1 Section 3.1

2. Let \( y = e^{rt} \). Substitution of the assumed solution results in the characteristic equation \( r^2 + 3r + 2 = 0 \). The roots of the equation are \( r = -2, -1 \). Hence the general solution is \( y = c_1 e^{-2t} + c_2 e^{-t} \).

4. Substitution of the assumed solution \( y = e^{rt} \) results in the characteristic equation \( 2r^2 - 3r + 1 = 0 \). The roots of the equation are \( r = 1/2, 1 \). Hence the general solution is \( y = c_1 e^{t/2} + c_2 e^t \).

9. Substitution of the assumed solution \( y = e^{rt} \) results in the characteristic equation \( r^2 + r - 2 = 0 \). The roots of the equation are \( r = -2, 1 \). Hence the general solution is \( y = c_1 e^{-2t} + c_2 e^t \). Its derivative is \( y' = -2c_1 e^{-2t} + c_2 e^t \).

11. Substitution of the assumed solution \( y = e^{rt} \) results in the characteristic equation \( 6r^2 - 5r + 1 = 0 \). The roots of the equation are \( r = 1/3, 1/2 \). Hence the general solution is \( y = c_1 e^{t/3} + c_2 e^{t/2} \). Its derivative is \( y' = c_1 e^{t/3}/3 + c_2 e^{t/2}/2 \). Based on the first condition, \( y(0) = 1 \), we require that \( c_1 + c_2 = 4 \). In order to satisfy the condition \( y'(0) = 1 \), we find that \( c_1/3 + c_2/2 = 0 \). Solving for the constants, \( c_1 = 12 \) and \( c_2 = -8 \). Hence the specific solution is \( y(t) = 12e^{t/3} - 8e^{t/2} \).

20. The characteristic equation is \( 2r^2 - 3r + 1 = 0 \), with roots \( r = 1/2, 1 \). Therefore the general solution is \( y = c_1 e^{t/2} + c_2 e^t \) with derivative \( y' = c_1 e^{t/2}/2 + c_2 e^t \). To satisfy the initial conditions, we require that \( c_1 + c_2 = 2 \) and \( c_1/2 + c_2 = 1/2 \). Solving for the coefficients, \( c_1 = 3 \) and \( c_2 = -1 \). This means that the specific solution is \( y(t) = 3e^{t/2} - e^t \). To find the stationary point, set \( y' = 3e^{t/2}/2 - e^t \). There is a unique solution, with \( t_1 = \ln(9/4) \). This implies that the maximum value is then \( y(t_1) = 9/4 \). To find the \( x \)-intercept, solve the equation \( 3e^{t/2} - e^t = 0 \). The solution is readily found to be \( t_2 = \ln 9 \approx 2.1972 \).

2 Section 3.2

3. \[
W(e^{-2t}, te^{-2t}) = \begin{vmatrix}
  e^{-2t} & te^{-2t} \\
-2e^{-2t} & (1-2t)e^{-2t}
\end{vmatrix} = e^{-4t}.
\]

5. \[
W(e^t \sin t, e^t \cos t) = \begin{vmatrix}
  e^t \sin t & e^t \cos t \\
e^t(\sin t + \cos t) & e^t(\sin t - \cos t)
\end{vmatrix} = -e^{-2t}.
\]

9. Write the equation as \( y'' + \frac{3}{t-4} y' + \frac{4}{(t-4)^2} y = \frac{2}{(t-4)^2} \). The coefficients are not continuous at \( t = 0 \) and \( t = 4 \). Since \( t_0 \in (0, 4) \), the largest interval is \( 0 < t < 4 \).

24. Yes. \( y_1' = -4 \cos 2t; \ y_2'' = -4 \sin 2t \). \( W(\cos 2t, \sin 2t) = 2 \).