A 0.440-kg block of wood hangs from the ceiling by a string, and a 0.0720-kg wad of putty is thrown straight upward, striking the bottom of the block with a speed of 5.74 m/s. The wad of putty sticks to the block. (a) Is the mechanical energy of this system conserved? (b) How high does the putty-block system rise above the original position of the block?

This problem has two distinct parts:

1. An inelastic collision (E not conserved)
2. Followed by motion of the combined masses under the influence of a conservative force (so energy is conserved).

Part 1. By momentum conservation during inelastic collision (F_{ext} = 0) better: F_{impact} = F_{grav} = F_{ext}

\[ p_i = mv_i = (m + M)v = P_f \]

so \( V = \left( \frac{m}{m + M} \right) V_e = \left( \frac{0.072 kg}{0.072 kg + 0.44 kg} \right) 5.74 m/s = 0.83 m/s \] where \( V \) is the speed of \( m + M \) the instant after the collision.

Part 2. Now energy is conserved, \( \Delta E = \Delta U + \Delta K \Rightarrow \Delta K = -\Delta U \)

or, \( K_f - K_i = K_i^0 - U_f \) where \( K_i \) is kinetic energy immediately after the collision and \( K_f \) is \( K \) at maximum altitude, \( h \).

So: \( K_i = U_f = (m + M)gh = \frac{1}{2} (m + M) V_e^2 \Rightarrow h = \frac{V_e^2}{2g} \approx 3.3 \text{ cm} \)
Elastic Collisions — some general comments

These are collisions that conserve both energy and momentum of the colliding bodies. From this condition, you text presents the relations:

\[
\begin{align*}
V_{1f} &= \left( \frac{m_1 - m_2}{m_1 + m_2} \right) V_i \\
V_{2f} &= \left( \frac{2m_1}{m_1 + m_2} \right) V_i 
\end{align*}
\]

For case where particle 1 (with speed \( V_i \)) collides with particle 2 which is at rest

One can then show:

\[
V_{2f} - V_{1f} = \left( \frac{2m_1}{m_1 + m_2} - \frac{m_1 - m_2}{m_1 + m_2} \right) V_i = \left( \frac{m_1 + m_2}{m_1 + m_2} \right) V_i = V_i
\]

or, noting that \( V_i = V_{1i} \) and \( V_{2i} = 0 \),

\[
|V_{2f} - V_{1f}| = |V_{2i} - V_{1i}|
\]

In words, this implies (for this special case)

"The speed of separation after a 1-D (head-on) collision is always equal to the speed of approach of the two bodies before collision."

It can be shown (using the equations in Prob. 74 of Ch 9) that this is generally true — for both bodies moving initially, and viewed from any inertial frame, (but only for a 1-D collision)."
Clearly true for \( v_{1x} = v, v_{2x} = 0 \) and \( m_1 = m_2 = m \)

\[ \begin{align*}
\text{A.} & \quad o_1 \rightarrow o_2 \quad \text{initial} \\
& \quad o_1 \leftarrow o_2 \quad \text{at rest} \quad \text{final} \\
\text{Let } M \gg m & \quad \Rightarrow o_1 \rightarrow o_2 \quad \text{final} \\
\text{But for } M \quad o_1 \leftarrow o_2 \quad \text{final} \\
\text{Now if } v_{1x} = -v_{2x} = v & \quad \Rightarrow \text{both moving} \\
\text{D.} & \quad o_1 \rightarrow o_2 \quad \text{initial} \\
& \quad o_1 \leftarrow o_2 \quad \text{final} \\
\text{In each case approach speed = separation speed.} \\
\text{For these simple cases, elastic collision behavior is transparent.} \\
\text{\textit{*the ones illustrated here}}
\end{align*} \]

\[ \text{Demos: Each of the cases A, B, C, D -- and a combination of 4 balls that compound the gain in speed of the smallest ball suggested in D.} \]