1 Section 3.4

2. The characteristic equation is $9r^2 + 6r + 1 = 0$, with the double root $r = -1/3$. The general solution is $y(t) = c_1e^{-t/3} + c_2te^{-t/3}$.

4. The characteristic equation is $4r^2 + 12r + 9 = 0$, with double root $r = -3/2$. The general solution is $y(t) = (c_1 + c_2t)e^{-3t/2}$.

6. The characteristic equation is $r^2 - 6r + 9 = 0$, with the double root $r = 3$. The general solution is $y(t) = c_1e^{3t} + c_2te^{3t}$.

23. Set $y_2(t) = t^2v(t)$. Substitution into the ODE results in

$$t^2(t^2v'' + 4tv' + 2v) - 4t(t^2v' + 2tv) + 6t^2v = 0.$$  

After collecting terms, we end up with $t^4v'' = 0$. Hence $v(t) = c_1 + c_2t$, and thus $y_2(t) = c_1t^2 + c_2t^3$. Setting $c_1 = 0$ and $c_2 = 1$, we obtain $y_2(t) = t^3$.

28. Set $y_2(x) = e^xv(x)$. Substitution into the ODE results in

$$v'' + \frac{x-2}{x-1}v' = 0$$

This ODE is linear in the variable $w = v'$. An integrating factor is

$$\mu = e^{\int \frac{x-2}{x-1}dx} = \frac{e^x}{x}$$

Rewrite the equation as $\left[ \frac{e^x}{x} \frac{d}{dx} \right] = 0$, from which it follows that $v'(x) = c(x-1)e^{-x}$.

Hence $v(x) = c_1xe^{-x} + c_2$ and $y_2(x) = c_1x + c_2e^x$. Setting $c_1 = 1$ and $c_2 = 0$, we obtain $y_2(x) = x$.

2 Section 3.5

The numbers as in 9th edition. The 10th edition numbers are Section 3.5 2, 5, 11, 17, 19, 20.

2. The characteristic equation for the homogeneous problem is $r^2 + 2r + 5 = 0$, with complex roots $r = -1 \pm 2i$. Hence $y_c(t) = c_1e^{-t}\cos2t + c_2e^{-t}\sin2t$.

Since the function $g(t) = 3\sin2t$ is not proportional to the solutions of the homogeneous equation, set $Y = A\cos2t + B\sin2t$. Substitution into the given ODE, and comparing the coefficients, results in the system of equations $B - 4A = 3$ and $A + 4B = 0$. Hence $Y = \frac{12}{17}\cos2t + \frac{4}{17}\sin2t$. The general solution is $y(t) = y_c(t) + Y$.

3. The characteristic equation for the homogeneous problem is $r^2 - 2r - 3 = 0$, with roots $r = -1, 3$. Hence $y_c(t) = c_1e^{-t} + c_2e^{3t}$. Note that the assignment $Y = Ate^{-t}$ is not sufficient to match the coefficients. Try $Y = Ate^{-t} + Bt^2e^{-t}$. Substitution into the differential equation, and comparing the coefficients, results in the system of equations $-4A + 2B = 0$ and $-8B = -3$. This implies that $Y = \frac{3}{16}te^{-t} + \frac{3}{8}t^2e^{-t}$. The general solution is $y(t) = y_c(t) + Y$. 

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9. The characteristic equation for the homogeneous problem is \( r^2 + \omega_0^2 = 0 \), with complex roots \( r = \pm \omega_0 i \). Hence \( y_h(t) = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t \). Since \( \omega \neq \omega_0 \), set \( Y = A \cos \omega t + B \sin \omega t \). Substitution into the ODE and comparing the coefficients results in the system of equations \((\omega_0^2 - \omega^2)A = 1\) and \((\omega_0^2 - \omega^2)B = 0\). Hence
\[
Y = \frac{1}{\omega_0^2 - \omega^2} \cos \omega t.
\]
The general solution is \( y(t) = y_c(t) + Y \).

15. The characteristic equation for the homogeneous problem is \( r^2 - 2r + 1 = 0 \), with a double root \( r = 1 \). Hence \( y_h(t) = c_1 e^t + c_2 te^t \). Consider \( g_1(t) = te^t \). Note that \( g_1(t) \) is a solution of the homogeneous problem. Set \( Y_1 = Ate^t + Bte^t \) (the first term is not sufficient for a match). Upon substitution, we obtain \( Y_1 = t^3 e^t / 6 \). By inspection, \( Y_2 = 4 \). Hence the general solution is \( y(t) = c_1 e^t + c_2 te^t + t^3 e^t / 6 + 4 \). Invoking the initial conditions, we require that \( c_1 + 4 = 1 \) and \( c_1 + c_2 = 1 \). Hence \( c_1 = -3 \) and \( c_2 = 4 \).

17. The characteristic equation for the homogeneous problem is \( r^2 + 4 = 0 \), with roots \( r = \pm 2i \). Hence \( y_h(t) = c_1 \cos 2t + c_2 \sin 2t \). Since the function \( \sin 2t \) is a solution of the homogeneous problem, set \( Y = At \cos 2t + Bt \sin 2t \). Upon substitution, we obtain \( Y = -\frac{3}{4} t \cos 2t \). Hence the general solution is \( y(t) = c_1 \cos 2t + c_2 \sin 2t - \frac{3}{4} t \cos 2t \). Invoking the initial conditions, we require that \( c_1 = 2 \) and \( c_2 - \frac{3}{4} = -1 \). Hence \( c_1 = 2 \) and \( c_2 = -1/8 \).

18. The characteristic equation for the homogeneous problem is \( r^2 + 2r + 5 = 0 \), with complex roots \( r = -1 \pm 2i \). Hence \( y_h(t) = c_1 e^{-t} \cos 2t + c_2 e^{-t} \sin 2t \). Based on the form of \( g(t) \), set \( Y = Ate^{-t} \cos 2t + Bte^{-t} \sin 2t \). After comparing coefficients, we obtain \( Y = te^{-t} \sin 2t \). Hence the general solution is
\[
y(t) = c_1 e^{-t} \cos 2t + c_2 e^{-t} \sin 2t + te^{-t} \sin 2t.
\]
Invoking the initial conditions, we require that \( c_1 = 1 \) and \( -c_1 + 2c_2 = 0 \). Hence \( c_1 = 1 \) and \( c_2 = 1/2 \).