The Search for Stationarity in Real Exchange Rates: An Unobserved Component Regime-Switching Approach

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Abstract

The Purchasing Power Parity Puzzle raised in Rogoff (1996) questions how the observed large short-term volatility in real exchange rates (RERs) can be reconciled with their high persistence, when in theory, volatile (e.g. monetary or financial) shocks should be short-lived and persistent real shocks should not occur so frequently. Against this backdrop, this paper analyzes over 100 years of RER behavior using models that allow for both stationary and nonstationary components, rather than forcing the data to choose one over the other. Exploring a variety of regime-switching specifications, we consistently find that modeling the RER as two Unit Root processes with variances of different size best represents its behavior for our long horizon data for 16 countries. There are distinct active and quiet periods for each of our country pairs, with major transitions to noisy periods correspond well to key historical events (such as wars and the collapse of the Bretton Woods). We then show that the likelihood of the transition correspond to various economic fundamentals, such as GDP per capita differences, commodity price levels and volatilities, and trade openness. Finally, not only do the estimates from our individual country by country regressions cover a large range, we also observe differing fundamental forces driving the switches between states across countries. This suggests one should proceed with caution when analyzing real exchange rate behavior in a panel framework.

JEL Classification: F3, C3, C1

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1. Introduction

The Law of One Price and the subsequent concept of Purchasing Power Parity (PPP) have been heavily studied with an extensive literature base. The modern use of PPP states that the Real Exchange Rate (RER) returns to some constant mean after being disturbed by any shocks. In his original work, Karl Gustav Cassel advocated using PPP in order to set relative gold parities across countries. It plays a central role in modern theories on exchange rate determination, such as in the Frenkel-Bilson and Dornbusch-Frankel models. Acknowledging inevitable short run fluctuations in price levels and the impracticality of the absolute version of PPP, economists can forecast the RER on longer horizons should this parity hold. An accurate measure of the RER is necessary because it is widely used to make cross country comparisons by equalizing the unit of measure.

In Rogoff (1996), he points out the following issue: "How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?" This is what has become known as the PPP puzzle.[1] He goes further by giving a consensus estimate of 3 to 5 years for the half-life of shocks to the RER. In Murray and Papell (2005a), they show that the confidence interval for the half-lives are wide enough to include infinity. Despite many attempts in the past decades to address this issue, the common view remains that it is very difficult to reduce the half-life estimates of a RER shock from a long 3-5 years.

[1] Recently, there has been some work done in exploring the underlying reasons for this persistent volatility. MacDonald and Ricci (2002) argue that the size and competitiveness of the distribution sector of an economy impacts the price adjustment mechanisms of its tradables sector. Obstfeld and Rogoff (2000) and Imbs, Mumatz, and Ravn (2002) show that transaction costs could impact the price levels of the individual markets as well. IMRR(2005a) argues that an aggregation bias in the data yields larger half-lives than the individual sectors would produce. This is questioned in Chen and Engel (2005) but later reiterated in IMRR (2005b).

The Post-Bretton Woods sample size is too limited to distinguish between a Unit Root process and its alternatives, so many have resorted to the use of long horizon data. Lothian and Taylor (1996) show that using a long data set, they can find PPP to hold, but the half-life estimates are still long: 6 years for the US-UK RER and 3 years for the US-France RER. Using the ADF and DF-GLS Unit Root Tests, Taylor (2000) shows that PPP holds over the past century between the US and UK. Mainly, the difficulty stems from having a relatively short amount of available data to distinguish among alternative time series models that may look very similar in small samples.[2] For example, the current literature fails to come to an agreement regarding the stationarity (or lack thereof) of RERs. While both hypotheses have theoretical backings, the data constraints simply have not allowed a settled answer to emerge. [3]

To resolve this problem, others have turned to pooling the data in order to increase the sample size and thus the power of the tests. Frankel and Rose (1996) and Wu (1996) use panel estimation to find stationarity and a half-life of around 4 years. However, Canzoneri, Camby, and Diba (1996) fail to reject the Unit Root hypothesis in their panel framework and thus fail to find PPP holding.

Others propose different classes of models to characterize the RER because the lasting effect of the shocks remains a puzzle. This leads researchers to explore other ways of modeling the RER. In Hegwood and Pappell (1998), they apply endogenous structural breakpoint tests and then find stationarity and short half-lives within each of the regimes. Diebold, Husted, and Rush

[2] In order to increase sample size, Frankel and Rose (1996) and Wu (1996) use panel estimation to find stationarity and a half-life of around 4 years. However, Canzoneri, Camby, and Diba (1996) fail to reject the Unit Root hypothesis in their panel framework and thus fail to find PPP holding.

[3] Engel (1999) and Murray and Pappell (2000) question the power of the tests in the small samples allowed by RER data. Other estimation methods are proposed in Murray and Pappell (2005a and 2005b) and Amara and Pappell (2004). They conclude that while PPP may hold, the persistence of shocks to the RER process remains high. In addition to the standard Unit Root and Cointegration analyses to determine the existence of PPP, other methods, such as exploring Impulse Response Functions have been used in Cheung and Lai (2000).

(1991), Cheung (1993), and Cheung and Lai (1993) use long memory and fractionally integrated models to represent RERs to show that it is indeed stationary. One method of reducing the half-life of a process would be to detrend the series. Papell and Prodan (2004) investigate its theoretical implications and whether it is econometrically justifiable to do so.

We follow the fashion of using long horizon data in our analysis. This paper analyzes RER behavior using models that allow for both stationary and nonstationary components, rather than trying to force the data to choose one over the other. Theoretically, we motivate such a model setup with the observation that real shocks may prove to have permanent effects on RERs, such as through the Balassa-Samuelson effect, whereas monetary and financial shocks tend to constitute volatile yet transitory disturbances. Allowing both types of shocks to affect the RER in different time periods, our models provide the flexibility for better understanding and characterization of the dynamic processes driving RERs. In this endeavor, we also hope that this more proper characterization of shocks may help reduce the half-life of the transitory components of the RER processes to 1 or 2 years, which would be an accepted deviation from purchasing power parity that theory affords through price stickiness.

We propose one model, AR(1) with Unit Root Model, that characterizes the RER as a stationary process with occasional permanent shocks and a second model, 2 Unit Root Model, that has "quiet" and "noisy" periods for the RER. This second model proxies persistent stationary shocks using small permanent shocks that come into effect during the quiet periods. Both models turn out to be identical in finite sample estimations.[4] The setup permits the possibility that the transitory processes never switch on so that no stationarity ever exists. The

[4] The model presented in this paper comes from an extensive series of permanent and stationary models in the business cycle literature. For example, Banerjee, Lumsdaine, and Stock (1992) and Lumsdaine and Papell (1997) present models allowing for multiple trends. Our model takes the Innovation Regime Switching (1; 1, 0) model of Kuan, Huang, and Tsay (2005) and extends it onto the RER framework. In that paper, they compare this model to other similar ones, such as the STOPBREAK Model of Engle and Smith (1999).

AR(1) with Unit Root Model also allows the outcome that once we control for the occasional real disturbances in the economy, characterized by a Unit Root process, the half-life of the remaining stationary process would fall below the 3-5 year interval found in previous research. To estimate the models, we employ the classical maximum likelihood techniques for estimating regime switching models. Our dataset includes annual data starting as far back as 1870 for 16 major OECD countries.[5]

First, we find that the autoregressive coefficient in the AR(1) with Unit Root Model is close to 1, so the half-life of shocks to the RER still remains large despite allowing a separate permanent component. The estimations show that the 2 Unit Root Model—2 unit root processes with variances of different size—best represents the behavior of the RER for our long horizon data. There are distinct active and quiet periods for each of the countries. We show that the model is robust to variations, such as allowing for structural breaks, additional ARMA processes, and I(2) specifications. Our model differs from the other regime-switching attempts because we model only 2 distinct variances. Rather than trying to create a model that best "fits" the data, our model remains very simple and allows a very clear interpretation.[6]

We then lessen the puzzle by using a more rigorous analysis in which we tie fundamentals—such as GDP/Capita Differences, Commodity Price Levels and Volatilities, and

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[5] Sample selection is clearly an issue because of shocks to the RER processes. Grilli and Kaminsky (1989) deal with the historical background of RER through a long set of data and conclude that the RER volatility depends on its historical setting and not on the nominal regime. Diebold, Husted, and Rush (1991) choose to use data from the Gold Standard because those regimes represent the greatest amount of international cooperation, which is necessary for PPP to hold. Frankel and Rose (1996) use only post-World War II data in their panel study because the data exhibits a clear shift before and after the war. Cheung and Lai (1998) claim that developed countries are less likely to exhibit stationarity than their developing counterparts.

[6] There have been a few other regime switching models applied in the RER context. Engel and Hamilton (1990) show long swings in the dollar and Engel (1994) presents a Markov Switching model with switching means and variances. Engel and Kim (1999) have a 3-state model with switching variances for the stationary process and a nonstationary process that is always present. Bergman and Hansson (2000) estimate post-Bretton Woods RERs under a Markov Switching AR(1) process with state-dependent intercepts and coefficients. Frömmel, MacDonald, and Menkhoff (2002) model the RER as 3 distinct regimes driven by fundamentals. See Hegwood and Papell (1998), Diebold, Husted, and Rush (1991), Cheung (1993), Cheung and Lai (1993), and Papell and Prodan (2004) for other approaches.

Trade Openness—into our models. We regress the filtered probabilities on fundamentals, include explanatory variables directly into our models, and allow the regimes to have time-varying transition probabilities. The results suggest that fundamentals definitely play a role in explaining RER processes, but their effect is unclear because of the limited amount of data available. Certain variables, such as the volatility of commodity prices and differences in GDP/Capita, do influence most bilateral RERs. We can attribute changes in regimes to these underlying series, such as the Balassa-Samuelson effect that we proxy using GDP/Capita differences.

Our approach also encompasses other models, such as the Quasi-PPP result in Hedgwood and Papell (1998). Using our model as the data generating process, we can reproduce the Quasi-PPP results. On the flipside though, our model does not pick up separate regimes if we use the Quasi-PPP model as the true data generating process. This can also be interpreted as heteroskedasticity being misidentified as structural breaks and structural breaks not being mistaken as heteroskedasicity.

Lastly, we argue that pooling the data may be too restrictive. Though each individual country exhibits heteroskedasticity in the RER process, the level of the variances differs greatly. If a model restricts the parameter estimates to be the same across countries, the pooled estimates of variance estimates would not properly reflect the processes of each series. The fundamentals used to enhance our models affect each RER series differently. Not only do the variables have varying significance for different country-pairs, but they also have different parameter estimates.

The following section presents our Models and Estimation Methodology. Section 3 covers the results and discussion for the AR(1) with Unit Root Model and the 2 Unit Root Model; we also include a few robustness checks for the latter. In Section 4, we directly model fundamentals to explain the behavior of RERs as described by our models. Finally, Section 5 concludes and offers extensions to our project.

2A. Models

For the AR(1) with Unit Root Model, we motivate a form that allows for both stationary and nonstationary components because shocks could affect the process in different ways depending on their inherent nature. For example, real shocks may prove to have permanent effects on real RERs whereas monetary shocks are merely transitory disturbances. This suggests that the model must allow for only one type of shock each period if we wish to make this distinction. If this characterization of shocks reduces the half-life of the transitory components of the RER processes to 1 to 2 years, it would ease the concerns raised by the purchasing parity puzzle.

In the AR(1) with Unit Root Model, the series is composed of a permanent process and a stationary process, but only one of them switches on at any given time. For our annual data, a half-life of 1 to 2 years coincides with an AR coefficient of between 0.5 and 0.7.

AR(1) with Unit Root Model:

$$y_t = x_t + z_t$$
$$x_t = x_{t-1} + S_t v_t$$
$$z_t = \varphi z_{t-1} + (1-S_t) e_t$$
$$v_t \sim N(0, \sigma_v^2), \quad e_t \sim N(0, \sigma_e^2)$$

For the AR(1) with Unit Root Model, the real exchange rate, y_t, is characterized by a Unit Root nonstationary process, x_t, and an AR(1) process with a coefficient of φ, which is defined to be between -1 and 1 so that it is nonexplosive. The shocks v_t and e_t, for the Unit Root process and AR(1) process respectively, are uncorrelated and distributed normally with mean zero and variance σ_v^2 and σ_e^2. S_t is a state parameter that takes the value of 0 or 1.

Next, we propose the 2 Unit Root Model, which is the following:

2 Unit Root Model $y_t = y_{t-1} + \varepsilon_t$,

$\varepsilon_t \sim N(0, \sigma_{St}^2)$ [7]

This model imposes the RER to follow a process of unit roots with different variances. The 2 Unit Root Model is equivalent to forcing the AR coefficient in the AR(1) with Unit Root Model, φ, to be equal to 1. In finite samples, these two models are the same because it is very difficult to distinguish persistent transitory shocks from small permanent shocks.

In both models, the unique feature is that the state variable, S_t, determines the allocation of the shock at time t. If $S_t = 0$ in the AR(1) with Unit Root Model, only the stationary shock, e_t, enters y_t. In other words, for the AR(1) with Unit Root Model, the process becomes $y_t = x_0 + \varphi z_{t-1} + e_t$ where e_t is the transitory shock. This is merely a stationary AR process with a level shift, x_0. (Note that $x_{t-1} = x_0$ for all t because no shocks enter the system through time). For the opposite case where $S_t = 1$, only the permanent process comes into effect and yields $y_t = \varphi z_0 + x_{t-1} + v_t$, where v_t is the permanent shock. Here, we have a Unit Root plus a constant, φz_0. (Note that $z_{t-1} = z_0$ for all t because no shocks enter the system through time).

The 2 Unit Root Model imposes the RER to follow a Unit Root process but allows the variance to change each period. If σ_0^2 or σ_1^2 is close to 0 while the other is not, it suggests "quiet" and "active" periods for the shocks. Since it is difficult to distinguish a highly persistent AR process from a unit root process with low variance in small samples, this specification again could represent a switching between a transitory and permanent process for RERs.[8] Here, different types of shocks would be featured in the different variances of the identical unit-root processes.

[7] This is the same as $y_{t-1} + S_t v_t + (1 - S_t)\varepsilon_t$ with $v_t \sim N(0, \sigma_v^2)$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

[8] See Hamilton (1994), page 444, for a detailed explanation of this equivalence.

Unlike many other models that incorporate both permanent and transitory components for RERs, the models above suggest that the shocks are mutually exclusive. There will either be a permanent (noisy) shock *or* a transitory (quiet) shock but not both in the same period. Though this may seem restrictive, it allows for an easier explanation of the models and interpretation of the empirical results. Note that it is entirely possible to express the above models in a general form of an ARMA model with state-dependent coefficients.

2B. Estimation Methodology

To estimate the AR(1) with Unit Root Model and the 2 Unit Root Model, we employ the classical estimation technique for regime switching models. Using the algorithms provided in Nelson and Kim (1998), we first put the models in state-space form[9]:

$$y_t = H_{S_t}\beta_t + A_{S_t}z_t + e_t$$

$$\beta_t = \tilde{\mu}_{S_t} + F_{S_t}\beta_{t-1} + G_{S_t}v_t$$

$$\begin{pmatrix} e_t \\ v_t \end{pmatrix} \sim N\left(0, \begin{matrix} R_{S_t} & 0 \\ 0 & Q^*_{S_t} \end{matrix}\right)$$

Then, we estimate the parameters of interest by numerically maximizing the likelihood functions constructed by their algorithms.

For the AR(1) with Unit Root Model, its state-space representation is as follows:

$$y_t = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} x_t \\ z_t \end{pmatrix}$$

(Measurement Equation of the form $y_t = H\beta_t$)

[9] See Nelson and Kim (1998) for an explanation of the equations.

$$\begin{pmatrix} x_t \\ z_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} x_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} S_t & 0 \\ 0 & (1-S_t) \end{pmatrix} \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix}$$

(Transition Equation of the form $\beta_t = F\beta_{t-1} + G_{S_t} v_t$)

Note that $R_{S_t} = 0$ and $Q^*_{S_t} = \begin{pmatrix} \sigma^2_{v_t} & 0 \\ 0 & \sigma^2_{\varepsilon_t} \end{pmatrix}$

The 2 Unit Root Model can be represented as the AR(1) with Unit Root Model where ϕ=1, which implies only the transition needs to be revised:

$$\begin{pmatrix} x_t \\ z_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} S_t & 0 \\ 0 & (1-S_t) \end{pmatrix} \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix}$$

This would mean that the standard form of the 2 Unit Root Model is a combination of 2 permanent processes with distinct variances. Here, the processes are state-dependant, but it can be shown to have the same effect as a single unit-root process with state-dependant variances.

$$y_t = x_t + z_t$$

$$x_t = x_{t-1} + S_t v_t$$
$$z_t = z_{t-1} + (1-S_t)\varepsilon_t$$

$$\Rightarrow y_t = x_{t-1} + z_{t-1} + S_t v_t + (1-S_t)\varepsilon_t$$

$$= y_{t-1} + S_t v_t + (1-S_t)\varepsilon_t$$

$$v_t \sim N(0, \sigma_v^2) \text{ and } \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

The programming is done in Gauss 6.0 using its add-on optimization package. We choose to use the BFGS (Broyden, Fletcher, Goldfarb and Shanno) search algorithm as the numerical optimization method. The programs converge within a couple minutes with the exception of the large sample validation simulations used to double-check the asymptotic integrity of them. We use the dataset from David Papell and Ruxandra Prodan, which contains the RER of the United

States with other developed countries from 1860-1996 on an annual basis. The RER is constructed as $q_t = s_t + p_t - p_t^*$, where s_t, p_t, and p_t^* are the logarithms of the nominal exchange rate (foreign price of the US Dollar), domestic and foreign price levels, respectively. We update the dataset through 2004 (if available) using the IFS database and include the following countries in our analysis: Australia, Belgium, Canada, Denmark, Germany, Finland, France, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

3. Results and Discussion

First, we run the AR(1) with Unit Root Model to see whether or not it can reduce the half-life of the shocks to the RER. Various models that have attempted this have not managed to do so. In the AR(1) with Unit Root Model, the shocks are either permanent or transitory each period—there cannot be both types. This allows for the interpretation of the existence of real shocks *or* monetary shocks in each period. The model stems from the trend-cycle literature for economic output such as the work of Kuan, Huang, and Tsay (2005) in which they applied a similar characterization for GDP.

The following tables show the validation of the Gauss program itself. Note that it depicts single instances of simulated data series. This is to show the potential bias in a single, small-sample series such as what we have for RERs.

Table: Program simulations—AR(1) with Unit Root Model

	Prob(UR|UR)	***Prob(St|St)***	φ	$\sigma_{UR}2$	$\sigma_{ST}2$
True Value	0.7	0.8	0.55	0.4	0.8
Estimated (N=150)	0.8420	0.8799	0.6606	0.3663	0.8065
	(0.0816)	(0.0605)	(0.1026)	(0.0492)	(0.0827)
Estimated (N=3000)	0.6808	0.7940	0.5079	0.3931	0.8066
	(0.0377)	(0.0303)	(0.0231)	(0.0141)	(0.0182)

*Notes: Standard Errors are in parentheses.

The probability of a Unit Root (stationary) state this period given that the previous period was a Unit Root (stationary) state is PR(UR|UR) (PR(ST|ST)). σ_{UR}^2 and σ_{ST}^2 are the variances of the Unit Root process and of the stationary process respectively. The results suggest that the program correctly estimates the true parameters even in our small sample (N=150) example. As expected, the estimates become closer to the true parameter values as the sample size increase to 3000.

Table 1 shows the above program's estimations for the log RER data series for our 16 countries. Since our data frequency is annual, an estimate of 0.5 to 0.7 for the AR coefficient, φ, would fall in the range of a 1-2 year half-life. As the results indicate, the large majority of the countries exhibit half-lives longer than 2 years. For Portugal, Finland, and Belgium—countries with half-lives that fall into that range, their probabilities of remaining in the stationary process are lower than those of the other countries. This implies that their RER processes are dominated by the Unit Root process in which the shocks to the series are permanent. Many of the countries have results like the United Kingdom whose AR coefficient is 0.9406, which shows stationary shocks as dissipating very slowly. In fact, it could be "mistaken" for being a Unit Root itself. Another main point of interest is that the variances are quite different. With the exception of Japan, the stationary variance is much larger than the unit root variance. For the United Kingdom, σ_{UR}^2 is 0.0247 and σ_{ST}^2 is 0.1227. The hypothesis that accounting for permanent shocks will subsequently reduce the half-life of the remaining series does not hold.

This suggests that the AR(1) with Unit Root Model can be approximated by the 2 Unit Root Model in which the AR coefficient, φ, is set to be 1 so it becomes 2 unit root processes with different variances. It is very difficult to distinguish between highly persistent transitory shocks and very quiet permanent shocks in small samples. Given that the results from the AR(1) with Unit Root Model indicate the shocks to the stationary process last a long time, the characterization imposed by the 2 Unit Root Model is reasonable. The new model can be interpreted as having "quiet" and "noisy" periods.

Simulations show that the program can correctly identify the different variances. The following table shows the estimated parameters of a generated series for which we defined the parameter values.

Table: Program simulations—2 Unit Root Model

	Prob(HI|HI)	***Prob(LO|LO)***	$\sigma_{LO}2$	$\sigma_{HI}2$
True Value	0.7	0.8	0.4	0.8
Estimated (N=150)	0.6366 (0.3352)	0.4309 (0.2802)	0.3165 (0.1300)	0.6860 (0.1016)
Estimated (N=3000)	0.7732 (0.0515)	0.8210 (0.0536)	0.3858 (0.0219)	0.6784 (0.0296)

*Notes: Standard Errors are in parentheses.

PR(LO|LO) (PR(HI|HI)) is the probability that the current state is quiet (noisy) given the previous state was also quiet (noisy). σ_{LO}^2 and σ_{HI}^2 are the variances of the quiet state and noisy states respectively. Again, this is just a single simulation of a small sample size and of a larger sample size. The reason behind this is that we wish to show the potential variation in estimating a single, small-sample series. Monte Carlo simulations show that as the simulations increase, the average parameter estimates become increasing close to the true parameters. Nevertheless, from the above tables, we see that, for the small sample size, the true variance of the quiet process, 0.4000, is estimated to be 0.3165 with a standard error of 0.1300 by the program. As the sample size increases to 3000, the estimated parameter is 0.3858 with a standard error of 0.0219, which is not significantly different at the 95% confidence interval from the true parameter of 0.4.

Next, we check to make sure the program does not falsely break the processes into quiet and noisy states. We generate a single unit root process with normally distributed error terms to see how the program reacts. The results are below.

Table: 2 Unit Root Model run on a single Unit Root Process

	Prob(HI|HI)	*Prob(LO|LO)*	$\sigma_{LO}2$	$\sigma_{HI}2$
Estimated (N=150)	0.6505 (4.2793)	0.6184 (1.8384)	1.1173 (0.1422)	1.1172 (0.1328)
Estimated (N=3000)	0.6514 (0.3781)	0.6171 (0.3792)	0.1011 (0.0545)	0.1011 (0.0182)

*Notes: Standard Errors are in parentheses.

The program is attempting to characterize the single series into 2 series with different variances. Regardless of the sample size, the large standard errors surrounding the probabilities of staying in their respective states indicate it does not know in which state the process resides. The variances are all around 1, which is the true variance. So, given that the 2 identified processes are identical, it makes sense that the program does not know in which state it belongs.

Table 2 shows the program run for our 16 countries[10]. We see that the different variances are indeed present. For the US-UK Real Exchange Rate, the quiet and noisy periods have variances of 0.0252 and 0.1258 respectively. This large difference is indicative of the behavior of RERs for other countries too. The large probabilities for remaining in each state demonstrates that both processes come into effect during our long horizon data series. Characterizing the RER as having quiet and noisy permanent shocks is consistent with the results from the current literature showing very persistent shocks to the RER. Though a small shock in our model would be “permanent,” it is conceivable that the reversion of a very small deviation from a long-run mean would be difficult to observe given the low power of testing. The converse also makes sense, so the quiet periods can be viewed as a proxy for slow reversion.

Table 2 shows that there the high variance states are generally 4-5 times “noisier” than the low variance states. Though this relation holds across countries, the absolute levels vary

[10] Table 2B has the results for a few other country pairs.

greatly. For example, the quiet and noisy variances for the US-UK RER are 0.0252 and 0.1258 respectively. However, it is 0.0894 and 0.3606 for the US-Portugal RER. In other words, the overall levels of the variances are higher for US-Portugal than for US-UK even though both exhibit high and low states. This leads us to caution the use of pooled analysis where the estimations yield common parameter estimates for each panel.

Looking at the plots in Graph 1, we see the dramatic shifts between the 2 states. The y-axis represents the probability of being in a permanent state during the year indicated on the x-axis. However, for most of the countries, the graphs show that it occupies the quiet state most of the time and only becomes noisy on a few occasions. For example, Italy is shown to have quiet shocks with a couple of exceptions such as during the period around 1945 when it was involved in World War II. Later, we will show in more detail how the spikes indicating noisy periods coincide directly with historical events.

3A. Robustness Check: Model Variations

Next, we provide a series of Robustness checks for the 2 Unit Root Model. The first modification is to allow for a third process that is always on; we will call this new form the 2 Unit Root with AR(1) Model. Should the additional process prove to be relevant, it would hint that the 2 Unit Root Model is an over-simplification of the actual process and we have to explore additional specifications to properly portray the RER.

2 Unit Root with AR(1) Model:

$$y_t = x_t + w_t + z_t$$
$$x_t = x_{t-1} + S_t v_t$$
$$w_t = w_{t-1} + (1 - S_t)\, n_t$$
$$z_t = \phi z_{t-1} + e_t$$
$$v_t \sim N(0, \sigma_v^2),\ n_t \sim N(0, \sigma_n^2),\ e_t \sim N(0, \sigma_e^2)$$

The 2 Unit Root with AR(1) Model is merely the 2 Unit Root Model augmented with the process z_t that should pick up any remaining variation not attributed to the original model.

Table 3 shows that the estimated $\boldsymbol{\sigma_{ST}}^2$, the variance of the always on AR(1) process, is zero for all of the countries. This indicates that the z_t process does not come into play and the model reduces to our original 2 Unit Root Model. It does not matter that the autoregressive coefficients lack homogeneity because the processes do not exist. The variances for the switching Unit Root processes are 0.0243 and 0.1249 for the United Kingdom, which are very close to the estimates from the 2 Unit Root Model. This is the case for the other countries as well.

In addition to this augmentation of the model, we ran different variations for the 2 Unit Root Model and they all led to the conclusion that the original form is a robust specification of RERs. For example, when we allowed the 2 processes to both be stationary, the estimated autoregressive coefficients both approached 1. Making the model switch between UR and ARMA(1,1) processes had the AR coefficient approach 1 and the MA process nonexistent. A model for which the AR process is always on and the UR process occasionally switches on estimates the stationary coefficient to be close to 1; this then became an always on UR with low variance and instances of a UR process with high variance. In another variation, we added an I(2) process to simulate a double-drift, but those results did not differ much from our current model. , The addition of time trends to these models does not reduce the half-lives, which is consistent with the findings in the current literature. Lastly, the model is robust for data of other frequencies too; using Post-Bretton Woods monthly data, we observe the distinct regimes in our RERs. The unifying result in all of these variations implies a model exhibiting 2 highly persistent processes with different variances: the 2 Unit Root Model.

3B. Robustness Check: Quasi-PPP

In Hedgwood and Papell (1998), the authors find a short half-life for the shocks on RER and call their result "Quasi-PPP." They use endogenous structural breakpoint tests on series of RERs and then run a simple AR(1) regression on the data while allowing for structural shifts for the dates indicated by the breakpoint tests. Their resulting AR coefficient is low enough to fall into the 1-2 year range for PPP to hold in the short-run. If their result holds true, it means the lack of reversion exhibited by the 2 Unit Root Model is not consistent with their findings. In this exercise, we attempt to replicate their results given a data generating process from the 2 Unit Root Model. Our method is as follows:

1) Generate the 2 Unit Root Model data with persistent states and different variances
2) Confirm high ϕ in a simple AR1 model: $q_t = \phi q_{t-1} + v_t$, $v_t \sim N(0, \sigma_v^2)$
3) Run Bai and Perron (1998) programs to find multiple breakpoints
4) Estimate AR(1) model allowing for different levels for each regime:

$$q_t = \phi q_{t-1} + D_1 t_1 + \ldots + D_n t_n + v_t,\ v_t \sim N(0, \sigma_v^2),$$

where D_i is a dummy variable that can take the value of 0 or 1 and

t_i is the level shift for the period without breaks.

In our generated series of 135 observations, which is the same number we have for US-UK RERs, we find a high coefficient (0.9983) in the simple autoregressive model. Then, the multiple breakpoint test program finds 3 breaks that would coincide with the years 1881, 1901, and 1970. This is somewhat disturbing because we know our true data generating process is merely 2 Unit Root switching back and forth and not a process with 3 breaks.[11] Nevertheless, we

[11] Nunes, Kuan, and Newbold (1995) use simulations to show that unit root processes can generate "spurious breaks." Bai (1998) provides a note in which he presents a mathematical proof for this. Another issue for testing structural breaks in RERs is the low power in the Bai and Perron (1998) tests; this issue is addressed in Prodan (2005).

proceed and run the AR(1) regression allowing for level shifts in our 4 regimes. Using Eviews 5.1, our regression yields a φ of 0.63, which falls into the range of 1-2 years for the half-life on a shock to the system.

We have shown that the 2 Unit Root Model can be mistaken as Quasi-PPP, but can Quasi-PPP be mistaken as the 2 Unit Root Model? If the models are all equivalent in small samples, it would be impossible to identify the "true" model. We generate a series of data following the Quasi-PPP method. We assume 1) 3 breaks, 2) 4 levels that are 0.7, -0.5, 0.6, and -1 in that respective order, 3) AR(1) coefficient is 0.6, and 4) errors are *iid* (0,1). Then, we generate data for sample sizes of 150 and 1000 for both the AR(1) with Unit Root Model and the 2 Unit Root Model. The break dates occur on observations 30, 60, 90 for the small sample and on 100, 500, and 750 for the large sample.

Table: AR(1) with Unit Root Model—Quasi-PPP results

	P(UR|UR)	***P(ST|ST)***	φ	σ_{UR}^2	σ_{ST}^2
N = 150	0.8625	0.9595	0.6053	1.4928	0.8975
	(0.1343)	(0.0279)	(0.1184)	(0.2744)	(0.0859)
N = 1000	0.0001	0.9853	0.5984	1.4062	0.9798
	0.0000	(0.0073)	(0.0299)	(0.3251)	(0.0215)

*Notes: Standard Errors are in parentheses.

The autoregressive coefficients are estimated to be close to 0.6, which is the true parameter from the data generating process. As the sample size, N, increases, the unit root process fades into the background, which is shown by the estimate of P(UR|UR) as 0.0001.

Table: 2 Unit Root Model—Quasi-PPP results

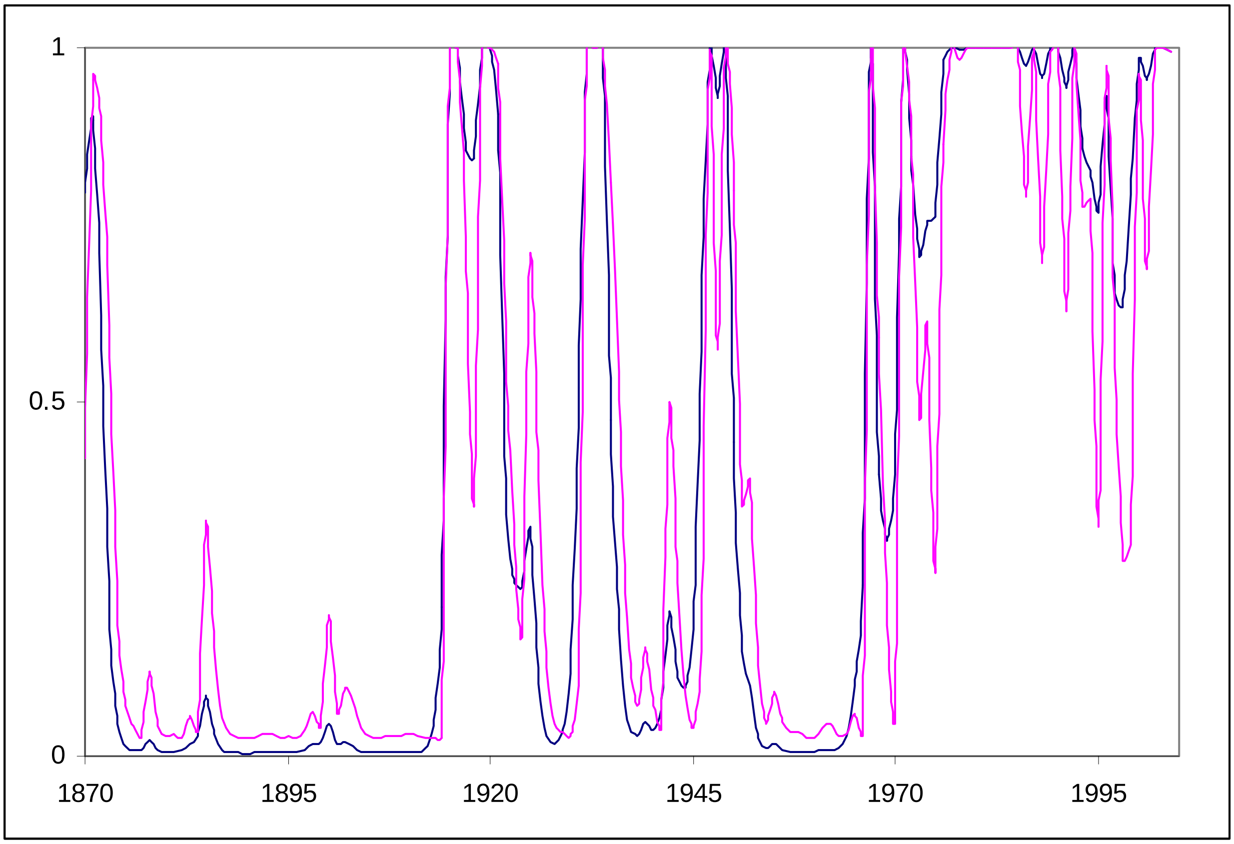
	P(UR1|UR1)	***P(UR2|UR2)***	σ_{UR1}^2	σ_{UR2}^2
N = 150	1.0000	0.0208	1.1955	1.2239
	(0.0043)	0.0000	(0.0700)	(10.1152)
N = 1000	0.5671	0.0001	1.0126	1.3354
	(0.1441)	0.0000	0.0000	(0.0805)

*Notes: Standard Errors are in parentheses.

The program for the 2 Unit Root Model only identifies a single Unit Root process for both the small sample and the large sample. Furthermore, the variances are not very different. The results we get from using RER data for these models are different from what we just simulated above. Our RER data yield high AR coefficients, 2 processes, and distinct variances. It does not appear that a Quasi-PPP data generating process can correctly replicate the empirical results brought forth by our models.[12]

3C. Historical Implications

Graph: Estimated Probabilities of High Variance Regime for 2 Unit Root Model (US-UK)



The above graph shows the plot of the US-UK RER for the 2 Unit Root Model, where the blue/solid lines are the smoothed estimates and the pink/dotted lines are the filtered estimates. Graph 2 in the Appendix shows plots for the other countries; again, the variations in the graphs are another indication that panel analysis may be too restrictive. The y-axis represents the

[12] This phenomena can also be interpreted as spurious structural breaks due to heteroskedasticity. On the flipside, structural breaks are not mistakenly attributed to be heteroskedastic processes.

probability of being in a noisy, high variance state at the date indicated on the x-axis. Our goal is to explain the noisy states indicated by the model. Using the smoothed estimates, we define a period to be in a noisy state should its probability exceed 0.5. Otherwise, it is in a quiet state. The following are the resulting noisy states with potential explanations.

1870-3: The United States government passes the "Fourth Coinage Act" in 1973 as a response to newly discovered Silver in the American West. The US leaves the bimetallism currency system where the dollar could be expressed in both Silver and Gold out of fear that the increased Silver supply would cause inflation.

1915-1921: This is a period of great instability for both the United States and the United Kingdom because of World War I. Furthermore, the world shifts towards an Anchored Dollar Standard in which the other currencies base themselves on the American Dollar.

1931-4: During the early 1930s, many governments change their currency away from being based on Gold. In 1931, the United Kingdom leaves the Gold Standard and the United States follows suit in 1933. The following year, the United States raises the price of gold from $20/oz to $35/oz. Another reason for the noise during this period is the Great Depression, where much of the stability is lost in the financial markets and the economy as a whole.

1946-9: After World War II, the United Nations held a conference and established the Bretton Woods institutions. *Wikipedia* describes it as: "Setting up a system of rules, institutions, and procedures to regulate the international monetary system, the planners at Bretton Woods established the International Bank for Reconstruction and Development (IBRD) (now one of five institutions in the World Bank Group) and the International Monetary Fund (IMF). These organizations became operational in 1946 after a sufficient number of countries had ratified the agreement."

1967: This spike does not have as clear of an explanation as the others. It could be a result from the American War in Vietnam or the formation of OPEC a few years earlier.

1971-present: The Bretton Woods agreement collapses after the US abandons it. Ever since, both the United States and the United Kingdom have been in a world of flexible exchange rates.

The persistent high volatility state since the end of Bretton Woods is consistent with current findings dubbed as the Exchange Rate Disconnect Puzzle. Economic fundamentals should, in theory, be closely related to exchange rates. As the previous graph indicates, the RER has experience large volatility since 1971, but this volatility is not reflected in the fundamentals. There is seemingly a "disconnect" between the exchange rate and the underlying, economic variables.[13] The plots of the 2 Unit Root Model in Graph A of the Appendix that show this.

[13] Flood and Rose (1995) have a thorough discussion of this tendency, but recent work –such as Cheung and Lai (1997), which uses two efficient unit root tests to find PPP holding—challenge this finding.

4. Modeling Fundamentals in the 2 Unit Root Model

In the previous section, we established the robustness of the models and showed how the different states coincide with historical events. Now, we will apply rigorous, econometric techniques to explain the behavior of the switching in the 2 Unit Root Model. First, we apply OLS and regress the filtered probabilities on fundamentals. Then, we include the fundamentals directly within the measurement equation to see if that reduces the residual volatility. Lastly, the model is allowed to have Time-Varying Transition Probabilities dependent on fundamentals. These exercises show that while fundamentals do influence the switching behavior of our model, the effect varies depending on the choice of bilateral exchange rates. This is more evidence that pooling the data may lead to spurious results.

4A. OLS Regressions

The filtered probabilities of being in a high variance state or a low variance state is the dependent variables. Even though there is little difference in the 2 series for our models, we use the filtered as opposed to the smoothed probabilities because the latter depicts an overall "trend" by utilizing the entire data sample whereas the former only uses the data up to the point in question to derive its estimate. We include both prices and volatilities of Gold, Silver, Oil, and The Economist Commodity Index. Then, we construct 3 series of dummy variables:

1) When the US is on the Gold Standard, DUM_GOLDUS takes a value of 1
2) When the UK is on the Gold Standard, DUM_GOLDUK takes a value of 1
3) If the US is involved in a war, the WAR dummy takes a value of 1.[14]

[14] We include DUM_GOLDUS (DUM_GOLDUK) when the US (UK) is one of the countries in the bilateral RER. For US-UK, we use DUM_GOLDUS but both dummies yield similar results because both countries were on the Gold Standard during nearly identical periods.

Other variables in the analysis include Inflation Volatility Differences, GDP/Capita Differences, and Average Openness, which is the average Total Trade / GDP between the 2 countries.[15] The data is obtained from Global Financial Services and Wikipedia. We log the prices and construct volatilities by taking the standard deviation of monthly data over the year in question.

The following table shows the OLS results for selected countries.[16] The variable inclusion criterion is based on Bayesian Model Averaging.

Table: OLS Regressions on Fundamentals for Various Country Pairs[17]

	Gold	Gold Volatility	Silver	Silver Volatility	Oil	Oil Volatility	Commodity Index	Commodity Volatility	Openness	War	Gold Standard	GDP/Capita Difference	Intercept	R-Sqaured
Canada - Germany	--	--	0.1241 (0.0405)	--	0.2360 (0.1335)	--	0.1335 (0.0407)	3.7104 (0.9894)	--	--	--	--	(0.7850) (0.1747)	0.5158
Canada - US	0.0009 (0.0002)	--	--	(1.2943) (0.3668)	--	--	--	2.2652 (0.7954)	--	(0.1495) (0.0546)	(0.1927) (0.0530)	1.4498 (0.2414)	(0.0479) (0.0985)	0.4193
France - UK	--	--	--	--	--	--	--	3.9612 (0.2959)	--	(0.2091) (0.0667)	--	0.2959 (0.1119)	0.6772 (0.0625)	0.2730
Germany - UK	--	--	0.1244 (0.0552)	--	0.3575 (0.0624)	1.1101 (0.2967)	--	4.2346 (0.9753)	--	--	(0.3464) (0.0861)	--	(0.6456) (0.1703)	0.6151
Switzerland - US	--	2.6854 (0.9287)	--	0.3085 (0.0743)	0.3251 (0.1144)	--	(0.1173) (0.0574)	--	--	--	(0.4912) (0.1226)	--	(0.2448) (0.2307)	0.3100
US - UK	--	--	--	1.4389 (0.3782)	--	--	--	3.5897 (0.8532)	1.9174 (0.3772)	--	(0.3231) (0.0590)	(0.3197) (0.1383)	0.2558 (0.0543)	0.6439

For the US-UK, Openness, US on the Gold Standard, Silver Volatility, and Commodity Index Volatility have significant effects on the probability of being in a low or high variance state. We note that each variable (except for Inflation Volatility Differences, which is not listed in the table) is significant under some country pair.[18] The lack of pattern in significance suggests the fundamentals are highly collinear, so it is difficult to distinguish their effects from each other. The results point out that fundamentals do have explanatory power over the movements of RER

[15] For our selected countries, Inflation Volatility Differences are available for only US-UK, and Openness is constructed for only US-UK and Switzerland-US.
[16] Results for other country pairs are available upon request.
[17] Notes: OLS regression with Standard Errors in Parenthesis. Missing value markers, "-", are inserted when either 1) no data available or 2) BMA selection drops the series.
[18] Inflation Volatility Differences is only available for US-UK from 1914 onwards.

volatility in our model, but which fundamentals depends on the bilateral RER being analyzed. This is another outcome that suggests pooling countries for analysis is not suitable.

4B. Parameters in the Measurement Equation

In this exercise, we incorporate the fundamentals directly in the measurement equation. Whatever remains is put in our original model in which we characterize the series as 2 Unit Root processes with distinct variances. If the fundamentals truly affect our RER series, it should pick up a significant coefficient in β and reduce the weight placed on the switching UR processes.

$$y_t = x_t + z_t + \beta * Fundamental_t$$
$$x_t = x_{t-1} + S_t v_t$$
$$z_t = z_{t-1} + (1-S_t)\, e_t$$
$$v_t \sim N(0, \sigma_v^2),\ e_t \sim N(0, \sigma_e^2)$$

The results for the US-UK RER series are below.

Table: Fundamentals in Measurement Equation (US-UK)

	PR(LO|LO)	*PR(HI|HI)*	σ_{LO}^2	σ_{HI}^2	β	**Likelihood Value**
Gold	0.9035 (0.0493)	0.8752 (0.0801)	0.0252 (0.0028)	0.1146 (0.0144)	0.1586 (0.1044)	194.5409
Gold Volatility	0.8890 (0.0468)	0.8055 (0.0834)	0.0253 (0.0033)	0.1368 (0.0182)	(0.4467) (0.3426)	193.1937
Silver	0.9128 (0.0419)	0.8858 (0.0632)	0.0243 (0.0027)	0.1241 (0.0132)	0.0364 (0.0202)	192.6870
Silver Volatility	0.9106 (0.0425)	0.8774 (0.0699)	0.0252 (0.0027)	0.1259 (0.0139)	(0.0099) (0.0776)	191.2102
Oil	0.9081 (0.0439)	0.8719 (0.0750)	0.0252 (0.0026)	0.1263 (0.0141)	0.0058 (0.0221)	191.2342
Oil Volatility	0.9098 (0.0429)	0.8762 (0.0715)	0.0252 (0.0027)	0.1259 (0.0141)	(0.0017) (0.0498)	191.2028
Commodity Index	0.8994 (0.0494)	0.8579 (0.0842)	0.0244 (0.0026)	0.1256 (0.0140)	0.0669 (0.0383)	192.6646

Commodity Volatility	0.9097 (0.0429)	0.8763 (0.0706)	0.0252 (0.0027)	0.1257 (0.0139)	(0.0113) (0.1184)	191.2050
Openness	0.8803 (0.0501)	0.7568 (0.1088)	0.0241 (0.0025)	0.1379 (0.0176)	2.6612 (0.4497)	184.0305
War	0.9113 (0.0429)	0.8855 (0.0615)	0.0231 (0.0025)	0.1226 (0.0127)	0.0282 (0.0087)	195.6347
Gold Standard	0.9122 (0.0406)	0.8778 (0.0670)	0.0241 (0.0024)	0.1290 (0.0139)	0.0432 (0.0177)	193.7624
GDP/Capita Difference	0.9008 (0.0474)	0.8329 (0.0950)	0.0257 (0.0033)	0.1312 (0.0164)	0.0166 (0.0627)	189.5344

*Notes: Standard Errors are in parentheses.

The coefficient on the fundamental, β, is not significant for any of the series, and the other parameter estimates are very close to the original estimates. Just out of curiosity, we run the Commodity Index as a fundamental for the US-Australia RER because Australia is a commodity economy.

Table: Commodity Index in Measurement Equation (Australia)

	PR(LO|LO)	***PR(HI|HI)***	σ_{LO}^2	σ_{HI}^2	β	**Likelihood Value**
Commodity Index	0.8961 (0.0462)	0.8869 (0.0605)	0.0317 (0.0044)	0.1226 (0.0143)	0.0299 (0.0647)	163.8431

*Notes: Standard Errors are in parentheses.

Again, the coefficient for the fundamental is not significant and the processes of the 2 Unit Root Model pick of the majority of the variation in the series.[19] The sample size is too small for the confidence intervals to be tight enough to yield significant regressors.

4C. Time Varying Transition Probabilities

[19] We ran this exercise for the AR(1) with Unit Root Model and the other models with AR(1) processes. Again, the AR(1) coefficient did not differ from the original results when we did not have the fundamental in the measurement equation.

In our current specification, the transition probabilities are constant for the duration of our data sample. It is quite conceivable that the behavior of the RER is linked directly to the performance of underlying, economic fundamentals. Here, we attempt to explain the switching states in terms of these fundamentals. We run this program for our US-UK series. If these additional parameters have a significant impact on our model, it hints that there are other processes that must be accounted for in addition to the two permanent processes in the 2 Unit Root Model. The basic methodology is to enhance the original program to allow the transition probabilities to vary based on other variables. The basic probit probabilities are:

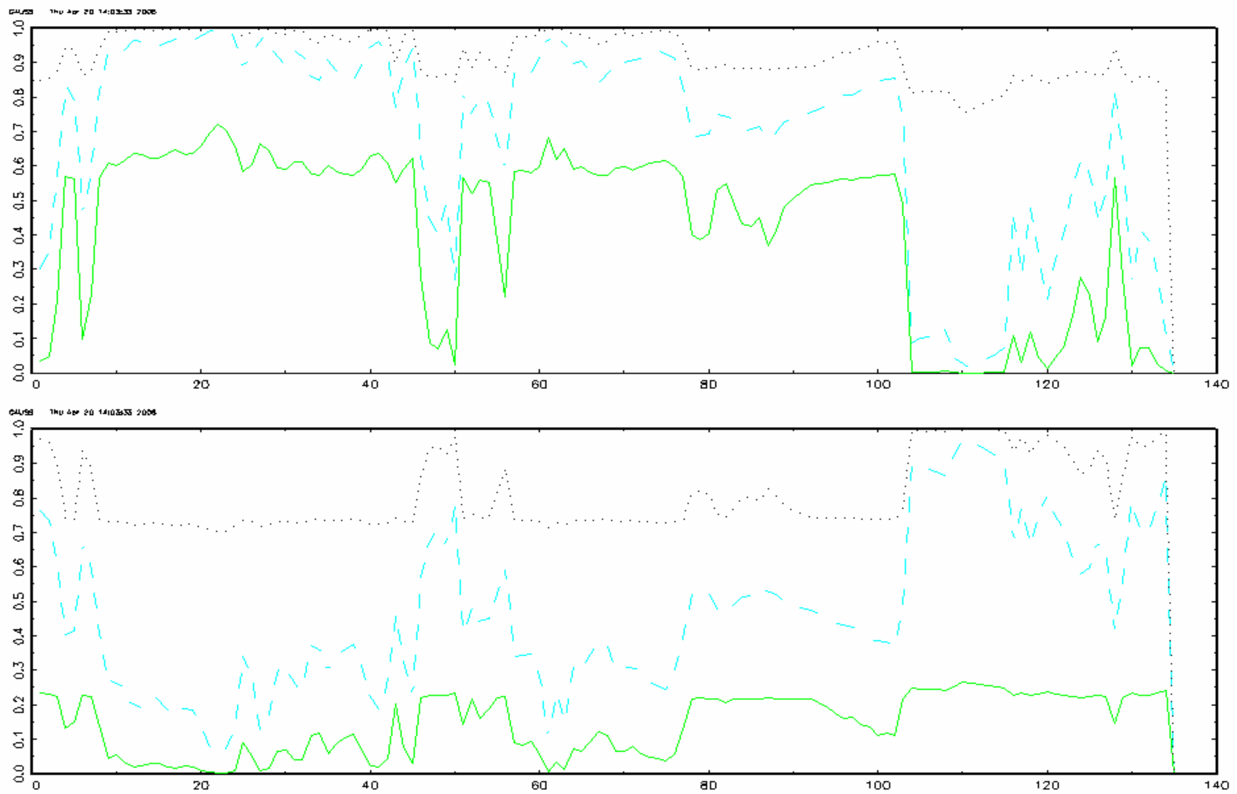
$$\text{PPR} = \text{PR (St=Hi | St=Hi)} = 1 - \frac{1}{1+\exp(p_0 + p_1 Z_t)}$$

$$\text{QPR} = \text{PR (St=Lo | St=Lo)} = 1 - \frac{1}{1+\exp(q_0 + q_1 Z_t)}$$

p_0, p_1, q_0, and q_1 are unconstrained parameters in the optimization and Z_t is the value of the fundamental at time *t*. The following grpoh shows the plots PPR (top) and QPR (bottom) with their 95% confidence bands for Oil.[20] Oil Prices show significant movement through time, which is represented by the x-axis where time 0 is 1870.

[20] Graph 2 in the Appendix shows the plots PPR (top) and QPR (bottom) with their 95% confidence bands for each of the fundamentals.

Graph: Time-varying Probabilities based on Oil Price Levels[21]



Even so, the confidence intervals are wide enough that a straight line can be drawn through the entire timeframe, so a constant probability could fit the 2 Unit Root Model. This is even more apparent for the other commodities, so the probabilities do not necessarily vary through time in our finite sample. The following table shows the numerical estimation produced by the program.

[21] Notes: Graphs is for US-UK only. Data runs from 1870 through 2004. Estimated values are dashed lines with an upper (lower) confidence estimate as dotted (solid) lines.

Table: Time-varying Transition Probabilities (US-UK)

	p_0	q_0	σ_{v2}	σ_{e2}	p_1	q_1	LLH Value
Gold	2.4745	1.7331	0.0252	0.1293	2.4317	2.9870	195.3219
	(0.6136)	(0.7046)	(0.0028)	(0.0147)	(1.1866)	(1.3996)	
Gold Volatility	0.4406	0.7762	0.0245	0.1297	(91.2717)	21.6333	196.5817
	(1.0719)	(0.7685)	(0.0027)	(0.0144)	(43.2762)	(24.4934)	
Silver	2.0829	1.5488	0.0246	0.1284	1.4715	0.3384	193.7223
	(0.5613)	(0.6453)	(0.0031)	(0.0145)	(0.7861)	(0.8208)	
Silver Volatility	2.5899	14.6758	0.0224	0.1139	(72.7028)	355.5128	201.5853
	(0.8714)	(7.1335)	(0.0022)	(0.0100)	(36.1170)	(197.9090)	
Oil	1.1290	(0.1143)	0.0245	0.1358	(3.7771)	2.4490	198.0088
	(0.4707)	(0.6389)	(0.0024)	(0.0158)	(0.5661)	(1.1434)	
Oil Volatility	2.7608	2.2264	0.0249	0.1236	(9.8680)	3.0652	193.3209
	(0.7493)	(0.6426)	(0.0027)	(0.0128)	(5.1328)	(0.8255)	
Commodity Index	2.6422	1.9619	0.0250	0.1248	3.3886	2.0477	192.1881
	(0.6693)	(0.6190)	(0.0029)	(0.0133)	(3.1249)	(1.8179)	
Commodity Volatility	1.0920	1.1860	0.0215	0.1216	(58.9334)	89.7928	197.4823
	(0.5184)	(0.8175)	(0.0031)	(0.0125)	(31.7486)	(51.6711)	
Openness	0.5839	0.7913	0.0248	0.1435	(58.1809)	19.5544	186.0224
	(0.9521)	(0.6960)	(0.0026)	(0.0176)	(35.8827)	(14.6952)	
War	2.4331	2.0640	0.0243	0.1240	(1.8569)	(1.9195)	193.5334
	(0.6458)	(0.6344)	(0.0027)	(0.0136)	(1.2003)	(1.1515)	
Gold Standard	1.8747	0.7391	0.0240	0.1266	2.2930	(2.4198)	195.8311
	(0.5694)	(0.9110)	(0.0027)	(0.0145)	(1.2689)	(2.1376)	
GDP/Capita Difference	1.6572	2.1023	0.1273	0.0254	(1.3025)	2.9605	192.5905
	(0.6876)	(0.5312)	(0.0145)	(0.0029)	(3.0261)	(2.5029)	

*Notes: Standard Errors are in parentheses.

From the table, we see that Oil yields significant coefficients for both probabilities. The coefficients and the probabilities move in opposite directions, so as Oil increases, PPR (probability of remaining in a high variance state) decreases and QPR (the probability of remaining in a low variance state) increases. In other words, as the log of Real Oil Prices increases, the probability of staying in (or moving to) a more volatile RER regime increases.

Nonetheless, the smoothed and filtered plots of the states barely changes after allowing for the additional parameters. In general, the confidence intervals for the probabilities are very wide, so there is not enough power in our sample size to determine whether or not the transition probabilities are indeed time-varying.

From our 3 ways of incorporating fundamentals into the regime-switching model, we find it difficult to pinpoint the exact fundamentals that determine the RER process. The OLS approach shows the fundamentals to be significant in determining the variance state of the RER, but there is much collinearity among the regressors. Nevertheless, certain series—such as GDP/Capita Differences—show up more than others. This suggests the Balassa-Samuelson effect may explain some of the variation in the RER. In general, the estimations reveal parameters to be of different values and significance for different country pairs, so the data should not be pooled.

5. Conclusion

Using over 100 years of data from 16 OECD countries, we find that the RER is best described as having both a noisy process and a quiet process. The 2 Unit Root Model has 2 nonstationary processes with variances of different magnitudes. The majority of the shocks to the RER are small, permanent ones that can be mistaken as very persistent, transitory disturbances. In the AR(1) with Unit Root Model, where we attempt to decompose the RER into a stationary and a permanent process, the autoregressive coefficient becomes large enough to suggest that the 2 Unit Root model holds. Other variations on the 2 Unit Root Model also show its robustness. For example, when we augment the model with a 3rd process, the additional process does not come into play, which is a sign of the integrity of the original specification.

We find that the inclusion of fundamentals diminishes the puzzle because they provide explanatory power for the changes of regimes. The fundamentals we use are commodity prices and volatilities, GDP/Capita differences, inflation differentials, war periods, openness, and gold standard periods; the significance of the variable depends on the bilateral RER used in the analysis. In addition to OLS regressions selected by BMA, we model the fundamentals as Time-Varying Transition Probabilities and also directly in the measurement equation. These latter 2 methods are someone inclusive because of the collinearity of the variables and the small sample size with which to work. Though the overall persistence of the RER series remains, it can be explained by other factors.

Our model is not only robust to variations but it also explains previous models, such as those using structural breaks. We show that our heteroskedastic model can be mistaken as Quasi-PPP, but the converse is not true. The 2 Unit Root model will not identify unique processes should the true model have structural breaks. Our specification yields regimes that are consistent with historical events. The model implies that shocks due to wars and currency regime shifts are permanent, whereas the other shocks simply deviate slightly around the long-run mean. After the collapse of Bretton Woods, our model consistently shows noisy periods. This supports the current literature exploring the Exchange Rate Disconnect Puzzle, where the large volatility observed in exchange rates is not reflected in economic fundamentals.

Lastly, we argue that pooling the data in order to increase the sample size, and thus power of tests, may be too restrictive. Even though each individual country exhibits this type of heteroskedasticity, the absolute levels of variance are different. In addition, depending on the country pair studied, different fundamentals are significant in characterizing regime shifts.

Despite our attempt to separate large shocks from small fluctuations, the PPP Puzzle still remains. Our model is in agreement with that literature that finds a long half-life for the shocks to RERs. Though we can attribute our noisy shocks to historical events for individual countries, one extension would be to place the data in a panel framework in search of common shocks, but we believe modeling it this way would be too limiting. However, it could be done for the countries with similar estimates, such as for Germany, France, Sweden, Switzerland, and the United Kingdom. Lastly, we could modify the model to include more states, but that may complicate the clean interpretation of our model that we currently have.

Works Cited

Amara, J. and D. Papell, 2004. "Testing for Purchasing Power Parity Using Stationary Covariates," working paper, University of Houston.

Bai, Jushan, 1998. "A Note on Spurious Break," *Econometric Theory* 14, 663-669.

Banerjee, A., R. Lumsdaine, and J. Stock, 1992. "Recursive and Sequential Tests of the Unit-Root and Trend-Break Hypotheses: Theory and International Evidence," *Journal of Business & Economic Statistics* 10 (3), 271-287.

Bergman, U.M. and J. Hansson, 2000. "Real Exchange Rates and Switching Regimes," working paper, Lund University.

Canzoneri, M, R. Cumby, and B. Diba, 1999. "Relative Labor Productivity and the Real Exchange Rate in the Long Run: Evidence For a Panel of OECD Countries," *Journal of International Economics* 47, 245-266.

Chen, S.S., and C. Engel, 2005. "Does 'Aggregation Bias' Explain the PPP Puzzle?" *Pacific Economic Review* 10 (1), 49-72.

Cheung, Y.W., 1993. "Long Memory in Foreign-Exchange Rates," *Journal of Business & Economic Statistics* 11, 93-101.

Cheung, Y.W. and K.S. Lai, 1993. "A Fractional Cointegration Analysis of Purchasing Power Parity," *Journal of Business & Economic Statistics* 11, 103-112.

Cheung, Y.W. and K.S. Lai, 1997, "Parity Reversion in Real Exchange Rates During the Post-Bretton Woods Period," working paper, University of California, Santa Cruz.

Cheung, Y.W. and K.S. Lai, 2000a. "On The Purchasing Power Parity Puzzle," *Journal of International Economics* 52, 321-330.

Cheung, Y.W. and K.S. Lai, 2000b. "On Cross-Country Differences in the Persistence of Real Exchange Rates," *Journal of International Economics* 50 (2), 375-397.

Clarida, R. and J. Gali, 1994. "Sources of Real Exchange Rate Fluctuations: How Important are Nominal Shocks?" NBER Working Paper 4658.

Diebold, F., S. Husted, and M. Rush, 1991. "Real Exchange Rates under the Gold Standard," *The Journal of Political Economy* 99 (6), 1252-1271.

Engel, C., 1994. "Can the Markov Switching Model Forecast Exchange Rates," *Journal of International Economics* 36, 151-165.

Engel, C., 2000. "Long-Run PPP May Not Hold After All," *Journal of International Economics* 51(2), 243-273.

Engel, C. and J. Hamilton, 1990. “Long Swings in the Dollar: Are They in the Data and Do Markets Know It?” *The American Economic Review* 80 (4), 689-713.

Engel, C. and C.J. Kim, 1999. “The Long-Run US/UK Real Exchange Rate,” *Journal of Money, Credit, and Banking* 31 (3), 335-356.

Engel, C. and K. West, 2004. “Exchange Rates and Fundamentals,” *Journal of Political Economy* 113 (3), 485-517.

Engle, R. and A. Smith, 1999. “Stochastic Permanent Breaks,” *The Review of Economics and Statistics* 81 (4), 553-574.

Flood, R. and A. Rose, 1995. “Fixing Exchange Rates: A Virtual Quest for Fundamentals,” *Journal of Monetary Economics* 36, 3-37.

Frankel, J. and A. Rose, 1996. “A Panel Project On Purchasing Power Parity: Mean Reversion Within and Between Countries,” *Journal of International Economics* 40, 209-224.

Friedman, M. and J. Schwartz, 1963. *A Monetary History of the United States*. Princeton, New Jersey: Princeton University Press.

Frommel, M., R. MacDonald, and L. Menkhoff, 2005. “Markov Switching Regimes in a Monetary Exchange Rate Model,” *Economic Modelling* 22 (3), 485-502.

Grilli,V. and G. Kaminsky, 1991, “Nominal Exchange Rate Regimes and the Real Exchange Rate: Evidence from the United States and Britain, 1885-1986.” *Journal of Monetary Economics* (27), 191-212.

Hamilton, J., 1994. *Time Series Analysis*. Princeton, New Jersey: Princeton University Press.

Hegwood, N. and D. Papell, 1998. “Quasi Purchasing Parity Puzzle,” *International Journal of Finance and Economics* 3, 279-289.

Imbs, J., H. Mumtaz, M. Ravn, and H. Rey, 2003. “Nonlinearities and Real Exchange Rate Dynamics,” *Journal of the European Economic Association* 1, 639-649.

Imbs, J., H. Mumtaz, M. Ravn, and H. Rey, 2005a. “PPP Strikes Back: Aggregation and the Real Exchange Rate,” *The Quarterly Journal of Economics* 120 (1), 1-43.

Imbs, J., H. Mumtaz, M. Ravn, and H. Rey, 2005b. “ ‘Aggregation Bias’ DOES Explain the PPP Puzzle,” NBER Working Paper 11607.

Kuan, C.M., Y.L. Huang, and R. Tsay, 2005. “An Unobserved-Component Model with Switching Permanent and Transitory Innovations,” *Journal of Business & Economic Statistics* 23 (4), 443-454.

Lothian, J. and Taylor, M., 1996. "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries," *The Journal of Political Economy* 104 (3), 488-509.

Lopez, C., C. Murray, and D. Papell, 2004. "State of the Art Unit root Tests and Purchasing Power Parity," working paper, University of Cincinnati.

Lumsdaine, R. and D. Papell, 1997. "Multiple Trend Breaks and the Unit-Root Hypothesis," *The Review of Economics and Statistics* 79 (2), 212-218.

MacDonald, R. and L. Ricci, 2001. "PPP and the Balassa Samuelson Effect: The Role of the Distribution Sector," IMF Working Papers.

Murray, C. and D. Papell, 2002. "The Purchasing Power Parity Persistence Paradigm," *Journal of International Economics* 84, 1-19.

Murray, C. and D. Papell, 2005a. "The Purchasing Power Parity is Worse Than You Think," *Empirical Economics* 30 (3), 783-790.

Murray, C. and D. Papell, 2005b. "Do Panels Help Solve the Purchasing Power Parity Puzzle?" *Journal of Business & Economic Statistics* 23 (4), 410-415.

Nunes, L., C.M. Kuan, and P. Newbold, 1995. "Spurious Break," *Econometric Theory* 11, 736-749.

Obstfeld, M. and K. Rogoff, 2000. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" NBER Working Paper 7777.

Papell, D. and R. Prodan, 2004. "Long Run Purchasing Power Parity: Cassel or Balassa-Samuelson?" working paper, University of Houston.

Prodan, R., 2005. "Potential Pitfalls in Determining Multiple Structural Changes with an Application to Purchasing Power Parity," manuscript, University of Alabama.

Rogoff, K., 1996. "The Purchasing Parity Puzzle," *Journal of Economic Literature* 34 (2), 647-668.

Taylor, A., 2002. "A Century of Purchasing-Power Parity," *The Review of Economics and Statistics* 84 (1), 139-150.

Wu, Y, 1996. "Are Real Exchange Rates Nonstationary? Evidence from a Panel-Data Test," *Journal of Money, Credit, and Banking* 28, 54-63.

Table 1: AR(1) with Unit Root Model

	Prob(UR|UR)	*Prob(St|St)*	φ	$\sigma_{UR}2$	$\sigma_{ST}2$	**Likelihood Value**
Australia	0.8881	0.8375	0.8440	0.0358	0.1288	164.0230
	(0.0562)	(0.1028)	(0.0681)	(0.0079)	(0.0209)	
Belgium	0.9751	0.6997	0.5277	0.0806	0.5832	80.4131
	(0.0150)	(0.1324)	(0.1255)	(0.0077)	(0.1273)	
Canada	0.9472	0.9224	0.8074	0.0314	0.0608	228.9060
	(0.0479)	(0.0660)	(0.1333)	(0.0046)	(0.0078)	
Denmark	0.9834	0.9673	0.9267	0.0350	0.1340	125.8168
	(0.0138)	(0.0202)	(0.0457)	(0.0038)	(0.0113)	
Germany	0.9398	0.9075	0.9580	0.0270	0.1280	169.5601
	(0.0299)	(0.0493)	(0.0190)	(0.0024)	(0.0145)	
Spain	0.9493	0.8634	0.9997	0.0530	0.1592	95.4154
	(0.0314)	(0.0667)	(0.0065)	(0.0087)	(0.0194)	
Finland	0.9125	0.7205	0.6673	0.0587	0.3224	85.5425
	(0.0353)	(0.1141)	(0.1390)	(0.0072)	(0.0508)	
France	0.8251	0.8292	0.7675	0.0434	0.1084	125.6131
	(0.0948)	(0.0787)	(0.1903)	(0.0092)	(0.0119)	
Italy	0.9846	0.9099	0.9985	0.0265	0.2211	109.3920
	(0.0122)	(0.0470)	(0.0010)	(0.0025)	(0.0223)	
Japan	0.9876	0.8856	0.9919	0.1130	0.0297	104.8256
	(0.0101)	(0.0601)	(0.0031)	(0.0088)	(0.0047)	
Netherlands	0.9379	0.9483	0.9442	0.0275	0.1241	166.6399
	(0.0308)	(0.0320)	(0.0257)	(0.0026)	(0.0117)	
Norway	0.9505	0.8880	0.8208	0.0327	0.1633	163.0848
	(0.0275)	(0.0606)	(0.0445)	(0.0028)	(0.0195)	
Portugal	0.9722	0.6299	0.5519	0.0753	0.3473	77.8230
	(0.0170)	(0.1511)	(0.0655)	(0.0061)	(0.0781)	
Sweden	0.9373	0.8877	0.8510	0.0263	0.1379	154.2002
	(0.0326)	(0.0480)	(0.0410)	(0.0029)	(0.0139)	
Switzerland	0.8495	0.8663	0.9610	0.0227	0.1477	127.9609
	(0.0597)	(0.0622)	(0.0142)	(0.0031)	(0.0150)	
United Kingdom	0.9175	0.8900	0.9406	0.0247	0.1227	194.3264
	(0.0417)	(0.0655)	(0.0351)	(0.0026)	(0.0135)	

*Notes: Standard Errors are in parentheses. RERs are in terms of USD.

Table 2: 2 Unit Root Model

	Prob(HI|HI)	***Prob(LO|LO)***	σ_{HI}^2	σ_{LO}^2	**Likelihood Value**
Australia	0.8911 (0.0570)	0.8966 (0.0463)	0.0311 (0.0043)	0.1227 (0.0131)	164.0491
Belgium	0.7660 (0.1703)	0.9916 (0.0091)	0.0951 (0.0070)	0.8250 (0.2683)	80.0729
Canada	0.8748 (0.0665)	0.7150 (0.1178)	0.0136 (0.0033)	0.0551 (0.0049)	231.6840
Denmark	0.9811 (0.0154)	0.9401 (0.0339)	0.0360 (0.0039)	0.1360 (0.0114)	121.6769
Germany	0.9102 (0.0465)	0.9406 (0.0309)	0.0271 (0.0028)	0.1257 (0.0142)	166.8682
Spain	0.8565 (0.0681)	0.9650 (0.0231)	0.0540 (0.0073)	0.1604 (0.0790)	87.9914
Finland	0.7297 (0.1071)	0.9005 (0.0451)	0.0561 (0.0113)	0.3302 (0.0565)	81.8098
France	0.8733 (0.0541)	0.7301 (0.1141)	0.0217 (0.0042)	0.1053 (0.0093)	130.1611
Italy	0.6247 (0.2397)	0.9671 (0.0191)	0.0829 (0.0060)	2.4071 (0.6779)	80.7979
Japan	0.1053 (3.9427)	0.9914 (0.0088)	0.1000 (0.0065)	6.2768 (4.4975)	96.0614
Netherlands	0.9519 (0.0292)	0.9416 (0.0289)	0.0291 (0.0027)	0.1264 (0.0120)	164.1804
Norway	0.8946 (0.0531)	0.9550 (0.0270)	0.0349 (0.0033)	0.1564 (0.0166)	154.3472
Portugal	0.7786 (0.1624)	0.9894 (0.0146)	0.0894 (0.0089)	0.3603 (0.1000)	73.4821
Sweden	0.9039 (0.0447)	0.9500 (0.0281)	0.0291 (0.0037)	0.1409 (0.0144)	147.4977
Switzerland	0.9112 (0.0500)	0.8803 (0.0512)	0.0258 (0.0034)	0.1443 (0.0146)	125.2769
United Kingdom	0.8770 (0.0702)	0.9098 (0.0427)	0.0252 (0.0027)	0.1258 (0.0138)	191.4932

*Notes: Standard Errors are in parentheses. RERs are in terms of USD.

Table 2a: 2 Unit Root Model: Other Country Pairs

	Prob(HI|HI)	*Prob(LO|LO)*	σ_{HI}^2	σ_{LO}^2	**Likelihood Value**
Germay/UK	0.9411 (0.0355)	0.9285 (0.0456)	0.0257 (0.0026)	0.1037 (0.0109)	169.3137
Japan/UK	0.9599 (0.0488)	0.0001 (0.0047)	0.0906 (0.0080)	0.2327 (0.1038)	91.8639
France/UK	0.7134 (0.1007)	0.8706 (0.0551)	0.0136 (0.0023)	0.1020 (0.0085)	142.1099
Australia/UK	0.8636 (0.0590)	0.8645 (0.0630)	0.0273 (0.0045)	0.1146 (0.0121)	171.9570
Canada/Germany	0.9114 (0.0465)	0.8944 (0.0544)	0.0346 (0.0044)	0.1202 (0.0136)	146.7804
Portugal/Japan	0.8322 (0.0777)	0.8479 (0.0806)	0.0488 (0.0085)	0.2019 (0.0225)	70.1062
Italy/Australia	0.9741 (0.0200)	0.8298 (0.0952)	0.0614 (0.0070)	0.2518 (0.0398)	79.5657
Switzerland/UK	0.7625 (0.0826)	0.7054 (0.1493)	0.0310 (0.0053)	0.1406 (0.0190)	125.1205
Spain/UK	0.9492 (0.0315)	0.8638 (0.0652)	0.0528 (0.0074)	0.1590 (0.0184)	95.4100

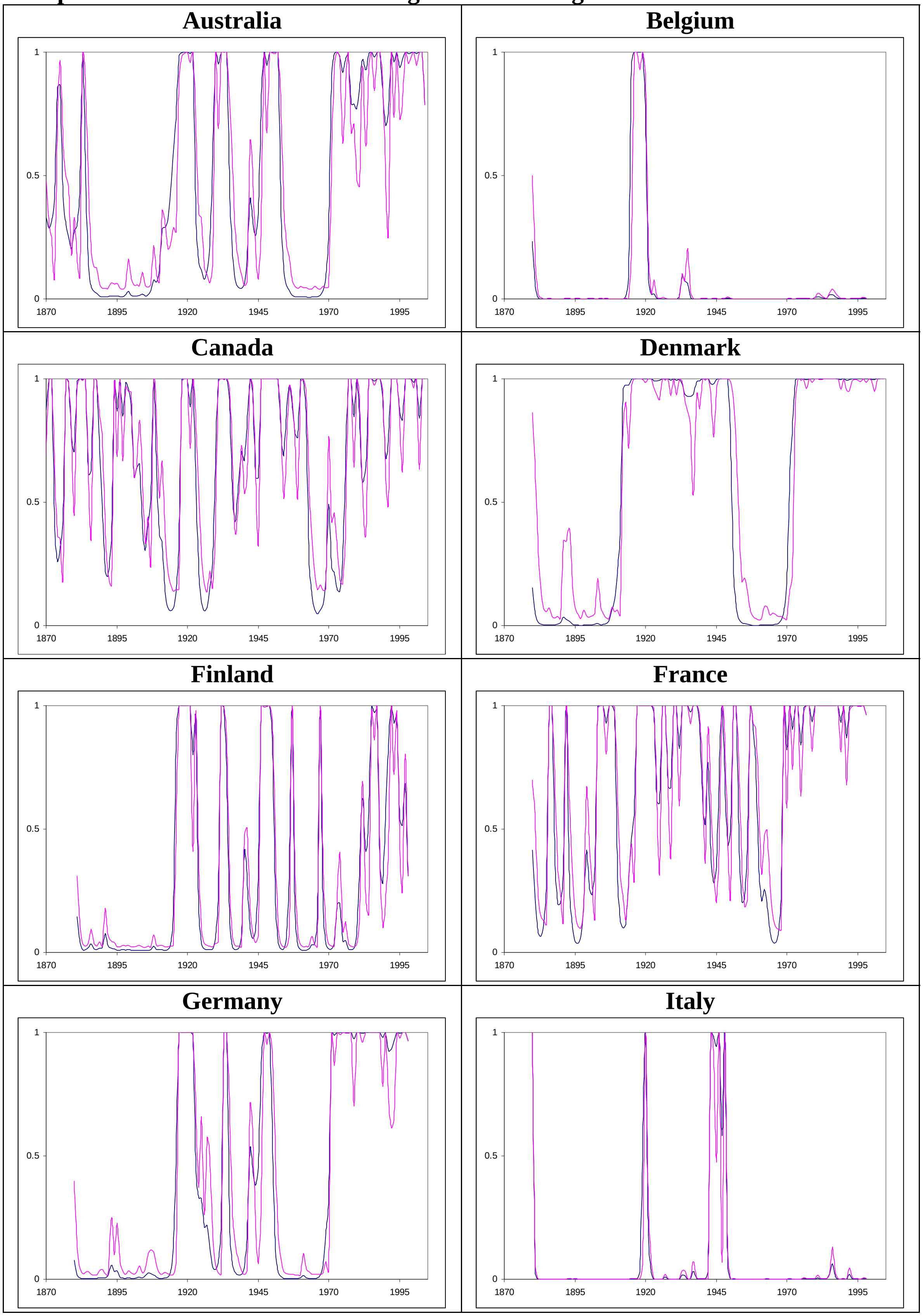
*Notes: Standard Errors are in parentheses. RERs for (country A / country B) are constructed as $q_t = s_t + p_t - p_t^*$, where s_t, p_t, and p_t^* are the logarithms of the nominal exchange rate (country A price in terms of country B currency), country A and country B price levels, respectively .

Table 3: 2 Unit Root with AR(1) Model

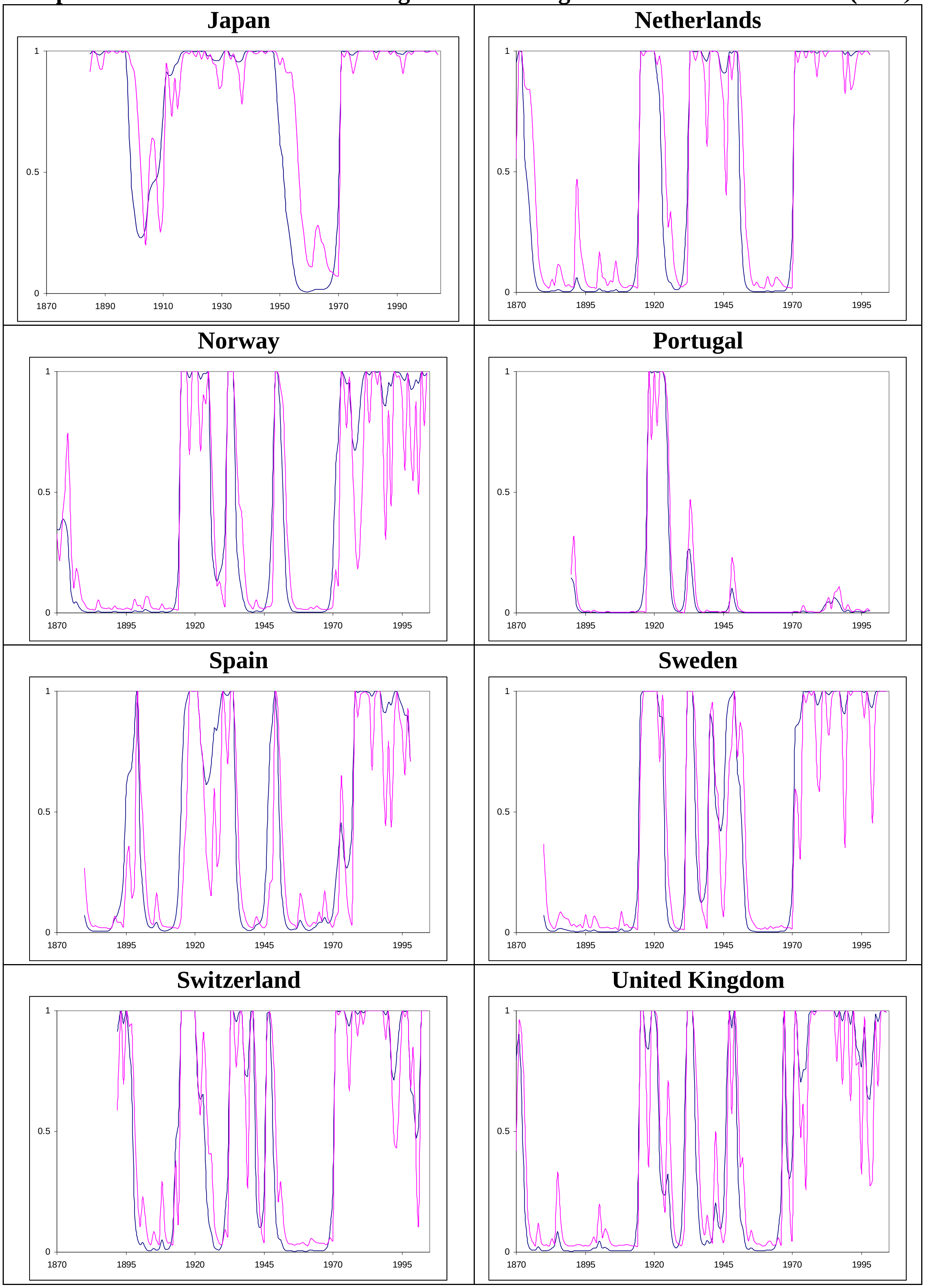
	Prob(HI|HI)	*Prob(LO|LO)*	ϕ	σ_{HI}^2	σ_{LO}^2	σ_{ST}^2	Likelihood Value
Australia	0.8912 (0.0571)	0.8966 (0.0463)	0.9998 (0.0008)	0.0311 (0.0043)	0.1187 (0.0132)	0.0000 0.0000	163.5387
Belgium	0.7862 (0.1649)	0.9908 (0.0096)	0.1507 (0.1985)	0.0943 (0.0071)	0.7957 (0.2470)	0.0000 (0.0020)	84.4400
Canada	0.7079 (0.1189)	0.8708 (0.0678)	0.4328 (0.3481)	0.0528 (0.0048)	0.0134 (0.0035)	0.0001 (0.0050)	231.4926
Denmark	0.9764 (0.0186)	0.9580 (0.0235)	0.0000 (0.0889)	0.1497 (0.0132)	0.0349 (0.0035)	0.0000 (0.0026)	132.6832
Germany	0.9392 (0.0312)	0.9118 (0.0467)	0.0000 (0.0357)	0.1226 (0.0145)	0.0271 (0.0028)	0.0000 0.0000	166.8948
Spain	0.9492 (0.0652)	0.8638 (0.0316)	0.0000 (0.0777)	0.1500 (0.0189)	0.0528 (0.0074)	0.0000 (0.0076)	95.4100
Finland	0.8924 (0.0514)	0.7435 (0.1070)	0.0203 (0.2884)	0.3188 (0.0580)	0.0543 (0.0137)	0.0000 (0.0024)	82.5820
France	0.7463 (0.1006)	0.8732 (0.0540)	0.9942 (0.0057)	0.1039 (0.0093)	0.0210 (0.0036)	0.0001 (0.0028)	131.6932
Italy	0.9790 (0.0153)	0.9098 (0.0450)	0.9980 (0.0015)	0.2205 (0.0026)	0.0266 (0.0025)	0.0001 0.0000	114.4503
Japan	1.0000 0.0000	0.0171 0.0000	0.0829 (0.1768)	0.0040 0.0000	0.0998 0.0065	0.0000 0.0000	93.3987
Netherlands	0.9577 (0.0244)	0.9522 (0.0299)	0.4947 (0.1069)	0.1249 (0.0128)	0.0295 (0.0026)	0.0000 (0.0010)	167.7789
Norway	0.9496 (0.0282)	0.9174 (0.0450)	0.7110 (0.0950)	0.1517 (0.0164)	0.0326 (0.0029)	0.0000 0.0000	160.2424
Portugal	0.9464 (0.0327)	0.7596 (0.1069)	0.1534 (0.0977)	0.2358 (0.0491)	0.0609 (0.0120)	0.0000 (0.0013)	82.8247
Sweden	0.9404 (0.0326)	0.9128 (0.0432)	0.0000 (0.0610)	0.1376 (0.0144)	0.0289 (0.0036)	0.0000 0.0000	149.3463
Switzerland	0.8805 (0.0517)	0.8909 (0.0574)	0.1483 (0.1475)	0.1429 (0.0153)	0.0259 (0.0034)	0.0000 0.0000	126.0131
United Kingdom	0.8937 (0.0647)	0.9233 (0.0383)	0.5120 (0.2006)	0.0243 (0.0024)	0.0124 (0.0139)	0.0000 0.0000	194.4131

*Notes: Standard Errors are in parentheses. RERs are in terms of USD.

Graph: Estimated Probabilities of High Variance Regime for 2 Unit Root Model



Graph: Estimated Probabilities of High Variance Regime for 2 Unit Root Model (*cont.)*



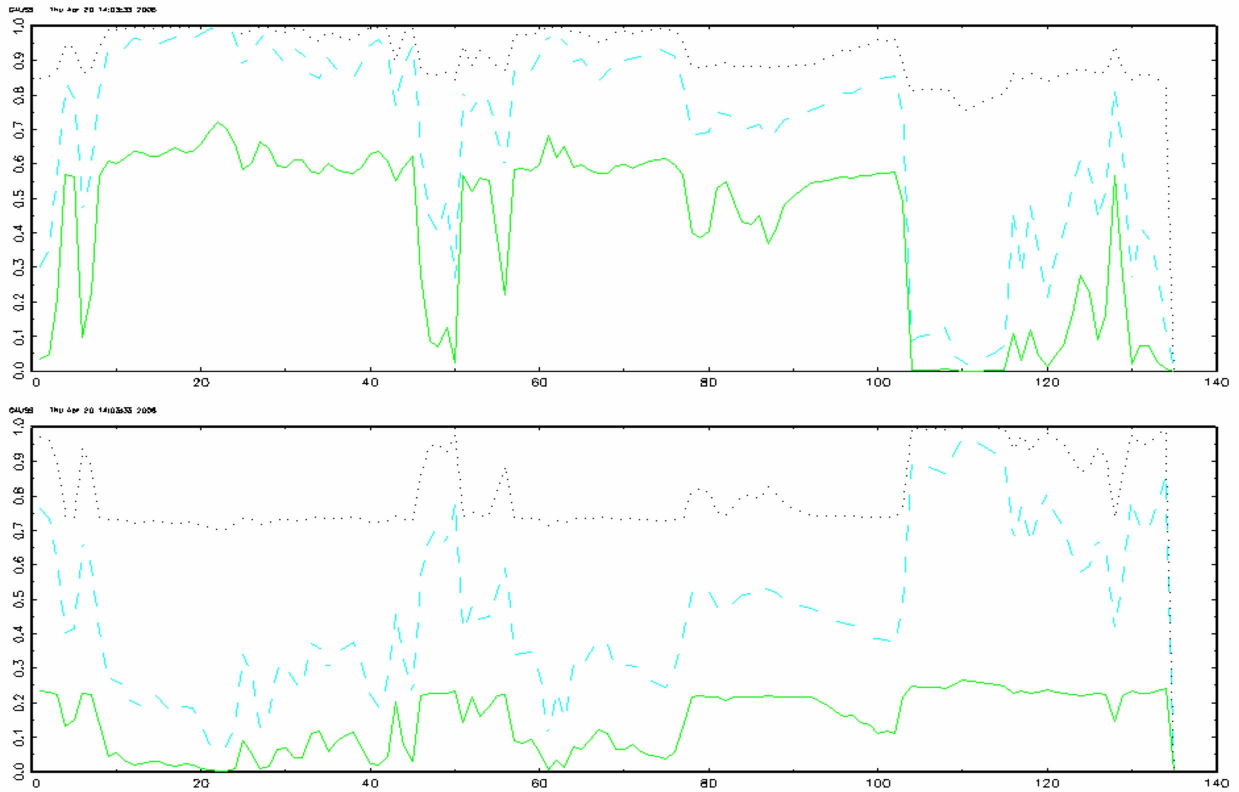
Graph: Time-varying Transition Probabilities (US-UK)

The following graphs show PPR on top and QPR on the bottom for the fundamental, Z_t. The estimated probabilities are in light blue with the dotted line (solid line) as the upper (lower) 95% confidence limit. Our data runs from 1870-2004. Note:

$$\text{PPR} = \text{PR (St=Hi | St=Hi)} = 1 - \frac{1}{1+\exp(p_0 + p_1 Z_t)}$$

$$\text{QPR} = \text{PR (St=Lo | St=Lo)} = 1 - \frac{1}{1+\exp(q_0 + q_1 Z_t)}$$

Oil:



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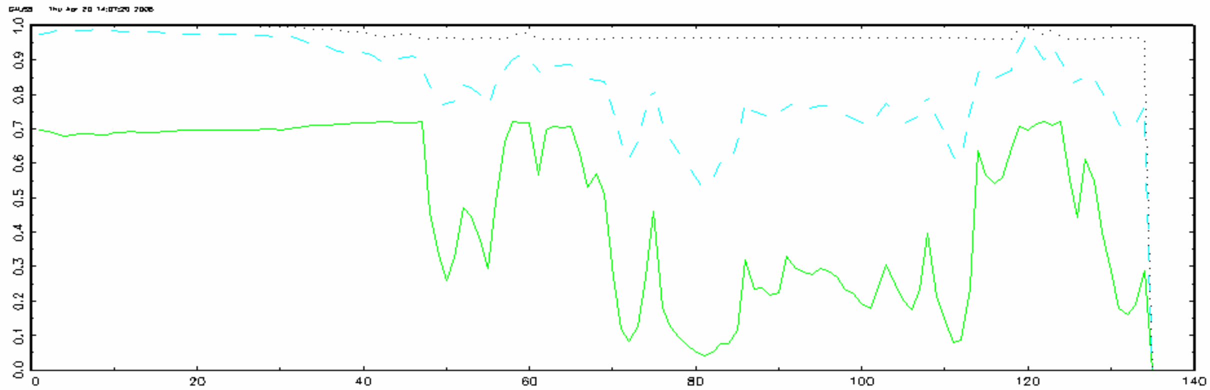
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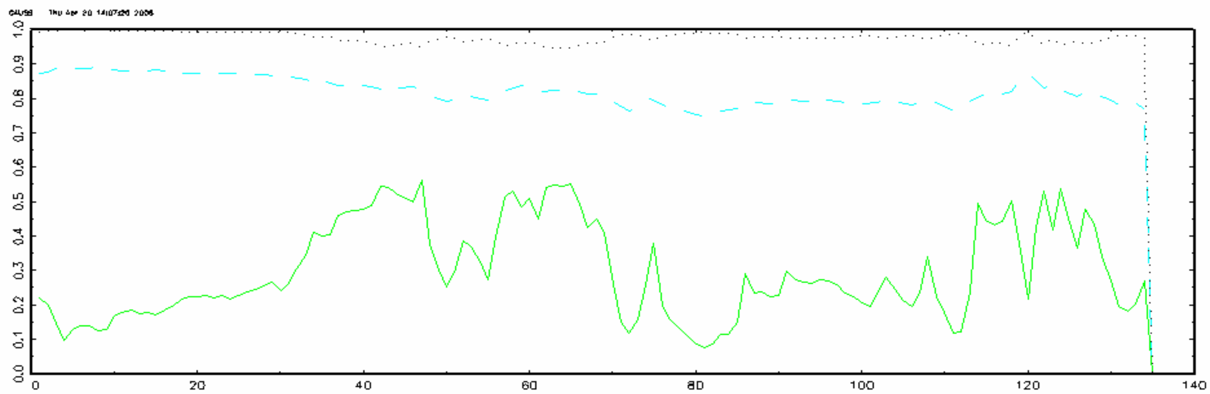
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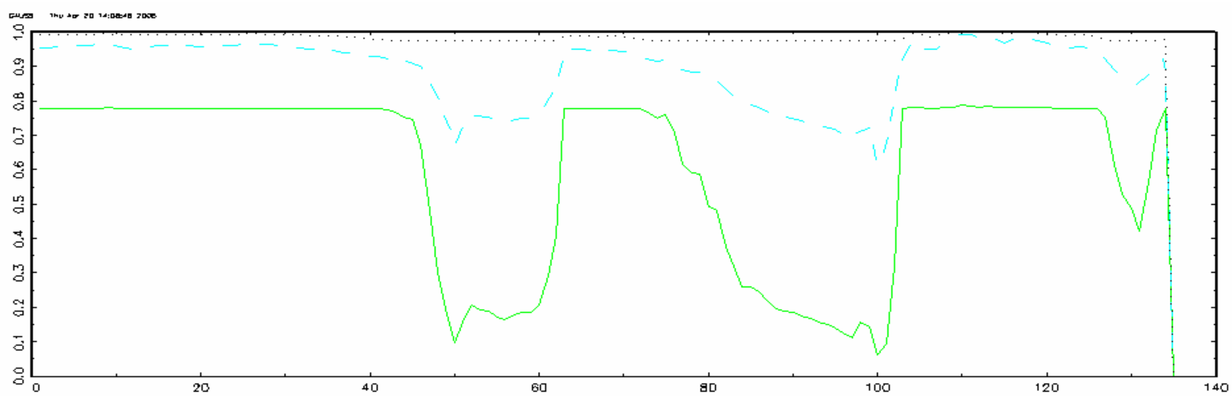
Gold: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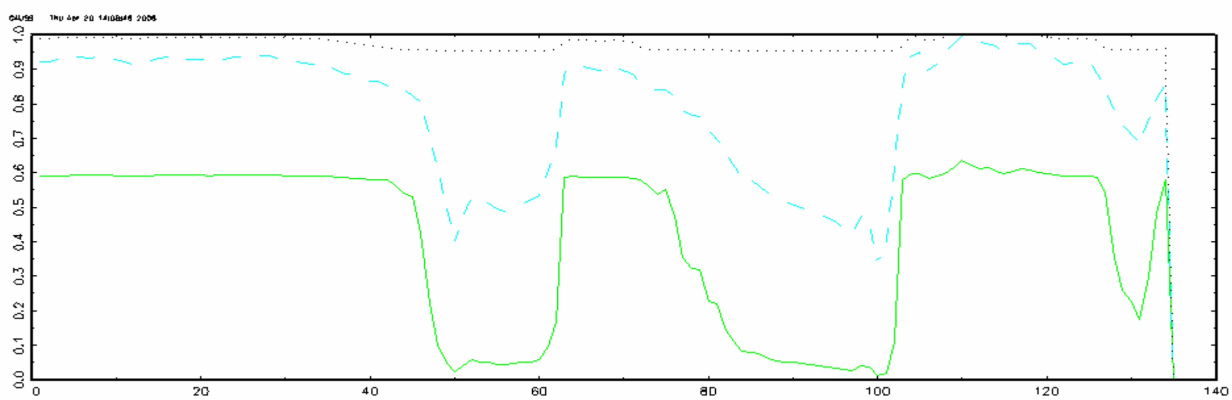
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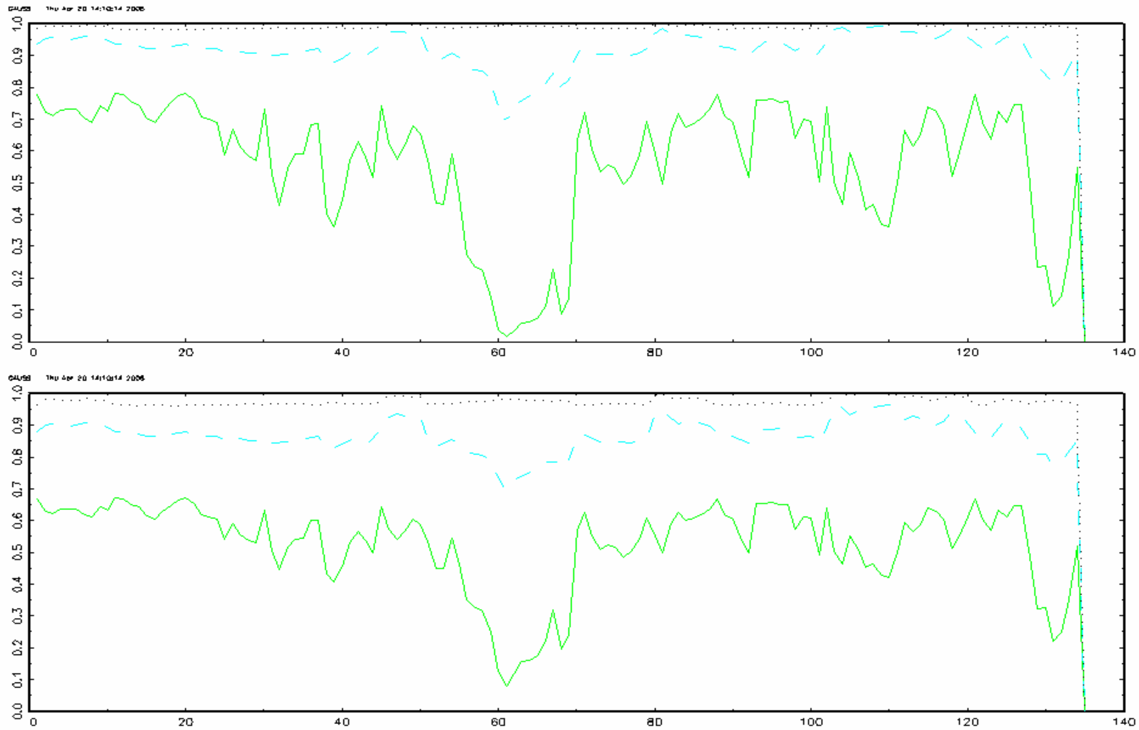
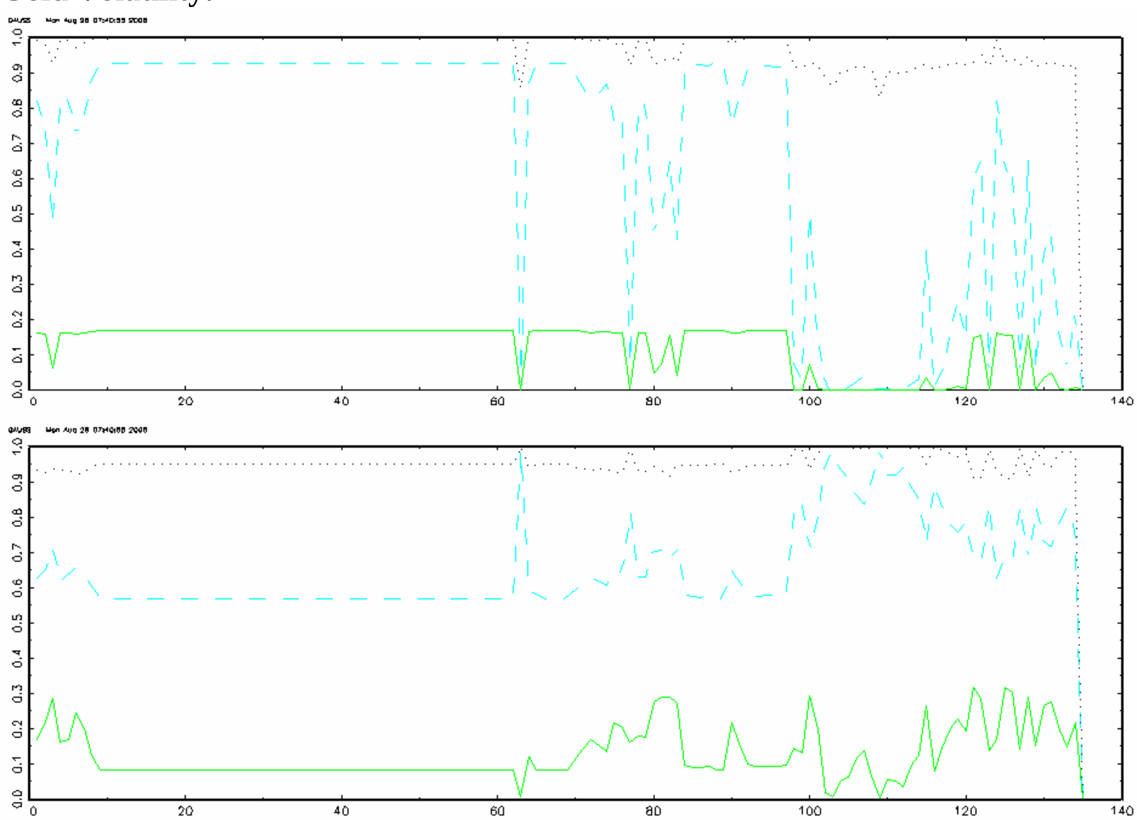
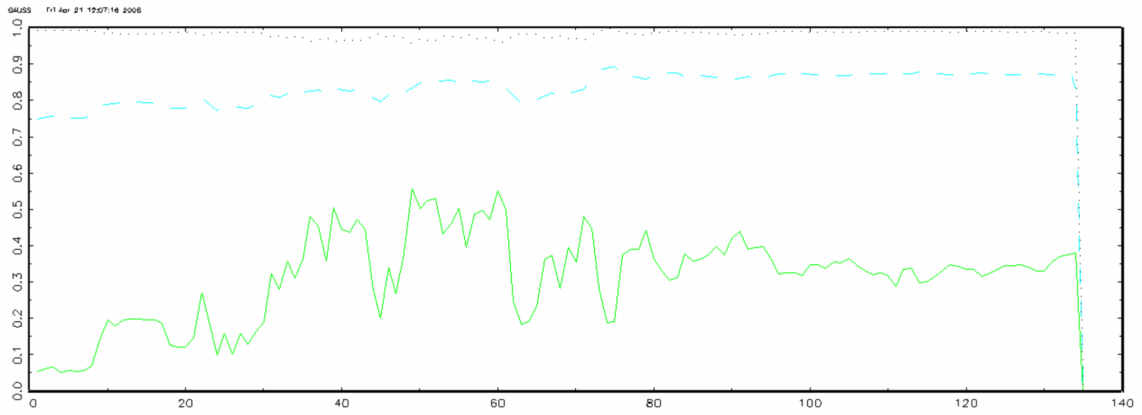
Economist Commodity Index:

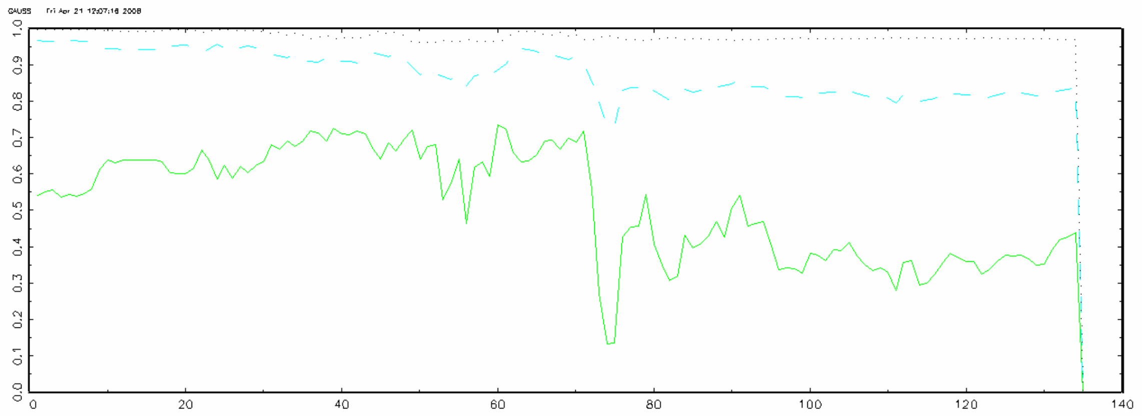


Gold Volatility:



GDP/Capita Volatility:





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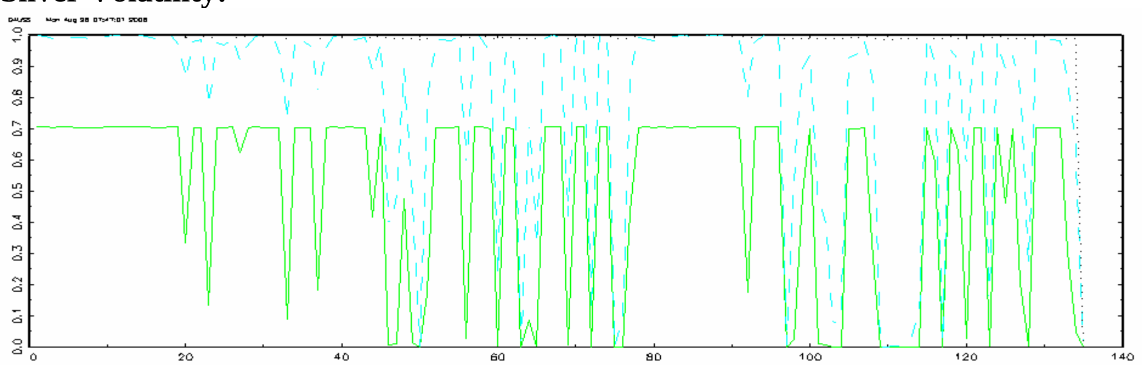
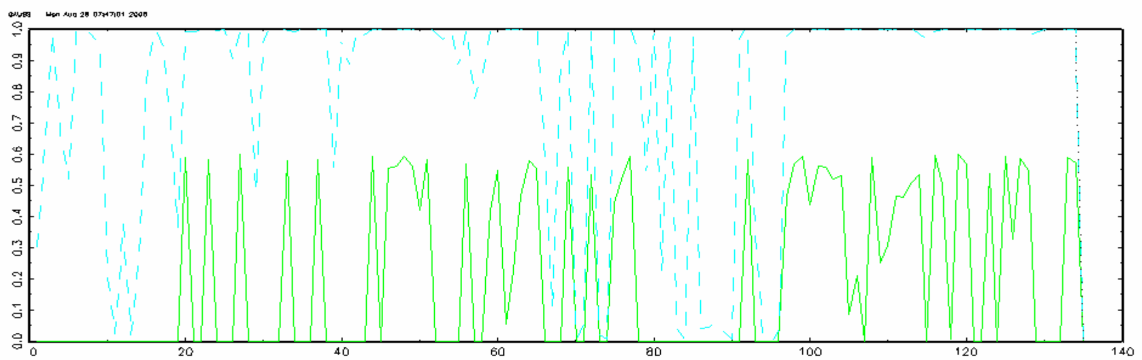
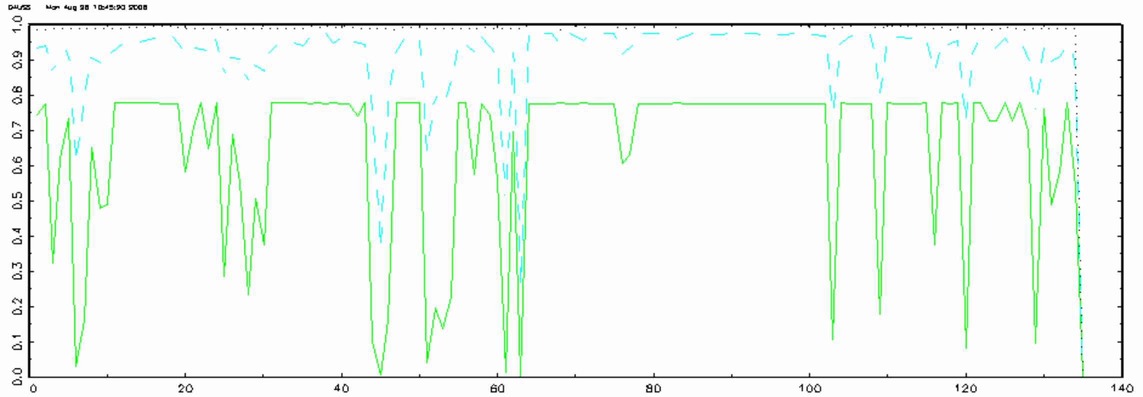
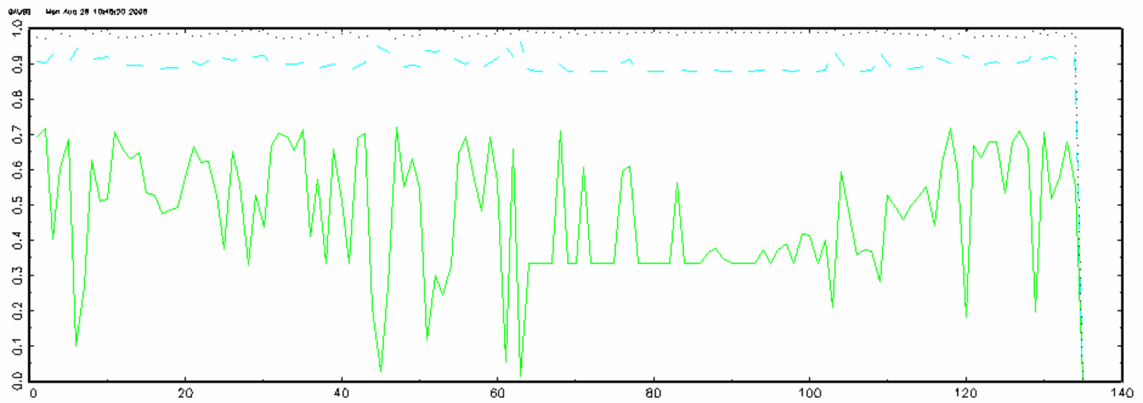


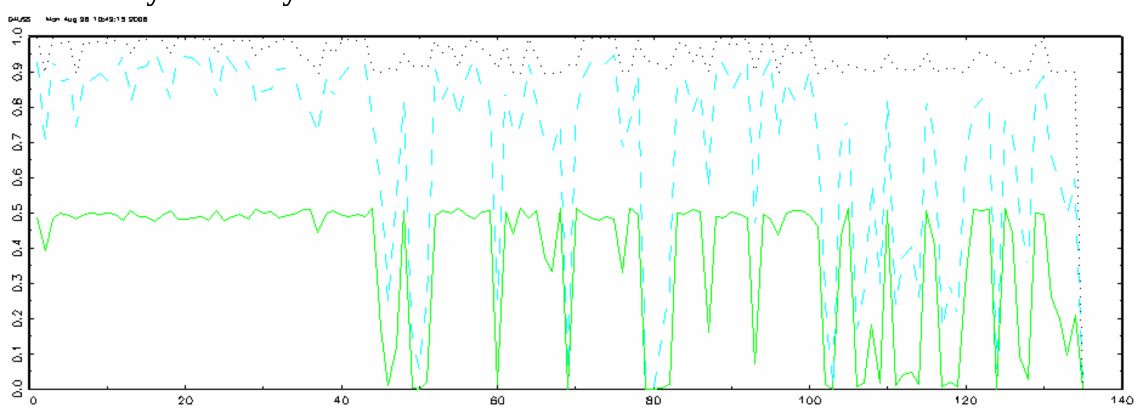


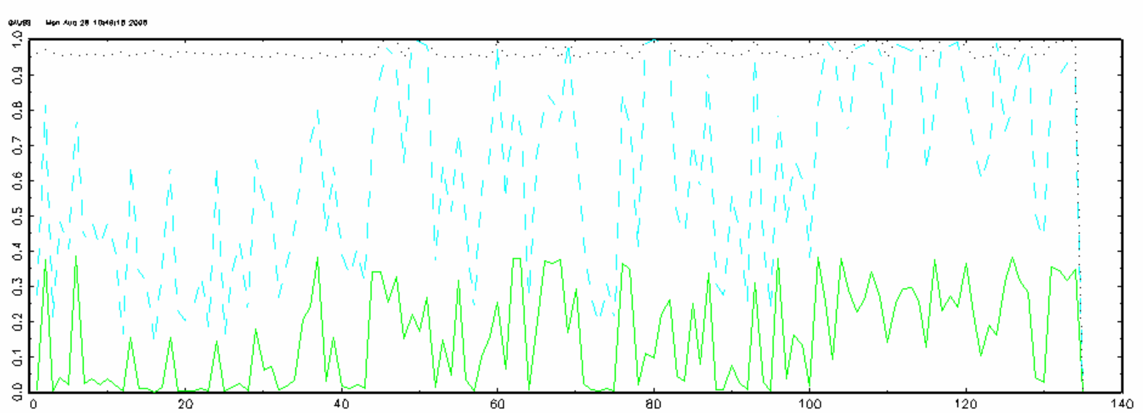
Oil Volatility:



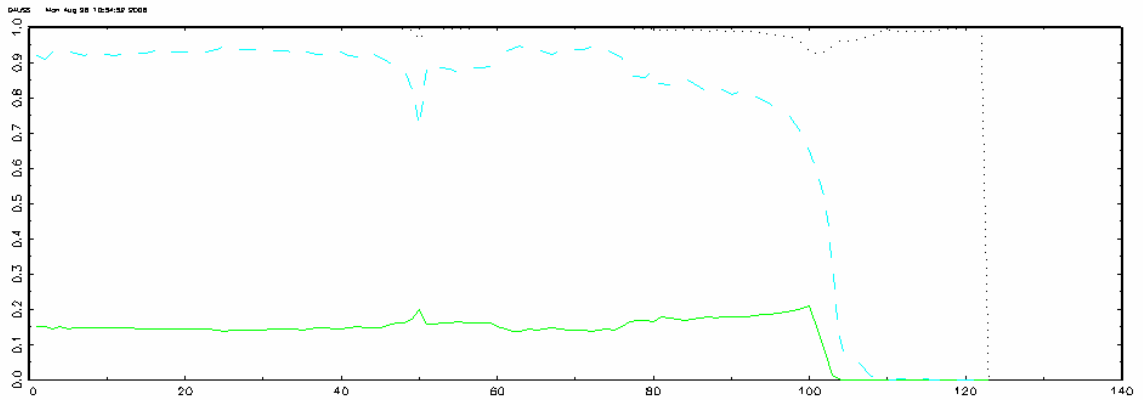


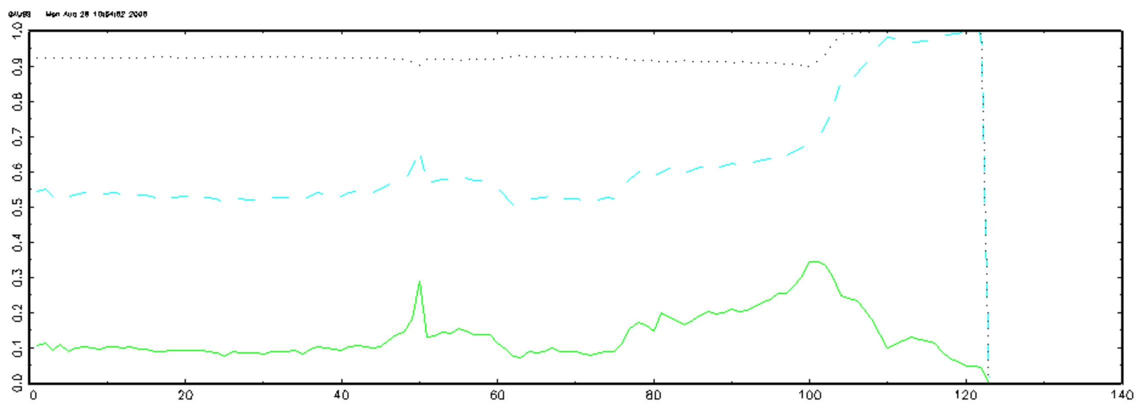
Commodity Volatility:





Openness:





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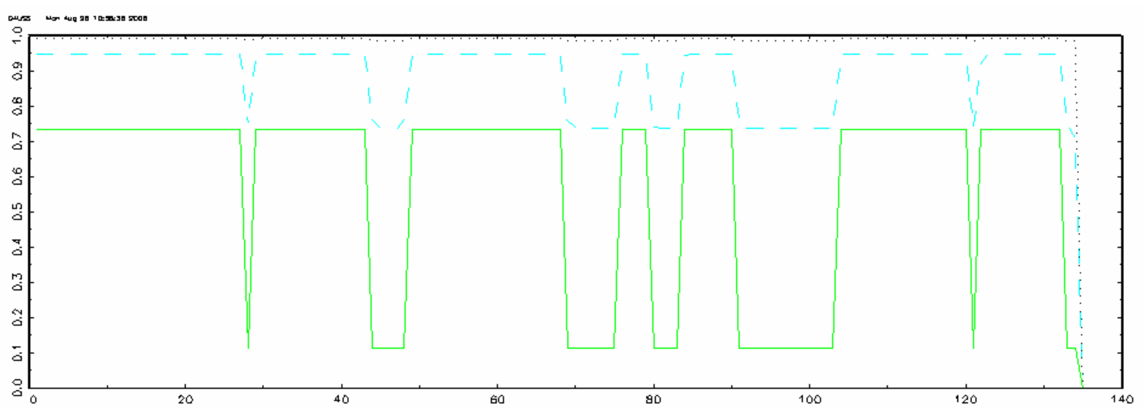

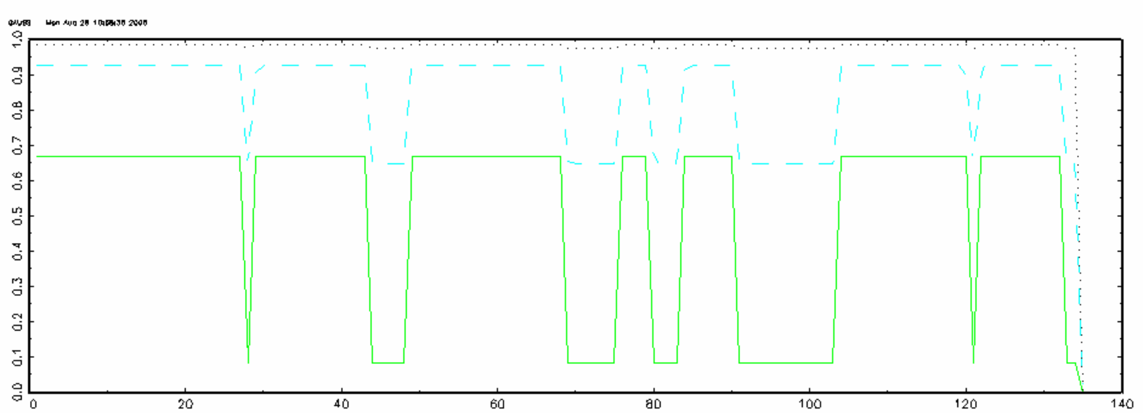

Gold Standard: