Question 1. (This question is entirely algebraic.) The splendid shape of the flask shown below is such that when it contains a volume $V$ of liquid, the level $h$ of the liquid is given over a wide range by $h = \alpha V^2$, where $\alpha$ is a constant.

A volume $V_0$ of water (of density $\rho$) is poured into the flask at room temperature.
(a) What is the total weight $W$ of the water? [1]

(b) What is the total force $F_{\text{bottom}}$ exerted on the bottom of the flask (area $A$) by the water? [2]

(c) Explain why $W$ and $F_{\text{bottom}}$ different. In what type of container would they be the same? [4]
The water is now warmed so that its density decreases by an amount \( \delta \rho \).
(d) What is the change in \( F_{\text{bottom}} \)? [3]

Explain how \( F_{\text{bottom}} \) can increase while the weight of the water stays the same. [2]

The water is allowed to cool back to room temperature. The stopper is now removed so that the water can flow out freely through the spout at the bottom, whose cross-sectional area is \( a \).
(e) For a volume \( V \) of water, at what speed is water ejected from the spout? [5]

(f) What assumptions did you make in this calculation? [2]

(g) Show that the total time taken for half the water to run out is \( \ln 2/\{a(2g \alpha)^{1/2}\} \). [6]

The flask is refilled and a rubber tube is attached to the spout, with its end dangling at vertical distance \( x \) below the bottom of the flask.
(h) How fast does the water now emerge from the flask? [2]

(i) Why is it faster than without the tube? [3]

**Question 2.**

**Part 1.**

(a) Show that an energy density associated with a free liquid surface leads to a force per unit length exerted by the surface (i.e., a surface tension). [4]

A cylindrical tube of diameter \( d = 0.1 \) mm is placed vertically with one end in open water:

(b) Sketch the contours of pressure throughout the water. [3]
(c) Show that the maximum height to which the water could be raised up the tube by capillary action is 0.3 m. (The surface tension of water at room temperature is $\gamma = 0.073$ N/m.) [5]

(d) Discuss whether the pressure in the water column could ever be negative, and if so under what circumstances. [2]

Part 2. A piece of ice is floating in a beaker of water held at 1 degree C.
(e) After the ice melts, is the water higher, lower, or the same as before? Explain. [2]

(f) Would the answer be different if the ice had air bubbles in it? Explain. [2]

(g) What would the answer be if the ice had sand in it? Explain. (Hint: think of the problem of throwing a stone out of a boat in a swimming pool). [4]

Part 3.
(h) A clever prairie dog knows how to get rid of unpleasant odors in his burrow. He makes one entrance raised above the ground and the other one flush, as indicated below. Sketch streamlines for a
steady wind blowing across the landscape, and explain why the burrow remains well ventilated by a steady draft. [5]

(i) If the diameter of the tunnel is 20 cm, and the average air speed is 1 m/s, will his ventilation system be turbulent? (Take the density of air to be 1 kg/m³ and the viscosity to be $1.5 \times 10^{-5}$ Ns/m², and use the Reynolds number of 2000 for flow along a pipe.) [3]

Part 4 - (4 bonus marks – don’t answer if you don’t have time)

(j) A partygoer takes a helium balloon onto a train and lets it float against the ceiling. When the train brakes suddenly as it approaches a hazard, does the balloon float towards the front or the rear of the carriage? Explain your answer.