{V''(r_0) \cdot (\Delta r)^2}{2}.
\]

Notice that the convexity correction is always positive since both \( V''(r) \) and \((\Delta r)^2\) are positive. When yield declines, the convexity correction increases the price increase, and when yield increases, the convexity correction decreases the price decrease.
Interest rate risk, ©Alan C. Hess, May 1991

<table>
<thead>
<tr>
<th></th>
<th>Rates increase</th>
<th>Rates decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Duration</strong></td>
<td>Value decreases</td>
<td>Value increases</td>
</tr>
<tr>
<td><strong>Convexity</strong></td>
<td>Value increases</td>
<td>Value increase</td>
</tr>
</tbody>
</table>

Apply the convexity correction to our examples.

Example 1. \[ V = \sum_{t=1}^{4} 100 \cdot (1.1)^{-t} = $316.99. \]

\[ V' = \sum_{t=1}^{4} -t \cdot 100 \cdot (1.1)^{-t-1} = -$686.18. \]

\[ V'' = \sum_{t=1}^{4} -t \cdot (-t - 1) \cdot 100 \cdot (1.1)^{-t-2} = $2434.12. \]

\[ \Delta V = -$686.18 \cdot (0.01) + \frac{2434.12}{2} \cdot (0.01)^2 = $6.98 \]

\[ \Delta V = +$6.86 + $0.12 = $6.98 \]

Compare this approximation to the exact value change of $6.98.

Given the low-cost availability of high-speed computers, it is not necessary to use duration and convexity to estimate asset prices since they can be calculated exactly. Instead, duration and convexity are used to understand the relationships between expected rates of return and risk. For example, of two bonds with the same duration and yield, would you rather have a bond with a high convexity or a low convexity?
### Convexity Correction Example

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Cash Flow</th>
<th>Discount factor @ 10%</th>
<th>Present value PV</th>
<th>Maturity times PV times (1+r)^2-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$100</td>
<td>0.91</td>
<td>90.91</td>
<td>90.91</td>
</tr>
<tr>
<td>2</td>
<td>$100</td>
<td>0.83</td>
<td>82.64</td>
<td>165.29</td>
</tr>
<tr>
<td>3</td>
<td>$100</td>
<td>0.75</td>
<td>75.13</td>
<td>225.39</td>
</tr>
<tr>
<td>4</td>
<td>$100</td>
<td>0.68</td>
<td>68.30</td>
<td>273.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>316.99</td>
<td>745.11</td>
</tr>
</tbody>
</table>

Predicted price change formula from p. 39 of Course Pack:

\[
DP = -\left(\frac{DV}{1+r}\right)\cdot DR + \left(\frac{CX\cdot DR^2}{2}\right)
\]

Predicted price change stated in terms of Excel cell locations:

\[
DP = -E6\cdot C3\cdot DR + \left(\frac{F6\cdot DR^2}{2}\right)
\]

Predicted price change for 1% increase in yield:

<table>
<thead>
<tr>
<th>Due to duration</th>
<th>-6.86</th>
</tr>
</thead>
<tbody>
<tr>
<td>Due to convexity</td>
<td>0.12</td>
</tr>
<tr>
<td>Total</td>
<td>-6.74</td>
</tr>
</tbody>
</table>

Predicted price at 11% equals initial price plus predicted price change:

\[
310.25
\]

Actual price at 11%:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.90</td>
<td>90.09</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0.81</td>
<td>81.16</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>0.73</td>
<td>73.12</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>0.66</td>
<td>65.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>310.24</td>
</tr>
</tbody>
</table>
If you are risk averse, you would rather have the bond with the high convexity because convexity decreases price risk. Thus, two bonds with the same duration will not sell at the same yield unless they have the same convexity. The bond with the higher convexity will have a lower yield.

3. Value versus time diagram

To insure that you will have a given amount of money at a set future date, invest in a portfolio of assets that has a duration equal to your holding period, which is the length of time between now and the future date. To see this we look at the value versus time diagram. The future value is the current value $V_0$ compounded at the current yield to maturity, $r_0$. This assumes that the yield does not change between now and the future date.

$$V_t = V_0(1 + r_0)^t$$

Assume that you follow this dictum and invest your money in a portfolio of assets whose duration matches your holding period. Will you still have enough money in the future if immediately after you invest interest rates increase unexpectedly to $r_1$? At the new interest rate the current value of the portfolio falls to $V'_0$. The new future value is

$$V'_t = V'_0(1 + r_1)^t.$$

The new future value differs from the original future value in two ways. First, the new future value starts from a lower current value since $V'_0 < V_0$. Second, the new future value grows at a faster rate than the original future value grew because $r_1 > r_0$. Is it possible for the higher compounding rate to overcome the lower starting value? Yes, it can be shown that

$$V'_D = V''_D. \quad (3)$$

$$V'_t = V'_0 \rightarrow (1 + r_0)^tV'_0 = (1 + r_1)^tV'_0'. \quad \text{All the variables on the right-hand-side are known except for } t. \quad \text{This equation can be solved for } t \text{ to obtain } t = \log V'_0/V_0/\log(1 + r_0)/(1 + r_1). \quad \text{After some manipulation this can be shown to imply } t = D.$$
The value of the portfolio after D compounding periods is the same even if the interest rate changes immediately after the portfolio is assembled.

Here is an example to show the crossing of the two time-to-value lines. Start with an asset that pays $1 per period for 10 periods. Calculate its present value in each period at 4%, which I use as the initial interest rate, and at 5%, which I use as the new interest rate. At 4%, the present value of the asset is $8.11. At 5% the present value is $7.72. As every finance student knows, the increase in the interest rate causes the value of the cash flow to decrease. Next, calculate the future value of the asset after each period at 4% and at 5%. When you calculate the future value at 4% you must use the present value at 4% which is $8.11. Similarly, when you calculate the future value at 5% you must use the present value of $7.72. Note in the following table that the ending value at 5% exceeds the ending value at 4%. At about the 5th year the two future values are equal.

<table>
<thead>
<tr>
<th>Time</th>
<th>Cash flow</th>
<th>Rate</th>
<th>Aggregate present value at each rate</th>
<th>Future value at 4%</th>
<th>Future value at 5%</th>
<th>Individual present value at 4%</th>
<th>Maturity times PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.01</td>
<td>$9.47</td>
<td>$8.11</td>
<td>$7.72</td>
<td>$0.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.02</td>
<td>$8.98</td>
<td>$8.44</td>
<td>$8.11</td>
<td>$0.96</td>
<td>$0.96</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.3</td>
<td>$3.09</td>
<td>$8.77</td>
<td>$8.51</td>
<td>$0.92</td>
<td>$1.85</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.04</td>
<td>$8.11</td>
<td>$9.12</td>
<td>$8.94</td>
<td>$0.89</td>
<td>$2.67</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.05</td>
<td>$7.72</td>
<td>$9.49</td>
<td>$9.39</td>
<td>$0.85</td>
<td>$3.42</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0.06</td>
<td>$7.36</td>
<td>$9.87</td>
<td>$9.86</td>
<td>$0.82</td>
<td>$4.11</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.07</td>
<td>$7.02</td>
<td>$10.26</td>
<td>$10.35</td>
<td>$0.79</td>
<td>$4.74</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0.08</td>
<td>$6.71</td>
<td>$10.67</td>
<td>$10.87</td>
<td>$0.76</td>
<td>$5.32</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>0.09</td>
<td>$6.42</td>
<td>$11.10</td>
<td>$11.41</td>
<td>$0.73</td>
<td>$5.85</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.1</td>
<td>$6.14</td>
<td>$11.54</td>
<td>$11.98</td>
<td>$0.70</td>
<td>$6.32</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td></td>
<td>$12.01</td>
<td>$12.58</td>
<td>$12.68</td>
<td>$0.68</td>
<td>$6.76</td>
</tr>
<tr>
<td>Sums</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$8.11</td>
<td>$41.99</td>
</tr>
<tr>
<td>Duration</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.18</td>
</tr>
</tbody>
</table>

I also show the calculation of the asset’s duration in the table. The duration is 5.18 periods. This is the same as the time when the two time-to-value lines cross. To see this more clearly, I plot the two time-to-value lines in the next chart.
Value versus Time Diagram

- Dotted line: Future value at 4%
- Solid line: Future value at 5%
4. The **value risk profile** relates the change in the market value of the bank’s equity to an unexpected change in the level of interest rates. A major difference between the interest rate risk of a bank and a bond is that the bond’s cash flow does not change when interest rates change but the bank’s cash flow does change.

There are two ways to represent the effects of changes in interest rates on a bank’s market value. One way is to start with the relationship among the values of the bank’s assets, $A$, liabilities, $L$, and equity, $V$.

\[ V = A - L \]

When interest rates change unexpectedly, the values of the bank’s assets and liabilities can both change. If they do not change by equal amounts, the value of the bank’s equity changes.

\[ \frac{\partial V}{\partial r} = \frac{\partial A}{\partial r} - \frac{\partial L}{\partial r} \]

We use the definition of duration to relate changes in the market values of the bank’s assets and liabilities to changes in the level of interest rates.

\[ D_A = -\frac{\partial A \cdot (1+r)}{\partial r \cdot A} \quad \text{and} \quad D_L = -\frac{\partial L \cdot (1+r)}{\partial r \cdot L} \]

Let us rewrite these to obtain

\[ \frac{\partial A}{\partial r} = -D_A \frac{A}{(1+r)} \quad \text{and} \quad \frac{\partial L}{\partial r} = -D_L \frac{L}{(1+r)} \]

These are standard duration expressions relating changes in the market values of the bank’s assets and liabilities to changes in the level of interest rates. We substitute these expressions into the value change expression to obtain our desired equation relating the change in value to the durations and values of assets and liabilities.

\[ \frac{\partial V}{\partial r} = \frac{\left( L \cdot D_L - A \cdot D_A \right)}{(1+r)} \]

When interest rates change unexpectedly, the values of the bank’s assets and liabilities can each change. The sizes of their changes depend on the durations and their initial values. There are several features of this expression that you should understand.
It is always the case that $A>L$. Otherwise the bank is not a going concern. It is bankrupt. The size of $L$ relative to $A$ is called the bank’s leverage. Banks get money from liabilities, which are primarily deposits, and from equity. It uses these monies to fund loans and to buy securities and other assets. The leverage ratios tell how much of the bank’s assets it funds with debt instead of equity. Banks are relatively highly levered compared to industrial companies. Banks often finance 90% of their assets with deposits.

The duration of assets does not have to equal the duration of liabilities. There are three possibilities.

- $LD_L>AD_A$. The duration of the bank’s liabilities exceeds the duration of its assets by a large enough amount to overcome the bank’s leverage. In this case the bank is said to be long-funded. This situation would occur if the bank issues long-term time deposits and uses the money to lend short-term or at variable interest rates. If interest rates rise the bank’s interest expense is relatively unchanged since it has long-term deposits. However, its interest revenue rises since it has short-term or variable rate assets that earn the higher interest rate. The bank’s net interest income rises. Of course, so does its cost of capital. If $LD_L>AD_A$, its net interest income rises more than its cost of capital and the market value of its equity rises.

- $LD_L=AD_A$. In this case the bank is said to be even-funded. If interest rates change the market value of its equity does not change. This could describe a bank that is funding short-term loans with short-term deposits. Since $A>L$, it has more loans than deposits. If $LD_L=AD_A$ and $A>L$, it must be that $D_L>D_A$. Therefore, its short-term assets exceed its short-term deposits. A rise in interest rates increases interest revenue and interest expense. However, interest revenue increases more so that net interest income rises. Of course, so does its cost of capital. In this situation the rise in net interest income just offsets the rise in the cost of capital leaving the bank’s market value of equity unchanged.

- $LD_L<AD_A$. In this case the bank is said to be short funded. It is financing long-term assets with short-term deposits. If interest rates increase, the bank must pay higher interest expense but does not receive higher interest revenue. Its net
interest income decreases. At the same time its cost of capital rises. Thus, its stock price falls.

The **banker’s dilemma** refers to the bank's choice of whether it should try to immunize its market value, its net income, or something else. To immunize means we set the duration of assets and liabilities so that unexpected changes in interest rates have no effect. The question is what do we want to be unaffected? Is it net income? Or is it market value of equity? As we shall see, it cannot be both.

- **Immunize net income.** Net interest income is the portion of net income that is responsive to changes in interest rates. What if the bank issues short-term deposits and makes short-term loans? If interest rates rise, its interest expense and interest revenue rise together and its net interest income is unchanged. However, its value falls because the cost of capital has increased. If the bank immunizes its net interest income its net income resembles the payments on a very long-term bond. Since long-term bonds have high durations, the market value of its equity is especially sensitive to changes in interest rates.

- **Immunize market value of equity.** The bank must have its net interest income rise when interest rates rise and fall when interest rates fall. These changes are required to offset the cost of capital effect. Interest rates in the United States often move in a procyclical pattern. If a bank immunizes its market value of equity, its earnings will also move in a procyclical pattern. You may read or hear about managers attempting to manage earnings because they think that this will increase their stock price. Aside from the fact that this notion is inconsistent with market efficiency, you also now see that it is also inconsistent with stock price immunization.

The **value added** solution to the bankers’ dilemma advises the bank to issue loans and deposits that transfers interest rate risk from its customers to itself, and then to manage this risk through the use of swaps, securitization, futures, and options on futures.

A second way to analyze the effects of a change in interest rates on the value of the bank’s equity is to write the equity value in terms of the bank’s cash flows and cost of
capital. This approach gives the details that were somewhat hidden in the first approach. The bank’s value equation is

\[ V = \sum_{t=1}^{\infty} NI_t (1 + k)^{-t}. \]

The bank earns interest revenue on its short-term assets, called rate sensitive assets, \( RSA \), and on its rate insensitive assets, \( RIA \). Similarly, it pays interest expense on its rate sensitive liabilities, \( RSL \), and on its rate-insensitive liabilities, \( RIL \).

\[ IR_t = RSA_t \cdot r_{ad} + RIA_t \cdot \bar{r}_a. \]
\[ IE_t = RSL_t \cdot r_{ld} + RIL_t \cdot \bar{r}_l. \]

The bar over the interest rate means it is fixed in the short run. The change in value caused by a change in the interest rate is

\[ \sum_{t=1}^{\infty} \frac{\partial V}{\partial r} = \sum_{t=1}^{\infty} (1 + k)^{-t} \frac{\partial NI_t}{\partial r} - \sum_{t=1}^{\infty} t(1 + k)^{-t-1} NI_t. \]

Using the expressions for interest revenue and interest expense, we can rewrite this as

\[ \sum_{t=1}^{\infty} (1 + k)^{-t} (RSA_t - RSL_t) - \sum_{t=1}^{\infty} t(1 + k)^{-t-1} NI_t. \]

The second summation is exactly the same one that appears in the value change equation when the asset’s cash flow does not change with the interest rate. The first summation is new. It shows the effect of a change in interest rates on an asset’s value when the asset’s cash flow changes with interest rates. The term \((RSA - RSL)\) is often called the \textit{GAP}.

This expression nicely illustrates the bankers’ dilemma. If the bank immunizes its net income, the first summation is zero. An increase in interest rates decreases the market value of the bank’s equity because of the cost of capital effect in the second summation. If the bank immunizes the market value of its equity, it is must set the two summations so they add to zero. Since the second summation has a minus sign on it, the first summation must be positive. This requires \( RSA > RSL \). Thus, to immunize market value, the bank’s net interest income must rise and fall with interest rates.
Measurement and Management of the Risks of Financial Institutions

Financial institutions include brokers, dealers and asset transformers.

Dealers and asset transformers are exposed to interest rate risk if they provide maturity intermediation, or hold and trade assets whose durations differ from the institution’s holding period.

We use the standard tools of finance to analyze an institutions ex ante exposure to interest rate risk. The analysis is called DEAR (daily earnings at risk) and VAR (value at risk).

As always, our starting point is the time value of money.

TVM (time value of money) $\rightarrow$ DCF (discounted cash flow) $\rightarrow$ $V = \sum_{t=1}^{T} C_t (1 + R)^{-t} \rightarrow$

![Value versus Yield](image)
Ex post interest rate risk is the change in value due to a change in the interest rate.

The formula for the bank’s risk profile is: \( \Delta V = \left( \frac{D}{1 + R} \right) \cdot V \cdot \Delta R \).

To measure ex ante interest rate risk, substitute the expected adverse change in interest rates, \( n \cdot \sigma_{\Delta R} \), for the ex post (realized) change in interest rates, \( \Delta R \).

\[ \Delta V = \left( \frac{D}{1 + R} \right) \cdot V \cdot n \cdot \sigma_{\Delta R} \]
The following graph is an Excel plot of the standard normal density function. It has a mean of zero and a standard deviation of one.
The following graph is an Excel plot of the change in the standard normal density function. I present it to show you how the inflection points of the standard normal density occurs at standard deviations of minus one and plus one.
A common task in finance is to find the mean and variance of two or more random variables. Here is a simple solution to this task.

Let X and Y be random variables, and Z = X + Y.

The expected value of Z is: \( E(Z) = E(X) + E(Y) \)

The variance of Z is: \( V(Z) = V(X) + V(Y) + 2\text{Cov}(X,Y) \).

The covariance may be rewritten as \( \text{Cov}(X,Y) = \text{Corr}(X,Y)\text{s.d.}(X)\text{s.d.}(Y) \), where s.d. is the standard deviation; the square root of the variance.

In your WEAR assignment, you must calculate the WEAR of equity. Because equity, Eq, is assets, A, minus liabilities, L, you can use the formula for the variance of the sum or difference of two random variables.

\[ \text{Eq} = A - L \]

\[ V(\text{Eq}) = V(A) + V(L) - 2\text{Cov}(A,L) = V(A) + V(L) - 2\text{Corr}(A,L)\text{s.d.}(A)\text{s.d}(L). \]
Ex post versus ex ante risk measurement.

We have used duration and convexity to measure the change in an asset’s price caused by a change in its yield to maturity, assuming a flat yield curve. This is ex post analysis since we estimated the change in the asset’s price for a known change in its yield. We now switch to ex ante analysis. The question is what is the likely change in the asset’s price given its duration and convexity and the likely change in its yield to maturity? We can answer this for a single asset for a single day, DEAR, for a single asset for multiple days VAR, and for a portfolio of assets for a single day, Portfolio DEAR.

<table>
<thead>
<tr>
<th>Ex post risk measure</th>
<th>Daily</th>
<th>Multiple Days</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>( \Delta P \approx -\frac{D}{1+r} P \cdot \Delta r )</td>
<td>( \Delta P \approx -\frac{D}{1+r} P \cdot n \cdot \sigma_{\Delta r} )</td>
<td>( \Delta P = -\left[DEAR^2_1 + DEAR^2_2 + 2 \rho \sigma_{1,2} \sigma_{DEAR_1} \sigma_{DEAR_2} \right]^{1/2} )</td>
</tr>
<tr>
<td>DEAR</td>
<td>( P ) is the asset’s initial price. ( r ) is the asset’s initial yield to maturity</td>
<td>( n ) is the number of standard deviations required to obtain the desired confidence interval</td>
<td>( N ) is the number of days in the planning horizon</td>
</tr>
<tr>
<td>VAR</td>
<td>( \Delta P \approx -\frac{D}{1+r} P \cdot n \cdot \sigma_{\Delta r} \cdot \sqrt{N} )</td>
<td>( \rho ) is the correlation coefficient</td>
<td></td>
</tr>
<tr>
<td>Portfolio DEAR</td>
<td>( \left[DEAR^2_1 + DEAR^2_2 + 2 \rho \sigma_{1,2} \sigma_{DEAR_1} \sigma_{DEAR_2} \right]^{1/2} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Convexity correction

\[ 0.5 \cdot CX \cdot (\Delta r)^2 \]
Topics in Bank Lending

1) Using present value analysis in the bank lending decision
   a) Estimating future cash flows
      i) Estimating market size and its growth
      ii) Assessing competition and market share
      iii) Estimating the borrower’s operating profit margin
      iv) Estimating the borrower’s overhead costs to be assigned to the project
   b) Estimating the borrowing rate
      i) Choosing the risk free rate
      ii) Estimating beta
      iii) Choosing the equity premium
      iv) Adding the bid to ask spread
   c) Using post-financing cash flows to incorporate the effects of the bid/ask spread

2) Distinguishing between the contractual and expected loan rates
   a) Effects of adverse selection and moral hazard
   b) Estimating default probabilities
      i) Start with the indifference condition: \( p(1+k) = 1+i \)
      ii) Estimate forward rates for Treasury \( i \) and corporate bonds \( k \)
      iii) Use bonds with risk equal to the borrower’s risk
      iv) Deduce the borrower’s repayment probability \( p \) from the forward rates

3) Using RAROC in the bank lending decision
   a) Estimate the annual income to the bank from the loan
      i) Interest income plus fee income
      ii) Amortize one-time fees over the life of the loan
   b) Estimate the contribution of the loan to the bank’s risk
      i) Use the ex ante duration model as in the DEAR model
      ii) Use the change in the credit risk premium as the interest rate shock
      iii) Compare the RAROC to the bank’s cost of capital
1) The net present value framework.
   a) Variable definitions:
      i) \( NPV \): the net present value of a series of cash flows that occur at different times.
      ii) \( C_t \): the cash flow that occurs at date \( t \).
      iii) \( k_t \): the risk-adjusted discount rate linking date \( 0 \), the date for which the \( NPV \) is being calculated, with date \( t \), the date when the cash flow is received. Note that the discount rate can have a different value for each cash flow.
      iv) \( T \): the date of the last cash flow.
      v) \( I_0 \): the initial investment required to generate the cash flow whose value is being determined.
   b) The net present value formula.

\[
NPV_0 = \sum_{t=1}^{T} C_t \left(1 + k_t\right)^{-t} - I_0
\]

The net present value at date \( 0 \) of a future cash flow \( C_t \) is the sum of the discounted value of each cash flow less the investment necessary to generate the cash flow.

c) To apply the NPV formula to a bank loan:
   i) \( I_0 \) is the amount of the loan.
   ii) \( C_t \) is the principal plus interest payments the borrower pays to the bank at date \( t \).
   iii) \( K_t \) is the interest rate on the loan.
   iv) \( T \) is the maturity date of the loan. This is the date when all interest and principal payments are scheduled to be made to the bank.

2) Cash flow forecast.

The bank must make forecasts independently of the borrower. It can either have the expertise to make its own forecasts or it can buy them from professional forecasters.

a) Forecasted cash flow equals forecasted market size in number of units times forecasted market share times forecasted spread between the selling price and variable costs. Cash flow = size times share times spread.
b) Market size forecast. Known as an industry forecast.
   i) Trend projections of past industry sales using Box-Jenkins models.
   ii) Cyclical deviations from trend forecasted using Box-Jenkins models.
   iii) Seasonal patterns that occur within a year forecasted using Box-Jenkins models.

c) Market share forecast. Known as competitive analysis.
   i) Number of incumbent competitors.
   ii) Ease of entry into the market.
   iii) Borrower’s production costs relative to incumbent’s.
   iv) International competition.

d) Spread forecast.
   i) Selling price per unit.
   ii) Average variable costs.
       (1) Wage rates
       (2) Number of employees
       (3) Productivity
   iii) Average fixed costs.
       (1) Capital costs
       (2) Production quantities

3) Cost of capital. This is the interest rate the bank charges on the loan.
   a) CAPM
   b) Bank’s spread.
   c) Adverse selection problems.

4) Consequences of the lending rate differing from the borrowing rate.
   a) Standard present value analysis assumes that the lending rate equals the borrowing rate. Since the borrowing rate is higher than the deposit rate, and the bank often requires a compensating balance deposit, it behooves us to investigate the consequences of the borrowing rate exceeding the lending rate.

b) A simple example.

\[ PV_0 = C_1 (1 + k_d)^{-1} + C_2 (1 + k_d)^{-2} \]
\[ k_d \text{ is the discount rate.} \]

\[ FV_2 = PV_0 (1 + k_c)^2 \]
\[ k_c \text{ is the compounding rate.} \]
Combining these two equations gives,

\[
FV_2 = \frac{(1 + k_c)^2 C_1}{1 + k_d} + \frac{(1 + k_r)^2}{(1 + k_d)^2}.
\]

Standard present value analysis assumes \( k_c \) equals \( k_d \). But, what happens if \( k_c \) does not equal \( k_d \)?

c) A spreadsheet example.
### Credit analysis pro formas

<table>
<thead>
<tr>
<th>Item</th>
<th>Year0</th>
<th>Year1</th>
<th>Year2</th>
<th>Year3</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market size in units demanded</td>
<td>8601.00</td>
<td>8859.03</td>
<td>9124.80</td>
<td>Assumed 3% growth</td>
<td></td>
</tr>
<tr>
<td>Market share</td>
<td>0.140</td>
<td>0.140</td>
<td>0.140</td>
<td>An assumed share</td>
<td></td>
</tr>
<tr>
<td>Price</td>
<td>3.000</td>
<td>3.093</td>
<td>3.189</td>
<td>Annual inflation since 1926: 3.1%</td>
<td></td>
</tr>
<tr>
<td>Average cost</td>
<td>2.600</td>
<td>2.681</td>
<td>2.764</td>
<td>Annual inflation since 1926: 3.1%</td>
<td></td>
</tr>
<tr>
<td>Margin</td>
<td>481.656</td>
<td>511.485</td>
<td>543.161</td>
<td>share<em>size</em>spread</td>
<td></td>
</tr>
<tr>
<td>Interest expense</td>
<td>0.1584</td>
<td>158.412</td>
<td>158.412</td>
<td>158.412 Loan rate*principal</td>
<td></td>
</tr>
<tr>
<td>Interest earnings</td>
<td>0.0924</td>
<td>0.000</td>
<td>25.607</td>
<td>54.453 Deposit rate * deposit amount</td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>333.333</td>
<td>333.333</td>
<td>333.333</td>
<td>Straight line depreciation</td>
<td></td>
</tr>
<tr>
<td>Overhead expenses</td>
<td>75.000</td>
<td>77.325</td>
<td>79.722</td>
<td>Annual inflation since 1926: 3.1%</td>
<td></td>
</tr>
<tr>
<td>NIBT</td>
<td>-85.090</td>
<td>-31.978</td>
<td>26.147</td>
<td>Margin+intrev-intexp-dep-oh</td>
<td></td>
</tr>
<tr>
<td>Taxes</td>
<td>-28.931</td>
<td>-10.873</td>
<td>8.890</td>
<td>NIBT*tax rate</td>
<td></td>
</tr>
<tr>
<td>NIAT</td>
<td>-56.159</td>
<td>-21.106</td>
<td>17.257</td>
<td>NIBT-taxes</td>
<td></td>
</tr>
<tr>
<td>Cash flow before financing</td>
<td>1000</td>
<td>435.587</td>
<td>445.033</td>
<td>454.549 Margin-oh-taxes</td>
<td></td>
</tr>
<tr>
<td>Pre financing NPV</td>
<td>0.069</td>
<td></td>
<td></td>
<td>NPV(loan rate,CF1:CF3)-loan</td>
<td></td>
</tr>
<tr>
<td>Cash flow saved to repay principal</td>
<td>277.174</td>
<td>312.228</td>
<td>350.590</td>
<td>CF-IE+IR</td>
<td></td>
</tr>
<tr>
<td>Cumulative deposit</td>
<td>277.174</td>
<td>589.402</td>
<td>939.992</td>
<td>Prior deposit + new cash flow</td>
<td></td>
</tr>
<tr>
<td>PV of cash after repaying principal and interest</td>
<td>-38.603</td>
<td></td>
<td></td>
<td>(CumDep3-loan)(1+loan rate)^-3</td>
<td></td>
</tr>
<tr>
<td>Risk free rate</td>
<td>0.051</td>
<td></td>
<td></td>
<td>Long-term govt rate since 1926</td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500 Index</td>
<td>0.113</td>
<td></td>
<td></td>
<td>Annual rate since 1926</td>
<td></td>
</tr>
<tr>
<td>Project beta</td>
<td>1.200</td>
<td></td>
<td></td>
<td>An assumed beta</td>
<td></td>
</tr>
<tr>
<td>Bank's operating costs</td>
<td>2236</td>
<td></td>
<td></td>
<td>(Salaries+LLP+other+taxes)</td>
<td></td>
</tr>
<tr>
<td>Bank's assets</td>
<td>40859</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank's beta</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank's capital ratio</td>
<td>0.1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Data in italics are input data. All other data are calculated in the spreadsheet.*
You know a basic way to estimate the required rate of return on a bank loan from the spreadsheet assignment on page 67 of the Course Pack. Saunders chapter 11 shows several modifications to the basic approach.

1) Page 299 shows how to modify the base rate to include the compensating balance requirement and the Federal Reserve’s reserve requirement.
   a) The compensating balance requirement raises the cost of the loan to the borrower because the borrower pays interest on the principal of the loan but only gets to use the principal less the compensating balance.
   b) The reserve requirement also raises the cost of the loan because the bank does not get to use all of its deposits to make loans. It must keep some of its cash as reserves against its deposits.

2) Page 314 shows one way to distinguish the contractual loan rate from the bank’s expected return on the loan. When a bank makes loans it anticipates that some borrowers will not repay them on time, if at all. A bank does not know a priori which borrowers will default. If it did it would not lend to the potential defaulters. The formula on Saunders page 314 shows a market-based way for a bank to estimate its expected return on its loans. It has three parts.
   a) First, the formula assumes investors are risk-neutral. We use the risk neutral indifference condition to estimate that probability \( p \) that the borrower will repay the loan. \[ p = \frac{1+i}{1+k} \]. The risk free rate \( i \) and the risky rate \( k \) you can find on Bloomberg or from other price discovery vendors. The risk free rate is the rate on a zero-coupon U.S. treasury security whose duration matches that of the loan. The risky rate is the market yield to maturity on a bond issued by a company with the same credit rating as the borrower, and ideally with the same duration as the loan. Sometimes as analysts you will find that the available data fall short of your ideal data and you must learn to improvise.
b) Second, the formula allows for the bank to require that the borrower post collateral for the loan. If the borrower defaults, the bank gets the collateral. If the borrower repays the loan, the borrower gets the collateral. \( \gamma \) is the ratio of the market value of the collateral to the principal and interest the borrower owes the bank.

c) Third, the bank’s expected return is:

i) the product of the probability of repayment \( p \) times the full amount the borrower owes the bank \( (1+k) \)

ii) plus the product of the probability of default \( (1-p) \) times the amount of money the bank receives if the borrower defaults \( \gamma(1+k) \).

d) Bank loans may have maturities longer than one year. Pages 315-317 show how to use the yield curve and risk neutral pricing to estimate the marginal and cumulative probabilities of default on multi-period loans.

3) Page 302 shows that as the contractual loan rate increases, the expected return first increases and then decreases. You should understand the arguments and be able to explain them using the concepts of adverse selection and moral hazard combined with Campbell and Kracaw’s argument about how to solve the principal-agent problem.

4) A borrower may default either because of idiosyncratic problems that beset the particular borrower or because of market-wide problems that affect many borrowers. By diversifying banks reduce their exposure to idiosyncratic risk. The RAROC model helps banks control their systematic risk from lending. If the economy worsens, more borrowers will default, and they may default by larger amounts. If the economy worsens, credit risk premiums increase. If banks have made loans with a variable base rate but a fixed risk premium, the market values of their loans decrease as the risk premium increases. RAROC provides a way for banks to anticipate and charge for possible increases in the credit risk premium. RAROC is similar to VAR. VAR includes a term to allow for possible increases in the risk free rate, and RAROC includes a similar term to allow for possible increases in the credit risk premium.

5) Bank loans have a one-sided risk. The borrower never pays more than the principal and interest, but he may pay less. Options also have one-sided risks. A bank’s position in a loan is comparable to writing a put option. When the loan is due, the
borrower has the option of repaying the loan and keeping the collateral, or defaulting on the loan and giving the collateral to the bank. In bad times the value of the collateral may decrease. The option pricing model provides a numerical estimate of the risk premium the bank should charge on a risky loan. See the following table.

a) The Benchmark column repeats the analysis from Saunders. Can you write an Excel program that does this?

b) The Change Repayment columns show the effect on the risk premium of decreasing the size of the loan while keeping the borrower’s collateral assets constant. Decreasing the loan decreases leverage. This changes h1 and h2 and decreases the premium.

c) The Change Maturity columns show the effect on the risk premium of changing the maturity of the loan. Maturity has two offsetting effects on the risk premium.

   i) As the maturity of the loan increases, the present value of the loan repayment decreases. If the maturity of the loan does not affect the value of the borrower’s assets, leverage decreases because leverage is the ratio of the present value of the repayment to the value of the borrowers’ assets. Thus, the premium decreases.

   ii) Alternatively, if we hold leverage constant at the benchmark value of 0.9, when maturity increases to two years, the risk premium increases. This is because the loan has become riskier to the bank. Remember that more bad events can happen over two years than over one year. You might think that more good things could also happen. That is correct. However, the bank does not get to share in the good things since the loan repayment is a fixed amount. The bank only gets to share in the bad events.

d) If the risk free interest rate increases from 5% to 10%, the risk premium decreases. This is because the present value of the loan repayment has decreased and the borrower’s leverage ratio has decreased. This may be an incomplete analysis. If the risk free rate increases, the value of the borrower’s assets may also decrease. If they decrease by the same amount that the repayment present value decreases, leverage will not change. They could decrease by the same percents if they have the same duration.
e) If the borrower’s risk, as measured by the standard deviation of the change in the value of its assets decreases, the risk premium decreases.

<table>
<thead>
<tr>
<th>Repayment B</th>
<th>Benchmark Change Repayment</th>
<th>Change maturity, allowing leverage to change</th>
<th>Change maturity holding leverage constant</th>
<th>Change Risk Free Rate</th>
<th>Change St. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maturity tau</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Risk free rate I</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Assets c</td>
<td>105692.2</td>
<td>105692.2</td>
<td>105692.2</td>
<td>105692.2</td>
<td></td>
</tr>
<tr>
<td>Leverage d</td>
<td>0.9000</td>
<td>0.8100</td>
<td>0.8561</td>
<td>0.9000</td>
<td></td>
</tr>
<tr>
<td>Stdev sigma</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>lower h1</td>
<td>-0.9380</td>
<td>-1.8160</td>
<td>-1.0003</td>
<td>-0.7057</td>
<td></td>
</tr>
<tr>
<td>upper h2</td>
<td>0.8180</td>
<td>1.6960</td>
<td>0.8306</td>
<td>0.5360</td>
<td></td>
</tr>
<tr>
<td>Premium</td>
<td>0.0133</td>
<td>0.0021</td>
<td>0.0090</td>
<td>0.0148</td>
<td></td>
</tr>
</tbody>
</table>

Saunders' option-based loan risk premium, chapter 11.
Standby Letters of Credit

Alan C. Hess, October 1998

A letter of credit is a financial contract in which the contract issuer promises to pay the contract owner a specified amount of money if specified events occur.

- A commercial letter of credit is a third-party guarantee of payment for the shipment of goods. Before an exporter ships goods he may require a letter of credit from a third party, typically a bank. The bank guarantees that if the goods arrive at the importer as promised, and the importer cannot pay for them, the bank will pay the exporter. A commercial letter of credit replaces the importer’s credit with that of the bank, which usually is an internationally known bank.

- A standby letter of credit is a third-party guarantee of payment to a lender on a debt instrument, such as a bond or a bank loan. A lender may have money to lend but be unwilling to take the risk of a particularly risky borrower. A third party, often a bank with a better credit rating than the borrower, may issue a standby letter of credit in which it promises to repay the lender if the borrower defaults. A standby letter of credit separates the funding of the loan from the credit risk of the loan.

Since a letter of credit stipulates a payment only if specified events occur, it is an option. Since it specifies that the issuer must pay the owner of if the events occur, it is a put option. The issuer of the letter of credit, say a bank, is short the put option and the owner of the letter of credit, the party that made the loan, is long the put option. Since the bank writes the letter specifically for the borrower, it is an over-the-counter option. The borrower pays the issuer the option premium, but is neither long nor short the option.

Here is an example of a standby letter of credit. A borrower with a low credit rating, say a little-known municipality, issues bonds. If the amount of the bond is small, it may not be worthwhile for institutional bond buyers to incur the expense of investigating the city’s credit worthiness. Thus, there may not be a ready market for the city’s bonds. A bank that does regular business with the city may have better information about the city’s finances, its ability to collect
taxes, its fiscal integrity, and its wisdom in spending money. This bank may issue of standby letter of credit in which it promises to pay the bond buyers what the city owes them if the city defaults.

- The city sells the bonds to the bond buyers at an interest rate that reflects the bank’s credit risk. The city’s cash and debt both increase by the amount of the bonds issued. The city’s cash falls by an additional amount equal to the premium it pays the bank to issue the standby letter of credit. The benefit to the city is that its interest expense is smaller than it would have been without the letter of credit. If the present value of the reduced interest over the life of the bond is smaller than the premium, the city gains from the letter of credit.

- The bond buyer records the bonds as an asset on its balance sheet replacing the cash it used to buy the bond. The market value of the bonds is the sum of the promised repayment of principal and interest from the city plus its long-position in the bank-issued put option. The value of option is the difference between the value of the bonds with the standby letter of credit minus their value without the standby letter of credit. In equilibrium this equals the option premium.

- The bank records the option premium as part of its noninterest revenue. It records its short position in the put option as an off-balance sheet liability. The standby letter of credit is a sequence of put options each of which expires at one of the bond repayment dates. At each repayment date, the exercise price of the option is the face value of the city’s repayment amount. This is the interest due plus perhaps some of the principal.

If the city makes all the required bond payments when they are due, the standby letter of credit expires unused. The option is out-of-the-money. If the city defaults, the market value of the bonds without the standby letter of credit falls below their face value. The option is in-the-money, the bond-holders exercise it, and the bank pays them the face value of the bonds. The bank books a loan to the city equal to the outstanding amount of the bonds. The city is not excused from its debt. The amount due is transferred from the bond-buyers to the bank. It may be that the bank and the city rewrite the loan and the bank writes off a portion of it as a bad debt.
Since a letter of credit is a put option, the issuer can use option-pricing theory to determine the fee to charge to write the letter. This is easier for a commercial letter of credit that entails a one-time shipment of goods. It is difficult for a standby letter of credit that guarantees a sequence of payments and is exercised at the first default. Five factors that determine the price of a put option and their effects on its price are:

1. The risk-free interest rate: Letters of credit are commitments to pay money in the future. The larger the interest rate, the lower is the present value of the commitment. Hence, the fee the bank charges is negatively related to the risk-free rate.

2. The maturity of the letter of credit: The longer the maturity, the lower the present value of the payment if the borrower defaults. Hence, the fee is negatively related to maturity.

3. The market value of the underlying financial instrument: When the city issues the bonds their market and face values are equal. After issue, if the primary borrower is in good financial health and can make his payments on time, the market value of the debt continues to equal its face value. However, if the borrower’s financial position weakens, the market value of his debt falls below its market value and the value of the standby letter of credit rises. Hence, the fee for the letter of credit is negatively related to the market value of the underlying loan.

4. The face value of the underlying loan: The greater the face value of the loan, other things the same, the greater is the chance that the borrower will not be able to make his payments. This increases the likelihood that the letter of credit will be exercised. Hence, the fee increases with the face value of the primary loan.

5. The volatility of the cash flow of the borrower: The riskier the borrower, the greater the chance he will not be able to repay. This increases the chance that the standby letter of credit will be exercised. Hence, as volatility increases, the fee increases.
Liquidity is the ability to obtain cash on very short notice. A bank supplies liquidity to its loan and deposit customers by making loans upon demand and redeeming deposits on notice. A bank has several sources of liquidity: cash, interest revenue from assets in excess of interest expenses, sales of U.S. government securities that are traded in active secondary markets, federal funds purchased, and Eurodollar borrowing.

Liquid assets have lower rates of return than illiquid assets such as bank loans. Banks reduce information costs by providing illiquid loans to borrowers and they reduce transaction costs by providing liquidity to their customers. Thus, the bank has a tradeoff between providing information services and providing transaction services.

The graph illustrates these tradeoffs. The straight line starting at the origin and sloping upward represents the foregone interest that could have been earned if the bank had more loans and less liquidity. The slope of this line is the interest rate on loans less the interest rate on liquid assets. The downward sloping curved line that becomes asymptotic to the horizontal axis represents the expected stockout costs that the bank incurs if it has insufficient liquidity to meet its customers' liquidity demands. The expected stockout costs depend on the probability of a stockout, which depends on the variance of the cash flow, and the cost of a stockout should one occur. The probability of a stockout decreases as the amount of liquidity held by the bank increases.

The total cost line is the vertical sum of the foregone interest and expected stockout costs. The optimal amount of liquidity for the bank is the amount for which total costs are minimized. Miller and Orr (QJE, 1966) present a model for cash management that can be applied to a bank. The cash balance fluctuates due to uncertain inflows and outflows. The optimal rule is to let the cash balance fluctuate randomly until it reaches either an upper limit or a lower limit. When it reaches the upper limit, the bank uses its cash to buy securities. When it reaches the lower limit, the bank sells securities to raise cash.

The cost minimizing return point is 
(0.75*cost per transaction * variance of cash flow/interest rate)\(^{0.33}\).

The optimal amount of liquidity
1. Increases with the cost per transaction, which increases the cost of a stockout.
2. Increases with the variance of the cash flow, which increases the probability of a stockout.
3. Decreases with the interest rate, which increases the cost of liquidity.
A financial futures contract is an agreement to make delivery (short position) at a future date specified in the contract, or to accept delivery (long position) at a future date, of a fixed amount of a specific grade or quality of the underlying cash market financial instrument at a specified price.

Financial futures contracts are currently traded on Treasury bonds, notes, and bills, on Eurodollars, and on other cash market instruments. The Wall Street Journal carries a daily listing of the contracts, their prices, trading volume, and open interest.

1. What are the cash flows associated with a futures contract?

   a. Margin. A futures margin is a performance bond to guarantee the futures market principal's performance of its contractual obligations. In many cases, buyers and sellers of futures contracts must deposit 3-10% of the futures contract full value. Full value means the value of the underlying cash instrument in each futures contract times the number of futures contracts held. The Wall Street Journal listing shows the values of the underlying contracts.

   b. Daily marking to market. Futures prices change from transaction to transaction during daily trading in the futures trading pits. At the end of the trading day, each principal's open futures position is valued at that day's settlement price. If the position gains in value, the principal has the right to draw down its margin account to its maintainence margin level. If the position loses value, the principal must either deposit additional margin or have the open position closed by the broker holding the margin account. The Wall Street Journal listing shows daily prices and their change from the previous trading day.

   c. Return of margin. When the principal closes the futures position by entering a futures trade that offsets its initial trade, the broker returns the margin to the principal.

2. How is the value of a futures contract affected by unexpected changes in cash market interest rates?

   On the expiration date of the futures contract, the short position has the obligation to deliver the underlying cash instrument, and the long position has the obligation to take delivery of the underlying cash instrument. Thus, the value of the futures position at delivery equals the value of the cash instrument. Since all traders know this, in the days prior to expiration, the value of the futures contract is the expected present value of the cash instrument at delivery. If
market interest rates increase, the present value of the cash instrument falls and so does the current value of the futures contract.

3. How can a futures contract be used to hedge the interest rate risk of a cash market position? The cash flows of the cash instrument occur less often than the daily cash flows of the futures contract. These seem quite different.

The cash flows associated with a long position in T bonds.

a. Initial purchase price.

b. Semi-annual coupons.

c. Sales price.

4. How are the T bonds cash flows affected by unexpected changes in market interest rates?

a. Coupon reinvestment risk.

b. Sales price risk.

5. The basis is the difference between the current price of the cash market instrument and the current futures price. At expiration the futures price converges to the cash price. Before delivery, the futures price is tied to the expected cash price on the delivery date. Thus, the value of the futures contract is tied to the value of the cash instrument on the delivery date of the futures contract. The basis, in contrast, is the difference between the current cash price and the current futures price. Thus, the basis is related to the difference between the current cash price and the expected future cash price. As this difference changes, so does the basis.
I) Measure the risk exposure of the cash position.

You can view most cash positions as a portfolio of assets and liabilities (long and short positions). A formula for the interest rate risk of the portfolio’s value is

$$\Delta V = \left( \frac{LD_L}{1+r_L} - \frac{AD_A}{1+r_A} \right) \Delta r \tag{1}$$

$\Delta V$ is the unexpected change in the value of the portfolio. This is what we are trying to offset.

$L$ is the market value of liabilities; the value of the short positions. If your portfolio consists only of assets, set $L$ to zero.

$A$ is the market value of assets; the value of the long positions. If your portfolio consists only of liabilities, set $A$ to zero. (Can you have a portfolio of only liabilities?

$D_L$ is the duration of liabilities.

$D_A$ is the duration of assets.

$r_L$ is the rate of interest on the liabilities.

$r_A$ is the rate of interest on the assets

$\Delta r$ is the unexpected change in interest rates.

II) Understand what it means to hedge.

A) If interest rates change unexpectedly the value of your portfolio changes unexpectedly. If the value falls, $\Delta V < 0$, you lose. If the value increases, $\Delta V > 0$, you win. If you are risk averse, you suffer more from a loss than you benefit from an equally-sized gain.

B) To hedge means to combine your cash portfolio with a portfolio of derivative financial instruments into a hedged portfolio whose potential losses are less than the losses on the cash portfolio. Ideally, we want the changes in the values of the cash and derivative
portfolios to exactly offset each other leaving no change in the value of the hedge portfolio.

\[ \Delta H = \Delta V + \Delta F \approx 0. \]  

(2)

\( H \) is the value of the hedge portfolio.

\( F \) is the value of the derivatives in the hedge portfolio.

III) Find the risk minimizing value of derivatives.

A) The duration matching approach.

1) Use the duration formula to write an equation for the risk exposure of the derivative position.

\[ \Delta F = -D_F \cdot N_F \cdot \Delta r_F / (1 + r_F). \]  

(3)

\( D_F \) is the duration of the assets that underlie the derivatives.

\( r_F \) is the interest rate earned on the asset that underlies the derivative.

2) Substitute equations (1) and (3) into equation (2) and solve for \( N_F \).

\[ N_F = \left( \frac{L_D \cdot A_D}{1 + r_L} - \frac{A_D}{1 + r_A} \right) \frac{\Delta r}{P_F D_F / (1 + r_F)} \]  

(4)

(a) You know or can estimate the value of every component of the numerator and denominator of the first ratio.

(b) You can use historical data to estimate the ratio of the changes in the interest rates on the cash and derivative instruments that constitute the numerator and denominator of the second ratio. Thus, you can implement equation (4).

B) The minimum variance approach
1) In this approach you first estimate the regression coefficient between changes in the value of the cash position and changes in the value of the derivative position. Since this regression coefficient looks like the beta of the capital asset pricing model, let us call it $B$.

2) Substitute your estimated value of $B$ into the following formula to get your optimal derivatives position.

$$ F = -B \cdot V $$

(5)

IV) Implement the hedge.

A) To implement the hedge you must either buy or sell derivatives in the amounts given by equations (4) or (5).

B) Note in equation (4) that if assets are greater than liabilities and if the duration of assets is greater than the duration of liabilities, the optimal derivative position is negative. This means you sell derivatives.

C) Note in equation (5) that if the value of your cash position, $V$, is greater than zero, your derivatives position is negative. You sell derivatives to have a negative position in them.

D) In general, if your cash position is long, your derivative position is short. If your cash position is short, your derivative position is long.

E) Since most cash positions are positive, hedgers usually want to sell derivatives.
<table>
<thead>
<tr>
<th></th>
<th>Cash Market</th>
<th>Derivative Market</th>
<th>Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Now</td>
<td>C1. What is the source of risk? How much risk is there?</td>
<td>D1. Which derivative should we use?</td>
<td>B1. Beginning basis equals current cash value minus current derivative value: C2-D2</td>
</tr>
<tr>
<td></td>
<td>C2. What is the value of the cash position that is at risk?</td>
<td>D2. What nominal value of the derivative should we use?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C3. What is the risk sensitivity of the cash position?</td>
<td>D3. What maturity of the derivative should we choose?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C4. What is the likely change in the value of the cash position?</td>
<td>D4. What are our cash outlays?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C5. What is the hedge horizon?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>At hedge horizon</td>
<td>C6. Record value of the cash position if interest rates change.</td>
<td>D5. Record value of the derivative if interest rates change.</td>
<td>B2. Ending basis equals ending cash value minus ending derivative value: C6-D5.</td>
</tr>
<tr>
<td></td>
<td>C7. When do these value changes occur?</td>
<td>D6. When do these value changes occur?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C8. When are they recorded?</td>
<td>D7. When do we record them?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D8. What additional transactions do we make?</td>
<td></td>
</tr>
<tr>
<td>Change</td>
<td>C9. Record change in cash value: C6-C2</td>
<td>D9. Record change in derivative value: D5-D2</td>
<td>B3. Record change in basis, which is change in cash value minus change in derivative value: C9-D9.</td>
</tr>
<tr>
<td></td>
<td>C11. Prepayment risk</td>
<td>D11. Reinvestment risk</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D12. Liquidity risk</td>
<td></td>
</tr>
</tbody>
</table>
Hedging Situations.

There are four situations in which a person or a business might want to hedge its interest rate risk.

1. A business owns an asset, either financial or real, that it plans to sell before it matures. If interest rates rise between now and when the asset is sold, the value of the asset falls and the business incurs a capital loss.
2. A business plans to buy an asset in the future. If interest rates fall in the interim, the price of the asset rises and the business has to pay more for it.
3. A business plans to borrow in the future. If interest rates rise, it must pay higher borrowing costs.
4. A business has variable rate debt. If interest rates rise it must pay higher interest costs.

The essence of hedging interest rate risk is to form a new portfolio that has less interest rate risk than the existing portfolio. The new portfolio consists of the existing portfolio plus a derivative financial asset. The key is to choose the correct amount of the derivative asset. A person who is deciding whether to use financial futures to hedge a cash position that is exposed to interest rate risk must make four decisions.

1. Whether to buy or to sell futures.
2. Which futures contract to buy or sell.
3. What maturity of the futures contract to buy or sell.
4. How many futures contracts to buy or sell.

A Treasury bond example.

This is an easy example since we will hedge a Treasury bond with a Treasury bond futures contract. I conduct the analysis using “A Framework for Planning and Recording a Hedge.”
C1. What is the source of risk? How much risk is there?

The source of risk is unexpected changes in the Treasury bond rate. To determine how much risk there is I obtained weekly interest rates on the constant maturity 20-year Treasury bond from FRED, the Federal Reserve Bank of St. Louis Economic Database. I calculated weekly changes in the yield for the years 1962-99 and plotted them in a histogram, Chart 1. The average weekly change is about zero. The standard deviation is 0.14%. If we pretend that the distribution is about normal, there is a 5 percent chance that the weekly change in the 20-year interest rate will exceed 28 basis points in absolute value.

We find the amount of risk by assuming that the weekly changes are independent. We find the standard deviation of cumulative changes by multiplying the square root of the number of weeks until we plan to sell the bond by the weekly standard deviation. Thus, if we plan to sell in four weeks, there is a 2.5 percent chance that rates rise 56 basis points.

C2. What is the value of the cash position that is at risk?

This is the current market value of the bond. Using today’s Wall Street Journal, I find the value of a 20-year Treasury bond to be about $138. This is because its coupon of 8.875% exceeds its yield of 5.66%.

C3. What is the risk sensitivity of the cash position?

As a first approximation this depends on the duration of the bond. Using standard duration calculation procedures, as I have illustrated in a table, the duration of the bond is 11.24 years. The risk sensitivity of the bond is minus its modified duration times its price. This is $137.8815 which equals -$1466.35.

C4. What is the likely change in the value of the cash position?
If interest rates rise 0.56 % over the next four weeks, the price of the bond falls by 
0.0056*$1466.35 = -$8.21 per $100 face value of the bond.

Assume for our example that a bank has $1 million in T bonds that it plans to sell. It has a 2.5% chance of losing $821 thousand.

C5. What is the hedge horizon?

In our example the hedge horizon is four weeks. In general it is the length of time between now and when we plan to make a transaction in the cash market.

D1. Which derivative should we use?

The bank has several choices:
1. It could hold the bonds until end of the hedge horizon and sell them at the then prevailing market price. This exposes it to price risk.
2. It could sell the bonds now and use the proceeds to buy one-month Treasury bills. This incurs spreads and commissions in selling the bonds and buying the bills. It also may incur lost interest if the bill rate is less than the bond rate.
3. It could sell a forward contract in which it promises to sell the bond in four weeks at today’s price.
4. It could sell a futures contract that matures on or after the hedge horizon. A futures contract is an exchange-traded forward contract. A futures contract has several advantages over a forward contract. The underlying asset is standardized to concentrate trading so as to reduce transaction costs.

Using the ten financial services as our guide, we can compare the forward and futures contract.
<table>
<thead>
<tr>
<th>Ten Financial Services</th>
<th>Forward Contracts</th>
<th>Futures Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search for counterparty</td>
<td>Higher cost because OTC</td>
<td>Lower cost because exchange traded</td>
</tr>
<tr>
<td>Valuation</td>
<td>Higher cost because each contract is unique and must be value separately</td>
<td>Lower cost because contracts are standardized and few in number</td>
</tr>
<tr>
<td>Price discovery</td>
<td>Higher cost because of unique contracts and OTC</td>
<td>Lower cost because futures are exchange traded with widespread reporting of prices</td>
</tr>
<tr>
<td>Monitoring</td>
<td>Higher cost because of many possible counterparties</td>
<td>Lower cost because counterparty is exchange which has equity, daily marking-to-market, and margin</td>
</tr>
<tr>
<td>Denomination intermediation</td>
<td>Lower basis risk because contract size can be customized to cash position</td>
<td>Higher basis risk because contract sizes are standardized</td>
</tr>
<tr>
<td>Maturity intermediation</td>
<td>Lower basis risk because expiration date can be customized to cash position</td>
<td>Higher basis risk because of few expiration dates</td>
</tr>
<tr>
<td>Diversification</td>
<td>not applicable</td>
<td>not applicable</td>
</tr>
<tr>
<td>Hedging</td>
<td>Lower basis risk</td>
<td>Higher basis risk</td>
</tr>
<tr>
<td>Liquidity</td>
<td>Low liquidity due to lower trading volumes in specific contracts</td>
<td>High liquidity due to high trading volumes in a few standardized contracts</td>
</tr>
<tr>
<td>Payment system</td>
<td>not applicable</td>
<td>not applicable</td>
</tr>
</tbody>
</table>
The purpose of hedging is to have changes in the value of the futures contract plus changes in the value of the cash position add to zero. Let $F$ be the value of the futures position, which we will determine as we proceed, and $C$ the value of the cash position, which is predetermined by our cash market transactions. We want to choose $F$ to satisfy the following condition:

$$\Delta C + \Delta F = 0. \tag{1}$$

There are two ways to choose $F$, duration analysis and the minimum variance hedge ratio. Let us look at each of these.

**Using Duration Analysis to Choose the Derivative**

We know from duration analysis that the change in the value of the cash position due to a change in the cash market interest rate, $r_C$, is:

$$\Delta C \approx -\frac{D_c \times C}{1 + r_c} \times \Delta r_c. \tag{2}$$

Let us apply the same formula to the change in the value of the futures contract.

$$\Delta F \approx -\frac{D_f \times F}{1 + r_f} \times \Delta r_f. \tag{3}$$

In words, the change in the value of the futures contract is the duration of the asset underlying the futures contract, $D_f$, divided by one plus the yield on the asset underlying the futures contract, $1 + r_f$, times the value of the futures contract, $F$, times the change in the interest rate, $\Delta r_f$, on the asset underlying the futures contract.

Next, we add the expressions for the cash and derivative value changes, set their sum equal to zero, and solve for the value of the futures position. The value of the futures position that minimizes our interest rate risk is:

$$F_D \approx -\frac{(D_c/1 + r_c) \times \Delta r_c \times C}{(D_f/1 + r_f) \times \Delta r_f}. \tag{4}$$

The term $-\frac{(D_c/1 + r_c) \times \Delta r_c}{(D_f/1 + r_f) \times \Delta r_f}$ is called the hedge ratio. While it may look imposing at first glance, it consists of terms you are familiar with.
The term, \( D_c / (1 + r_c) \), is the modified duration of the cash position. It tells how much the value of the cash position changes for each change in the cash interest rate. It is multiplied by the change in the cash interest rate. Hence, the numerator of the hedge ratio is the change in the value of the cash position per dollar held in the cash position.

The term \( (D_f / (1 + r_f)) \Delta r_f \) is the modified duration of the futures position times the change in the value of the interest rate on the asset underlying the futures contract. It gives the change in the value of the futures position for each dollar held in the futures position. Notice that the hedge ratio is independent of the size of the cash position.

The hedge ratio includes a minus sign. This means that the value of the futures position is opposite that of the cash position. If the cash position is long, the futures position is short.

Let us summarize the duration answer to the question as to which derivative to use. To hedge our cash position, we want to find a futures contract whose modified duration and yield changes are close to those of the cash instrument. These conditions will result in the value of the futures contract changing in approximate unison with changes in the value of the cash position.

Here is an illustration of this approach for the Treasury bond cash position. The two most actively traded interest rate futures contracts are the Eurodollar contract traded on the Chicago Mercantile Exchange and the Treasury Bond contract traded on the Chicago Board of Trade. There are other interest rate futures contracts, but we will use these in our example. Which one should we use? To apply our approach we first have to calculate the duration of the two futures contracts.

The T bond contract has 15 years to maturity or first call, and an 8% coupon. The current yield on a 15-year bond is 5.62%. It duration is 9.85 years. The Eurodollar contract is a three-month contract with duration of 0.25 years. Since the T bond contract’s duration is closer to that of the
cash position, we use it in our subsequent analysis. We also need to know the average value of
the ratio of the change in the cash market yield to the change in the futures market yield. For the
T bond contract this ratio averaged 0.87 for the 1990s.

The Minimum Variance Hedge Ratio Approach

The hedge ratio tells the number of futures contracts to buy or to sell to hedge a given cash
position. The minimum variance hedge ratio is the ratio that minimizes the variance of the return
on a hedge portfolio consisting of the cash position and the futures position. Here is the logic
underlying the minimum variance hedge ratio.

Definitions of variables used in the analysis.

\( R_c \): The daily change in the value of the cash position. This is the change we are trying to
hedge.

\( R_f \): The daily change in the value of the futures contract. This is determined by daily trading
in futures markets.

\( C \): The market value of the cash position. This is the current market value to be hedged.

\( F \): The market value of the futures position. This is what we have to determine.

\( V(R_c) \): The variance of the daily change in the value of the cash position. This measures the
amount of price risk we face.

\( V(R_f) \): The variance of the daily change in the value of the futures position.

\( Cov(R_c, R_f) \): The covariance between daily value changes in the cash and futures instruments.

This is the key relationship between the cash and futures markets that allows us to hedge.

\( H \): Value of a hedge portfolio of the cash position and futures.

\( R_h \): The daily change in the value of the hedge portfolio.

\( V(R_h) \): The variance of the daily change in the value of the hedge portfolio.
We start with a given cash position with market value $C$ and a risk $V(R_c)$ to which it is exposed. We enter into a futures transaction in the amount $F$ to hedge our cash position. In doing so we form a hedged portfolio whose daily change in value is

$$R_h = C \cdot R_c + F \cdot R_f.$$ (5)

We want to make the variance of the daily change in the value of our hedge portfolio as small as possible. If we can make $V(R_h)$ zero we will have achieved a perfect hedge. We will usually not be able to achieve a zero risk of our hedge portfolio. The variance of the daily change in the value of the hedge portfolio is

$$V(R_h) = C^2 \cdot V(R_c) + F^2 \cdot V(R_f) + 2CF \cdot Cov(R_c, R_f).$$ (6)

Note that all terms in the equation are known except for $F$. Our task is to find the value of $F$ that minimizes the variance of the daily change in the value of the hedge portfolio. The effect of a change in $F$ on $V(R_h)$ holding the value of the cash position constant is

$$\frac{dV(R_h)}{dF} = 2F \cdot V(R_f) + 2C \cdot Cov(R_c, R_f).$$ (7)

We set this expression equal to zero and solve for the variance minimizing value of $F$.

$$F^* = -\frac{Cov(R_c, R_f)}{V(R_f)} C.$$ (8)

The hedge ratio is the term $-\frac{Cov(R_c, R_f)}{V(R_f)}$. This looks like the $\beta$ you are familiar with from the CAPM.

To estimate this hedge ratio we regress daily changes in the value of the cash position, these are changes in the value of the 20-year bond, on daily changes in the value of the futures positions. First we use changes in the value of the 15-year bond. Chart 2 shows the data for the 1990s. The regression coefficient for the line is 0.935. This is the minimum variance hedge ratio for the T
Bond futures contract. The $R^2$ of the regression is 0.96. Second, we would like to use changes in the value of LIBOR, the 3-month London Interbank Offered Rate. Since FRED does not list this rate, we use the 3-month domestic CD rate instead. Chart 3 shows the data for the 1990s. The slope is 0.56 and the $R^2$ is 0.24. Since the change in the 20-year T bond rate is less correlated with the CD rate than with the Bond rate, we use the bond futures contract to hedge to our T bond cash position.

D2. What nominal value of the derivative do we use?

This is the value of $F$ from either the duration-based or minimum variance hedge ratios. Using the duration-based hedge ratio,

$$F_D = -\frac{11.24 / 1.0566 - 9.85 / 1.0562}{0.87 \times 1,378,815} = -1,367,287.$$

The Treasury bond futures contract is traded in $100,000 units. To hedge the cash position, sell 14 Treasury bond futures contracts.

Using the minimum variance hedge ratio approach,

$$F^* = -0.935 \times 1,378,815 = -1,288,581.$$

This approach tells us to sell 13 Treasury bond futures contracts.

D3. What maturity of the derivative should we choose?

The maturity of the derivative should be the first contract that expires after the end of the hedge horizon. Treasury bond futures contract mature in March, June, September and December. If you plan to sell you Treasury bonds in the cash market in October to raise cash for Christmas gifts, you should sell T bond futures that mature in December.

D4. What are our cash outlays?
We have to post margin. At a minimum this has to be enough money to cover one day’s price change in the bond contract. The CBOT limits the daily price change to $3,000 per contract. If we sell 13 contracts, we would have to post $39,000 in cash or liquid assets like Treasury bills with the exchange.

We have sold 13 Treasury bond futures for delivery in say, December, and have deposited $39,000 with the exchange. Now we wait for tomorrow.

C6. Record value of the cash position if interest rates change.

Let us say for this example that T bond interest rates fall 10 basis points. Using equation (2), the cash market value of our T bonds rise by approximately \(-11.24/1.0566 \times 1,378,815 \times (-0.001) = 14,663.5\).

C7. When do these value changes occur?

They occur continuously as the cash market interest rate changes.

C8. When are they recorded?

This depends on the accounting system. If assets and liabilities are marked-to-market daily, they are recorded on a daily basis. If they are only marked-to-market when they are sold, they are recorded at the hedge horizon.

D5. Record value of the derivative if interest rates change.

Let us say for our example that the T bond futures rate declines 5 basis points. Using equation (3) the value of our futures position falls by \(-9.85/1.0562 \times (-1,400,000) \times (-0.0005) = -6,530\).

D6. When do these value changes occur?
They occur at the end of each day’s trading as the exchange marks the contracts to market.

D7. When do we record them?

Daily as they occur.

D8. What additional transactions do we make.

We either add to or remove money from our margin account. In the case where the value of our futures position fell, we might have to add money to our account.

C9. Record change in cash value.

The cash value rose $14,663.5.

C10. Price risk

The realized price risk of the bond is $14,663.5. You can approximate the ex ante price risk using the Value at Risk model.

D9. Record change in derivative value.

This is -$6,530.

D12. Liquidity risk

We have the liquidity risk that we have may have to post additional margin on short notice.

B4. Basis risk
The change in the value of the cash position was not offset by the change in the futures position. This is due to basis risk. Basis risk has several sources:

- The maturity of the futures contract may not match the hedge horizon. This occurs because futures contracts have set expiration dates. The T bond futures contract has only four expiration dates per year.
- The change in the cash market interest rate may not equal the change in the futures market interest rate. This occurred by assumption in my example.
- Because futures contracts come in set denominations it may not be possible to obtain the optimal value of futures that the hedge ratio specifies.

Basis risk in the example is $14,663.5 - $6,530 = $8,133.5.
### Duration of a 20-yr Treasury bond with a coupon of 8 and 7/8 % and a yield of 5.66%.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Cash flow</th>
<th>Discount factor</th>
<th>Present value</th>
<th>Maturity times present value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4.375</td>
<td>0.972847</td>
<td>4.256207</td>
<td>2.128104</td>
</tr>
<tr>
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<td>4.14064</td>
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<tr>
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<td>4.375</td>
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<td>4.375</td>
<td>0.847751</td>
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<tr>
<td>3.5</td>
<td>4.375</td>
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<tr>
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<td>0.759359</td>
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<td>4.375</td>
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<td>8.5</td>
<td>4.375</td>
<td>0.626267</td>
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<td>9</td>
<td>4.375</td>
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**Sum**: 137.8815  1549.341

**Duration**: 11.23676
Weekly Percent Changes in the 20-Year Constant Maturity Treasury Bond Rate: Data from FRED for 1962-1999.

Average = 0.000516
Standard deviation = 0.140131
Weekly 1990-1999

Change in 15 Year Treasury Bond Rate vs. Change in 20 Year Treasury Bond Rate


Change in 3-Month CD Rate

Change in 20-Year T Bond Rate
What is the notional value of a plain vanilla interest rate swap that is required to hedge the market value of a bank’s equity?

Notation:

- \( A \): Market value of assets
- \( D_A \): Duration of assets
- \( L \): Market value of liabilities
- \( D_L \): Duration of liabilities
- \( E \): Market value of equity
- \( R \): Market interest rate
- \( D_V \): Duration of the variable-rate instrument underlying the swap
- \( D_F \): Duration of the fixed-rate instrument underlying the swap
- \( S \): Market value of the swap. Usually this is zero at origination of the swap.
- \( N_S \): Nominal value of the swap. This is the unknown whose value we have to find.

Basic formulas:

\[
\Delta E = \left(D_A - \frac{L}{A} D_L\right) A \cdot \frac{\Delta R}{1 + R}
\]

Change in the market value of equity.

\[
\Delta S = -(D_V - D_F) \cdot N_S \cdot \frac{\Delta R}{1 + R}
\]

Change in the market value of the swap.

Notice that my formula for the change in the market value of the swap is different from the formula at the bottom of p. 628 of the textbook. Why is my formula correct? Start with the definition of the long and short positions in a swap on p. 623 of the textbook. The buyer of the swap makes fixed-rate payments and receives variable-rate payments. You can view the buyer as buying a variable-rate asset and paying for it by borrowing at a fixed-rate. Compare the two equations above. The asset duration comes first, followed by the leverage ratio, followed by the liability duration. For the swap buyer, the asset duration is \( D_V \), the leverage ratio is one, and the liability duration is \( D_F \).

\[
\Delta E + \Delta S = 0
\]

The hedge condition.

Notice that my formula for the hedge condition differs from the one on p. 629 of the textbook. Do you know why mine is correct?

Solution for the nominal value of the swap:

Substitute the two duration formulas in the hedge condition and solve for \( N_S \).

\[
N_S = \frac{-\left(D_A - \frac{L}{A} D_L\right) A}{(D_V - D_F)}
\]

The optimal nominal value of the swap.

If the bank is short-funded, its leverage-adjusted duration gap is positive. Since the duration of the variable-rate asset is always less than the duration of the fixed-rate asset, the denominator is always negative. Thus, \( N_S \) is positive for a short-funded bank. What does this
mean? A positive position means you own the swap. If you own the swap you make fixed payments and receive variable payments.

What is the optimal swap position for a long-funded bank?
Previous Tests
1. According to Campbell and Kracaw’s theoretical analysis, which of the ten financial services listed in the Course Pack is one of the most important functions of a financial intermediary? (5 points)

2. Is the intermediary in Campbell and Kracaw’s analysis a broker or an asset transformer? Explain briefly. (5 points)

3. In the Campbell/Kracaw analysis, what is meant by the cost of honesty? (5 points)

4. Briefly explain how each of the three cost economies might or might not explain why a bank has a low cost of being honest. (15 points)

5. If honesty is costly, who pays the bank to be honest? (5 points)

6. What do the parties that pay the bank to be honest gain from an honest bank? (5 points)
7. Give a formula and explanation for the cost of dishonesty in the Campbell/Kracaw analysis. (5 points)

8. How might we measure whether a bank has a high or low cost of dishonesty? (5 points)

9. Give and explain a formula that shows how to decide whether a bank is honest or dishonest. (15 points)

10. The book explains that banks add value when they act as delegated monitors. Explain briefly why depositors might trust banks to be their delegated monitors. (10 points)
Once there, go to “Institution Search.” Find the name of a commercial bank that you would like to analyze. Be sure it has stock that is traded on an exchange. Copy the bank’s FDIC Certificate #. Go to 
to find income statements and balance sheets for your chosen bank. Paste the FDIC Certificate # into the appropriate cell. Print the year-end financial statements for each of the last three years. (Use the custom category)
Use the Bloomberg terminal in the Foster Library to estimate the ADJ beta of the bank’s holding company. Use daily data to estimate beta for each year. Use the beta for the previous year to calculate the cost of capital for the current year.
Obtain the risk free from H.15, a Federal Reserve table available via the WWW. Use daily data from the “Business” column. Look for the one-year, constant maturity rate labeled tcm1y. Use the rate on Dec. 31 of the previous year for your analysis of the current year.

Previous students handed in a written analysis of the comparative financial performance of your bank for the last two years. The report included the following items in the following order:
1) A one-page written discussion of your findings. You should write this discussion as an analyst who is preparing a report for the bank’s current and potential stockholders. You should follow the framework that I present in the course Pack in conjunction with WAMU. Double-spaced.
2) A quantitative analysis of the financial performance of your bank for the last two years using the EVA-du Pont profit decomposition procedures in the Course Pack and in the class lectures. Present your results in the Excel spreadsheet format that mimics the one in the course Pack. No more than two decimals in your ratios.
3) A copy of your bank’s income statements for the last two years and balance sheets for the last three years. The copy must be a copy of an original source document.
4) A copy of all the beta data, risk-free interest rate data, and equity premium data that you use in your report as well as their sources. You must leave a clear citation trail so that someone can replicate your work.
5) The report used to be due in my hands at the start of the first test. Now, a question like this is likely to appear on the test.
1) Write a formula for a bank’s economic profit. Define your terms. (5 points)

2) Is economic profit a forward-looking or backward-looking measure of performance? Explain briefly. (5 points)

3) State and briefly explain each of the four types of interest rate risk. (8 points)

4) Identify each component of your bank’s economic profit equation that is exposed to interest rate risk. Briefly explain which type of interest rate risk affects each profit component that you have identified as being exposed to interest rate risk. (10 points)

5) Write a formula for the market value of a bank’s equity. Define any new terms that are not in your answer to question #1. (5 points)

6) Is market value a forward-looking or backward-looking measure of financial performance? Explain briefly. (5 points)

7) Identify each component in your bank’s value equation that is exposed to interest rate risk. Briefly explain which type of interest rate risk affects each value component that you have identified as being exposed to interest rate risk. (10 points)
8) Write a formula that relates changes in the value of a fixed income asset to changes in interest rates. Define your terms. Explain how your formula works. (10 points)

9) Write a formula that relates changes in a bank’s value to unexpected changes in the level of interest rates. Define your terms. (10 points)

10) Explain how a bank that immunizes its market value against changes in interest rates may not be providing all of the services associated with its role as an asset transformer. (10 points)

11) A 5-year maturity, 15% annual coupon bond with a face value of $1,000 is priced to yield 12%.
   a) What is the bond’s price? (You may use your calculator) (1 point)

   b) If the yield to maturity increases to 13%, what is the bond’s price? (You may use your calculator) (1 point)

   c) Using just the information in your answers to parts a) and b), and the definition of duration, what is the bond’s duration? Show your work. (10 points)
12) Bank A has assets composed solely of a 10-year, 12%, $1 million loan with annual interest payments and the principal due at maturity. It is financed with a 10-year, 10%, $1 million deposit with annual interest payments and principal due at maturity. Bank B has assets composed of a 7-year, 12% zero coupon bond with a current market value of $894,006.20. Its yield to maturity is 12%. It is financed with a 10-year, 8.275% coupon, $1 million face value CD with a YTM of 10%. Analyze the interest rate risk exposure of each bank. (30 points)
Use the data in the above spreadsheet to answer the following questions.

1. Write the RAROC formula. Define your terms. (10 points)

2. Calculate and report the cash flows to the bank from the loan that it should use to calculate the loan’s RAROC. (6 points)

3. Calculate the duration of the loan. (10 points)

4. Calculate the average annual cash income to the bank from the loan. (5 points)
5. Assume that the maximum change in the equity premium is 5%. Calculate the risk capital (the RAROC denominator) that the bank should assign to the loan. (10 points)

6. Using the average annual cash flow to the bank from the loan, what is the loan’s RAROC? (10 points)

7. What is the loan’s internal rate of return? Show your work. (5 points)

8. Why does the loan’s RAROC differ from its internal rate of return? (10 points)

9. When would the bank use discounted cash flow analysis to decide whether to make the loan, and when would it use RAROC analysis? (5 points)
10. A firm with assets whose market value is $300,000 is issuing $200,000 in two-year debt. The risk free interest rate is 6% and the standard deviation of the rate of change in the value of the borrower’s assets is 40%. Using the options pricing model, what is the market value of the loan and the risk premium to be charged on the loan? (20 points)

Merton has shown that the default premium for a risky loan is

\[ k(\tau) - i = \left(-\frac{1}{\tau}\right) \ln\left(\frac{1}{d}\right) \left[ N(h_1) + N(h_2) \right] \]

where:

- \( k(\tau) \) is the rate on a risky loan
- \( i \) is the rate on a risk free asset
- \( \tau \) is the remaining life of the loan.
- \( d \) is the borrower’s leverage ratio.
- \( \ln \) is the natural logarithm
- \( N(h) \) is the probability that a random variable with a standard normal density function will be less than or equal to \( h \).
- \( \sigma \) is the standard deviation of the change in the value of the borrower’s assets.

\[ h_1 = -\left[\frac{1}{2}\sigma^2 \tau - \ln(d)\right] / \sigma \tau^{1/2} \]

\[ h_2 = -\left[\frac{1}{2}\sigma^2 \tau + \ln(d)\right] / \sigma \tau^{1/2} \]
11. If a loan’s RAROC is greater than the loan’s hurdle rate, prove mathematically that the NPV of the loan is positive. (10 points)
Finance 423, Banking, Test 4, Spring 2003, Professor Hess
1. A bank has assets of $200 million that have duration of 7 years and yield 10%. It has $150 million in liabilities with duration of 3 years and yield of 6%. The Treasury bond futures contract has a price quote of $96 per $100 face value for the 20-year 8% coupon bond that is the benchmark for the futures contract. The ratio of the price sensitivity of the futures contract to the price sensitivity of the cash market interest rates is 0.90.
   a. Use your calculator to calculate the implied yield on the bond that underlies the futures contract. (5 points)

b. Use the duration formula, the market and face values of the bond that underlies the futures contract, and your answer to part a to estimate the duration of the bond that underlies the futures contract. (10 points)

c. What is the bank’s price risk if interest rates on assets and liabilities each increase by 100 basis points? (10 points)

d. Construct and explain a macro-hedge for the bank using financial futures. (20 points)

e. Calculate the hedge’s basis risk if interest rates increase 100 basis points. (10 points)
2. Use the following data to set the interest rates on a swap.
Notional principal $100
Maturity 1 year
Floating index 6-month LIBOR
Fixed coupon To be determined
Payment frequency Semi-annual
Day count 30/360
Spot yield curve 6-month yield is 8%
1-year yield is 10%

a. Compute the floating rate payments. (15 points)

b. Calculate the break-even fixed-rate payment. (10 points)

c. Check your to see that the NPV of the swap is zero. (10 points)

d. Mark the swap to market after a 100 basis point increase in market rates. (20 points)

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