1 Reading

1. Hayashi, Chapters 3 and 6.

2. Hall, Chapter 2 and Chapter 3, section 5.


2 Derivations of Asymptotic Results

1. Hayashi, Chapter 3, Analytic Exercises, pages 243-250, #7 and 9. Note, these questions are very matrix algebra intensive but informative. Hayashi does a good job walking you through the steps.

2. Consider the simple AR(1) model

\[
    y_t = \rho y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2), \quad t = 1, \ldots, T
\]

\(|\rho| < 1, \quad y_0 \text{ is fixed.}\)

(a) Is \{y_t\} covariance stationary and ergodic? What are \(E[y_t]\) and \(\text{var}(y_t)\)?)

(b) Consider the sample mean \(\bar{y} = T^{-1} \sum_{t=1}^T y_t\). Show that \(\bar{y}\) is an unbiased and consistent estimator for \(E[y_t]\). For the consistency result, be sure to state the appropriate LLN.

(c) What is the asymptotic distribution of \(\sqrt{T} \bar{y}\)? Be sure to state the appropriate CLT to justify your result.

(d) How would you estimate the asymptotic variance of \(\sqrt{T} \bar{y}\)?
(e) The least squares estimator of $\rho$ is $\hat{\rho} = \left( \sum_{t=1}^{T} y_{t-1}^2 \right)^{-1} \sum_{t=2}^{T} y_{t-1}y_t$. Is $\hat{\rho}$ an unbiased estimator of $\rho$? Briefly explain.

(f) Show that $\hat{\rho}$ is a consistent estimator of $\rho$. Be sure to state the appropriate LLN to justify this result.

(g) Let $g_t = y_{t-1} \varepsilon_t$ and $I_t = \{y_t, y_{t-1}, \ldots, y_0\}$. Show that $\{g_t, I_t\}$ is a MDS.

(h) What is the asymptotic distribution of $\sqrt{T}(\hat{\rho} - \rho)$? Be sure to state the appropriate CLT to justify your result.

(i) How would you estimate the asymptotic variance of $\sqrt{T}(\hat{\rho} - \rho)$?

### 3 GMM Estimation with Serial Correlation

1. Hayashi, Chapter 6, Analytic Exercises, page 437-438, #9

2. Hayashi, Chapter 6, Empirical Exercises (pages 438 - 440): 1(b) - 1(f). You can use any software that can do OLS estimation with HAC standard errors (e.g. Eviews, Matlab, R, S+FinMetrics, Stata).