1) \[ \bar{V} = 2600 \text{ cm}^{-1} \quad k = \? \quad T = \? \quad \mu = \frac{m_H m_e}{m_H + m_e} = 1.64 \times 10^{-27} \text{ kg} \]

\[ k = \left( 2\pi c \bar{V} \right)^2 \mu = \frac{394}{m} \]

\[ T = 2\pi \sqrt{\frac{\mu}{k}} = 1.28 \times 10^{-14} \text{ s} \]

2) \[ V = \int dv = \int_0^{2\pi} \int_0^\pi \int_0^a r^2 \sin(\theta) \, dr \, d\theta \, d\phi \]

\[ = \int_0^{2\pi} d\phi \int_0^\pi \sin(\theta) d\theta \int_0^a r^2 dr \]

\[ = \left[ 2\pi \right] \left[ 2 \right] \left[ \frac{1}{3} a^3 \right] = \frac{4\pi}{3} a^3 \]

↑ volume of a sphere!

3) Electrons in a finite square well have more energy with increasing n-values. The electrons with larger n-values are less tightly held by the nucleus of the atom or molecule (they are less confined) than electrons with low n-values. They are able to tunnel further through a given potential than electrons of low energy and their overlap probability with other atoms is higher. The electrons with larger n-values are the valence electrons.

4) \[ V(x) = D e \left( 1 - e^{-Bx} \right)^2 \leftarrow \text{Taylor Expand around } x = 0 \]

\[ U(x) = U(0) + (x) \left( \frac{3U}{2x} \right)_{x=0} + (x)^2 \left( \frac{3U}{2x^2} \right)_{x=0} + \frac{x^3}{3!} \left( \frac{3^3U}{2x^3} \right)_{x=0} + \ldots \]

\[ \frac{\partial V}{\partial x} = De \left( 2be^{-Bx} - 2be^{-2Bx} \right) \quad \text{at } x = 0 \quad \frac{\partial V}{\partial x} \big|_{x=0} = 0 \]

\[ \frac{\partial^2 V}{\partial x^2} = De \left( -2B^2 e^{-Bx} + 4B^2 e^{-2Bx} \right) \quad \to \quad \frac{\partial^2 V}{\partial x^2} \big|_{x=0} = De \left( 2B^2 \right) \]
\[
\frac{2^3V}{2x^3} = De \left( 2\beta^3 e^{-\beta x} - 8\beta^3 e^{-2\beta x} \right) \rightarrow \frac{2^3V}{2x^3} \bigg|_{x=0} = - De (6\beta^3)
\]

\[
V(x) = x^2 2De \beta^2 - x^3 6De \beta^3
\]

Compare to \( V(x) = \frac{1}{2} kx^2 \) ← simplest approximation

\[ k = 2De \beta^2 = \boxed{484 \frac{N}{m}} \]

5) The harmonic oscillator is a good approximation for small displacements of \( x \). It is a good approximation for the vibrations that occur between atoms in a molecule or crystal. This approximation is good for H-Cl molecules at low energy, as the energy increases the displacement from equilibrium increases relative to the bond length (see 7.11).

The harmonic oscillator is a bad approximation at high energies, as can be seen from the divergence of the H.O. potential and the Morse potential from problem #4. At higher energies more terms from the Taylor expansion of the potential need to be taken into account.
for $D_e=1$, $B=1$, $k=2$