1. \[ P_{A,t-1} = 27, \quad P_{A,t} = 32 \]
   \[ P_{c,t-1} = 15, \quad P_{c,t} = 12 \]
   \[ \frac{P_{A,t} - P_{A,t-1}}{P_{A,t-1}} = \frac{32 - 27}{27} = 0.185 \quad \text{and} \quad \frac{P_{c,t} - P_{c,t-1}}{P_{c,t-1}} = \frac{12 - 15}{15} = -0.20 \]

2. \[ r_{A,t} = \ln(1 + 0.185) = 0.190 \quad \text{and} \quad r_{c,t} = \ln(1 - 0.20) = -0.223 \]

3. \[ \frac{P_{c,t}}{P_{A,t}} = \frac{12 + 3}{15} = 0.60 \quad \text{Dwivedi yield} = \frac{3}{15} = 0.20 \]

4. Initial wealth = 27 + 15 = 42
   \[ X_A = \frac{27}{42} = 0.64 \quad \text{and} \quad X_C = \frac{15}{42} = 0.36 \]
   \[ R_p = X_A P_A + X_C P_C = (0.64)(0.185) + (0.36)(-0.20) = 0.042 \]
   Check: Initial wealth = 42
   \[ \frac{P_{A,t} + P_{C,t}}{2} = \frac{44}{2} = 22 \]
   \[ \text{End of period wealth} = 44 \quad \text{and} \quad \text{difference due to rounding} \]

5. \[ r_p = \ln(1 + 0.042) = 0.045 \]
1. Normal Distribution

\[ X \sim N(0.01, (0.50)^2) \]

\[ \mu = 0.01 \]
\[ \sigma = 0.50 \]

Given the above diagram:

\[ \mu - \sigma = 0.01 - 0.50 = -0.49 \]
\[ \mu - 2\sigma = 0.01 - 1 = -0.99 \]
\[ \mu - 3\sigma = 0.01 - 1.5 = -1.49 \]

2. Because roughly 99% of the area is between \( \mu \pm 3\sigma \), the 0.5% quantile is roughly \( \mu - 3\sigma = -1.49 \).

3. Given the above sketch, about 2.5% probability is less than -0.99. So a guess for \( P(X < -1) \) is about 2.5%.

4. No, because the smallest value for a simple return is \( -1 \) (-100%). This distribution allows values less than -1 with a non-negligible probability.

5. Yes, because \( r = \ln(1 + R) \) or \( R = e^r - 1 \). A value for \( r < -1 \) implies a value for \( R > -1 \). E.g., let \( r = -2 \). Then \( R = e^{-2} - 1 = -0.864 \).
1. Graphically, the returns look quite normal — roughly symmetric histograms & densities and fairly linear qq-plots.

However, the histograms & boxplots indicate a slight negative skewness (long left tail)

The numerical summaries show negative sample skewness. The excess kurtosis values are small, which indicates that the tails of the distribution are close to normal.

Amun appears much riskier than Dow Jones because its SD value, 0.225, is 4 times larger than the SD value for Dow Jones, 0.051.

2. The plug-in principle estimates are

<table>
<thead>
<tr>
<th></th>
<th>Amun</th>
<th>Dow Jones</th>
</tr>
</thead>
<tbody>
<tr>
<td>\hat{\mu}</td>
<td>0.018</td>
<td>0.0041</td>
</tr>
<tr>
<td>\hat{\sigma}^2</td>
<td>0.051</td>
<td>0.003</td>
</tr>
<tr>
<td>\hat{\sigma}</td>
<td>0.226</td>
<td>0.051</td>
</tr>
<tr>
<td>\hat{\sigma}_j</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>\hat{\rho}_j</td>
<td>0.469</td>
<td>0.469</td>
</tr>
<tr>
<td></td>
<td>Amazon</td>
<td>Dow Jones</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>( \hat{\mu} )</td>
<td>0.018</td>
<td>0.004</td>
</tr>
<tr>
<td>( SE(\hat{\mu})_{\text{boot}} )</td>
<td>0.030</td>
<td>0.006</td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>0.226</td>
<td>0.051</td>
</tr>
<tr>
<td>( SE(\hat{\sigma})_{\text{boot}} )</td>
<td>0.021</td>
<td>0.005</td>
</tr>
</tbody>
</table>

The values of \( SE(\hat{\mu})_{\text{boot}} \) are large compared to the estimate \( \hat{\mu} \) for both Amazon and Dow Jones. The \( SE(\hat{\mu})_{\text{boot}} \) values are about 2x the \( \hat{\mu} \) values.

The approximate 95% confidence intervals are

\[ \hat{\mu} \pm 2 \times SE(\hat{\mu})_{\text{boot}}. \]

These intervals will contain both positive and negative values for \( \mu \). Hence, there is a lot of uncertainty about the values for \( \mu \).

The values of \( SE(\hat{\sigma})_{\text{boot}} \) are much smaller than the estimates of \( \sigma \) for both Amazon and Dow Jones (about 10x smaller). The approximate 95% confidence intervals, \( \hat{\sigma} \pm 2 \times SE(\hat{\sigma})_{\text{boot}} \), are very narrow and imply that \( \sigma \) is well estimated for both Amazon and Dow Jones.
4. Analytic formulas for SE

\[ \text{SE}(\hat{\mu}) = \frac{\hat{\sigma}}{\sqrt{n}}, \quad \text{SE}(\hat{\sigma}) = \frac{\hat{\sigma}}{\sqrt{2n}} \]

Amazon

\[ \text{SE}(\hat{\mu})_{\text{boot}} = 0.030 \]

\[ \frac{0.220}{\sqrt{61}} = 0.029 \]

\[ \frac{0.051}{\sqrt{61}} = 0.0065 \]

Dow Jones

\[ \text{SE}(\hat{\mu})_{\text{boot}} = 0.006 \]

\[ \frac{0.220}{\sqrt{12.61}} = 0.020 \]

\[ \frac{0.051}{\sqrt{12.61}} = 0.0046 \]

The bootstrap SE values are very close to the analytic formulas. As a result, the bootstrap distributions of \( \hat{\mu} \) and \( \hat{\sigma} \) are symmetric.

5. $100,000 initial investment in Dow Jones.

\[ \hat{\sigma}_{0.05} = \hat{\mu} + \hat{\sigma}(-1.645) \]

\[ = 0.004 + (0.051)(-1.645) \]

\[ = -0.0799 \]

\[ \exp(\hat{\sigma}_{0.05}) - 1 = -0.0769 \]

\[ \text{Var}_{0.05} = \frac{\$100,000}{(0.0769)} = \frac{\$}{76290} \]

i.e. you chance to lose \$76290

\[ \frac{\$76290}{100000} = 0.0769 \]
VI. Portfolio Theory

1. \( \mu_p = 0.5 \mu_A + 0.5 \mu_B \)
   \[
   \mu_p = 0.5 (0.05) + 0.5 (0.07) = 0.06
   \]
   \[
   \sigma_p^2 = (0.5)^2 (0.10)^2 + (0.5)^2 (0.20)^2 + 2 (0.5)(0.5)(0.0)
   \]
   \[
   = 0.0125
   \]
   \[
   \sigma_p = 0.112
   \]

2. \( \mu_p \)

   \[
   x_A = 1, x_B = 0
   \]
   \[
   x_A = 0, x_B = 1
   \]
   \[
   \text{Slope } = \frac{\mu_A - \mu_B}{\sigma_A} = \frac{0.05 - 0.02}{0.10} = 0.3
   \]

3. \( \mu_p = x_A \mu_A + x_B (\mu_B - \mu_A) \)

   \[
   \text{Solve: } 0.10 = \mu_p = 0.02 + x_B (0.07 - 0.02)
   \]
   \[
   \implies x_B = \frac{0.10 - 0.02}{0.07 - 0.02} = \frac{0.08}{0.05}
   \]
   \[
   = 1.6
   \]
4. \[ \sigma_p = \psi \cdot \sigma_p \]

Solve \[ 0.10 = \sigma_p = \psi \cdot 0.20 \]

\[ \Rightarrow \psi = \frac{0.10}{0.20} = 0.5 \]

5. The global minimum variance portfolio solves

\[ \min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j x_i x_j + \sum_{i=1}^{n} \sum_{j=i+1}^{n} 2 \sigma_i \sigma_j x_i x_j \]

The analytic solution is

\[ x_A = \frac{\sigma_B^2 - \sigma_{AB}}{\sigma_A^2 + \sigma_B^2 - 2 \sigma_{AB}} \]

\[ x_A = \frac{0.20^2 - 0}{(0.10)^2 + (0.20)^2 - 0} \]

\[ = 0.80 \]

\[ x_B = 1 - x_A = 1 - 0.80 = 0.20 \]

\[ \mu_p^{\min} = (0.80)(0.05) + (0.20)(0.07) \]

\[ = 0.054 \]