Econ 422
Summer 2006
Midterm Exam Solutions

This is a closed book exam. However, you are allowed one page of notes (double-sided). Answer all questions. For the numerical problems, if you make a computational error you may still receive full credit if you provide the correct formula for the problem. There are 20 questions, and each question is worth 5 points. Total points = 100. You have 1 hour and 50 minutes to complete the exam. Good luck.

I. Intertemporal Consumption and Investment Decisions (25 points, 5 points each)

Mia lives in a two period Fisherian world. Her utility function for consumption streams can be written as $U(C_0, C_1) = C_0^{0.25}C_1^{0.75}$ hence her marginal rate of substitution is

$$MRS = -\frac{U_0}{U_1} = -\frac{C_1}{3C_0}$$

Her endowment of present and future resources is $(100,000, 100,000)$. She can borrow or lend at a market real interest rate of 3 percent.

a. What is her current wealth?

$$W_0 = Y_0 + \frac{Y_1}{1 + r} = 100,000 + \frac{100,000}{1.03} = 197,087.38$$

b. Write down the equation for her budget constraint. What is the slope of the budget constraint line?

$$BC: C_0 + \frac{C_1}{1 + r} = W \Rightarrow C_0 + \frac{C_1}{1.03} = 197,087.38$$

*The slope of the budget constraint is $-(1 + r) = -1.03$*

c. Derive the marginal rate of substitution.
\[
MRS = -\frac{U_0}{U_1}, \quad U_0 = \frac{\partial U}{\partial C_0} = \frac{1}{4}C_0^{-3/4}C_1^{3/4}, \quad U_1 = \frac{\partial U}{\partial C_1} = -\frac{3}{4}C_0^{1/4}C_1^{-1/4}
\]

\[
\Rightarrow -\frac{U_0}{U_1} = \frac{1}{4}C_0^{-3/4}C_1^{3/4} - \frac{3}{4}C_0^{1/4}C_1^{-1/4} = -\frac{C_1}{3C_0}
\]

d. What is her optimal consumption in the present and in the future?

Optimal consumption satisfies \(MRS = -(1 + r)\) and the budget constraint.

\[
MRS = -(1 + r) : \quad \frac{C_1}{3C_0} = -1.03 \Rightarrow C_1 = 3.09C_0
\]

\[
BC : C_0 + \frac{C_1}{1 + r} = W \Rightarrow C_0 + \frac{3.09C_0}{1.03} = 197,087.38
\]

\[
\Rightarrow 4C_0 = 197,087.38 \Rightarrow C_0 = 49,271.85
\]

\[
C_1 = 3.09C_0 \Rightarrow C_1 = 152,250
\]

e. What financial transaction is required to attain the optimal consumption stream?
Indicate what is exchanged for what, when, and indicate the amounts involved.

Since \(C_0 = 49,271.85 < Y_0 = 100,000\), the individual saves 50,728.15 and lends in the capital market at 3% interest per period. The future value of this saving is 50,728.15(1.03) = 52,250. Future consumption is the future endowment of 100,000 plus the return to savings of 52,250 for a total of 152,250.

**II. Present value computations (25 points, 5 points each)**

a. Suppose you invest $10,000 today in a money market fund earning 5% per year. How much will you have in your account after 10 years, 20 years and 30 years?

\[
FV = $10,000(1.05)^T.
\]

\[
T = 10 : FV = $16,288.95
\]

\[
T = 20 : FV = $26,532.98
\]

\[
T = 30 : FV = $43,219.42
\]

b. Now suppose you invest $10,000 today and every year thereafter in a money market fund earning 5% per year. How much will you have in your account after 10 years, 20 years and 30 years?

*Here the cash flows are \(C_t = 10,000\) for \(t = 0, 1, \ldots, T\). Hence, we have a finite annuity with payments starting in the initial period (not the first period as usually assumed). We*
compute future value in 2 steps. First compute the PV of the cash flows and then use the relationship \( FV = PV*(1 + r)^T \). The PV of the cash flows are

\[
PV = 10,000 + 10,000*PVA(r, T) = 10,000(1 + PVA(r, T))
\]

where \( PVA(r, T-1) \) is the PV of a finite annuity that pays $1 for \( T \) periods.

Now,

\[
PVA(r, T) = \frac{1}{r} \left[ 1 - \left( \frac{1}{1+r} \right)^T \right]
\]

\( T = 10 : PVA(0.05, 10) = \$7.72 \)
\( T = 20 : PVA(0.05, 20) = \$12.46 \)
\( T = 30 : PVA(0.05, 30) = \$15.37 \)

Our final results are

\( T = 10 : FV = 10,000*(1 + PVA(r, T))(1 + r)^T = \$142,067.87 \)
\( T = 20 : FV = 10,000*(1 + PVA(r, T))(1 + r)^T = \$357,192.52 \)
\( T = 30 : FV = 10,000*(1 + PVA(r, T))(1 + r)^T = \$707,607.90 \)

c. Finally, suppose you invest $10,000 today in a money market fund earning 5% per year. Going forward, you decide to increase your contributions to the money market fund by 3% each year. That is, in year \( t \) you invest \( 10,000*(1.03)^t \). How much will you have in your account after 10 years, 20 years and 30 years?

Here we have a finite growing annuity with payments starting in the initial period. We do the computation as in the part \( c \). First compute the PV of the cash flows and then use the relationship \( FV = PV*(1 + r)^T \). The PV of the cash flows are

\[
PV = 10,000 + 10,000*PVGA(r,g,T) = 10,000(1 + PVGA(r,g,T))
\]

where \( PVGA(r,g,T) \) is the PV of a finite growing annuity that pays $1 for \( T \) periods.

Now,

\[
PVGA(r, g, T) = \frac{1}{r-g} \left[ 1 - \left( \frac{1+g}{1+r} \right)^T \right]
\]

\( T = 10 : PVGA(0.05, 0.03, 10) = \$8.75 \)
\( T = 20 : PVGA(0.05, 0.03, 20) = \$15.96 \)
\( T = 30 : PVGA(0.05, 0.03, 30) = \$21.92 \)
Our final results are

\[ T = 10 : FV = 10,000 \times (1 + PVA(r,T))(1 + r)^T = 158,778.07 \]
\[ T = 20 : FV = 10,000 \times (1 + PVA(r,T))(1 + r)^T = 450,126.21 \]
\[ T = 30 : FV = 10,000 \times (1 + PVA(r,T))(1 + r)^T = 990,559.38 \]

d. Suppose you invest $100 for 2 years and achieve a future value of $120. What is the annual rate of return on this investment assuming annual compounding? What is the annual rate of return assuming that interest is compounded semi-annually?

\[ PV = \frac{FV}{(1 + r)^T} \Rightarrow r = \left( \frac{FV}{PV} \right)^{1/T} - 1 \]
\[ r = \left( \frac{120}{100} \right)^{1/2} - 1 = 9.54\% \]
\[ PV = \frac{FV}{(1 + \frac{r}{2})^{2T}} \Rightarrow r = 2 \times \left[ \left( \frac{FV}{PV} \right)^{1/2T} - 1 \right] \]
\[ \Rightarrow r = 2 \times \left[ \left( \frac{120}{100} \right)^{1/4} - 1 \right] = 9.327\% \]

e. Suppose Washington Mutual grants you a $500,000, 30 year mortgage with a fixed annual interest rate of 6%. Assuming the mortgage requires monthly payments, what is your monthly payment amount?

\[ PV = C \times PVA(r/12, T) \Rightarrow C = PV / PVA(r/12, 12 \times T) \]
\[ \Rightarrow C = 500,000 / PVA(0.06/12, 12 \times 30) = 500,000 / 166.79 = 2,997.75 \]

III. Bond Pricing and the Term Structure of Interest Rates (25 points, 5 points each)

The following is a list of prices for zero coupon bonds of various maturities:

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Price of zero coupon bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$966.18</td>
</tr>
<tr>
<td>2</td>
<td>$924.56</td>
</tr>
<tr>
<td>3</td>
<td>$876.30</td>
</tr>
<tr>
<td>4</td>
<td>$838.56</td>
</tr>
</tbody>
</table>

The bond prices are quoted as a percentage of par (face) value, and the par value is $1,000. Please answer the following questions:
a. Calculate the spot rates associated with each bond, and plot the term structure of interest rates.

Spot rates are computed using

\[ r_{0,T} = \left( \frac{FV}{PV} \right)^{1/T} - 1 \]

\[ r_{0,1} = \left( \frac{1000}{966.18} \right) - 1 = 0.035 \]

\[ r_{0,2} = \left( \frac{1000}{924.56} \right)^{1/2} - 1 = 0.040 \]

\[ r_{0,3} = \left( \frac{1000}{876.30} \right)^{1/3} - 1 = 0.045 \]

\[ r_{0,4} = \left( \frac{1000}{838.56} \right)^{1/4} - 1 = 0.045 \]

b. Calculate the implied 1-year forward rates, \( f_{t-1,t} \), for \( t=2, 3, 4 \).

The implied forward rates may be computed using the formulas

\[ f_{t-1,t} = \frac{(1 + r_{0,t})}{(1 + r_{0,t-1})^{t-1}} - 1 = \frac{P_{0,t-1}}{P_{0,t}} - 1 \]

\( f_{1,2} = 0.0450 \)

\( f_{2,3} = 0.0551 \)

\( f_{3,4} = 0.0450 \)

c. If \( f_{4,5} = 0.045 \), compute the price of the zero coupon bond that matures in 5 years.

Here we use the no arbitrage relationship

\[ f_{t-1,t} = \frac{P_{0,t-1}}{P_{0,t}} - 1 \Rightarrow P_{0,t} = \frac{P_{0,t-1}}{(1 + f_{t-1,t})} \]

\[ \Rightarrow P_{0,5} = \frac{P_{0,4}}{(1 + f_{4,5})} = \frac{838.56}{(1 + 0.045)} = 802.45 \]

d. The figure below shows the current Treasury yield curve (taken yesterday from the Yahoo! Bonds page).
If the expectations hypothesis of the term structure holds, what does the information in the yield curve say about the course of future short-term interest rates?

*Under the expectations hypothesis, the implied forward rates are the best forecasts of future spot rates. The flat term structure implies constant implied forward rates. So the current term structure predicts constant future short rates of about 5%.*

e. Using the information in the term structure from part d above, what is the approximate price of a 4 year coupon bond making annual coupon payments with an annual coupon rate of 5% and a face value of $1,000?

*Since the term structure is flat at about 5% (constant market yield at all maturities), the bond will price at par value (1000) since the coupon rate is also 5%.*

**IV. Stock Price Valuation and Capital Budgeting (25 points, 5 points each)**

Suppose Exxon’s stock is forecast to pay a dividend of $2.15 at the end of the year, and then the dividend is forecasted to grow at 11.2% per year forever. The required rate of return (market capitalization rate) on Exxon stock is 15.2% per year.

a. What is the current price of Exxon stock? What is the price of Exxon’s stock at the beginning of next year?

*Here, Exxon’s stock behaves like a growing perpetuity with first payment 2.15 and growth rate 11.2%. Using a market discount rate of 15.2%, the current stock price is*

\[
PV = \frac{D}{r - g} = \frac{2.15}{0.152 - 0.112} = 53.75
\]
b. If an investor were to buy Exxon stock now and sell it after receiving the $2.15 dividend a year from now, what is the capital gain in percentage terms? What is the dividend yield, and what is the total rate of return on the investment?

After receiving the dividend, Exxon’s stock again behaves like a growing perpetuity. However, now the first dividend payment is $2.15 \times 1.112 = 2.3908$. The sales price of the stock is then

$$PV = \frac{D}{r - g} = \frac{2.3908}{0.152 - 0.112} = 59.77$$

The capital gain is

$$gain = \frac{P_1 - P_0}{P_0} = \frac{59.77 - 53.75}{53.75} = 0.112$$

The dividend yield is

$$yield = \frac{D}{P_0} = \frac{2.15}{53.75} = 0.04$$

The total holding period return is then

$$HPR = gain + yield = 0.152$$

Consider two mutually exclusive projects called project A and project B, with the following cash flows:

<table>
<thead>
<tr>
<th>Project/Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-100</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>B</td>
<td>-120</td>
<td>70</td>
<td>97</td>
</tr>
</tbody>
</table>

The net present values of the projects, as a function of the discount rate $r$, are plotted in the figure below.
Please answer the following questions:

c. Based on the figure, approximately what are the internal rates or return (IRRs) for projects A and B?

\[ IRR \text{ is the interest rate at which } NPV = 0: \]

\[ IRR_A \approx 26\% \]
\[ IRR_B \approx 24\% \]

d. If the discount rate is \( r = 8\% \), which project is chosen by the NPV rule and which project is chosen by the IRR rule?

\[ NPV \text{ rule: choose project with highest } NPV > 0 \]
\[ IRR \text{ rule: choose project with highest } IRR > r \]

At \( r = 8\% \), \( NPV_B > NPV_A \) so NPV rule says choose project B. However, \( IRR_A > IRR_B \) so IRR rule says choose project A.

e. If the discount rate is \( r = 22\% \), which project is chosen by the NPV rule and which project is chosen by the IRR rule?

At \( r = 8\% \), \( NPV_B > NPV_A \) so NPV rule says choose project B. However, \( IRR_A > IRR_B \) so IRR rule says choose project A.
At \( r = 22\% \), \( NPV_A > NPV_B \) so NPV rule says choose project \( A \). However, \( IRR_A > IRR_B \) so IRR rule says choose project \( A \).