Asset Return Computations

1 Simple net returns

Assume we observe the price of an asset at the end of a period where the period could be days, months, or years, etc. We could then say that at the end of period \( t \), the asset price is \( P_t \) and at the end of the previous period, \( t - 1 \), the price was \( P_{t-1} \). We define the simple net return as the percentage change in the price:

\[
R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 = \% \Delta P_t
\]

2 Log returns

If we let \( R_t \) denote the simple net return, then we can define the continuously compounded return, \( r_t \), as:

\[
r_t = \ln(1 + R_t) = \ln \left( \frac{P_t}{P_{t-1}} \right)
\]

where \( \ln(\cdot) \) is the natural log function. We call this the continuously compounded return because we can manipulate the above equation into:

\[
P_t = P_{t-1}e^{r_t}
\]

so that \( r_t \) is the continuously compounded growth rate in prices between periods \( t-1 \) and \( t \) whereas \( R_t \) is the simple growth rate in prices between periods \( t-1 \) and \( t \) without any compounding. Note that the continuously compounded return is often referred to as the log return.

3 Annualized returns and annualized volatility

We can annualize continuously compounded returns and return volatility (the standard deviation of returns) using the following formulas:

\[
\begin{align*}
    r_A &= 100 \cdot T \cdot \text{mean}(r_t) \\
    \sigma_A &= 100 \cdot \sqrt{T} \cdot \text{sd}(r_t)
\end{align*}
\]

where:

\[
\begin{align*}
    r_A &= \text{annualized continuously compounded return} \\
    \sigma_A &= \text{annualized volatility} \\
    T &= \text{number of time periods per year} \\
    r_t &= \text{continuously compounded return series}
\end{align*}
\]