The Search for Stationarity in Real Exchange Rates: An Unobserved Component Regime-Switching Approach

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Abstract

The Purchasing Power Parity Puzzle questions how the observed large short-term volatility in real exchange rates can be reconciled with their high persistence. Exploring a variety of regime-switching specifications, we find that modeling the RER as a Switching AR(1) and Unit Root Model best represents its behavior for our long horizon data for 16 countries. We then show that the likelihood of the transition corresponds to various economic fundamentals, such as GDP per capita differences, commodity price levels and volatilities, and trade openness. Not only do the estimates from our country-by-country regressions cover a large range, we also observe differing fundamental forces driving the switches between states across countries. This suggests one should proceed with caution when analyzing real exchange rate behavior in a panel framework

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1. Introduction

The PPP Puzzle asks whether the large and frequent short-term shocks to the Real Exchange Rate (RER) and the persistence of these shocks can be supported by theory. Empirical evidence suggests the half-life of RER shocks to be 3 to 5 years, but traditional theory predicts a quicker reversion to the process due to goods market arbitrage (Rogoff, 1996). The inconsistency between theory and evidence leads us to believe that the RER best be described by a regime switching model. There may be 2 distinct states: real shocks could prove to have permanent, lasting effects whereas monetary and financial shocks could tend to constitute volatile yet transitory disturbances. Our results indicate regime shifts in line with historical events and strong evidence of both types of states for many country pairs.

This regime switching approach allows us to overcome many of the present problems in current approaches in the literature. One of the main issues in testing for PPP is the power of the Unit Root tests in small samples.[1] Post-Bretton Woods data is often used in PPP analysis to have a sample without nominal exchange rate regime shifts, but it contains less than 40 years of data.[2] To circumvent this problem, researchers frequently choose one of two types of approaches to increase sample size: use long-horizon data series or pool the data for use in a panel framework. Some examples of long-horizon data include Lothian and Taylor (1996), Rogoff (1996), and Taylor (2000). Not only do their efforts fail to reduce the half-life of shocks down to the desired

[1] Engel (1999) and Murray and Papell (2000) explore the power of tests in small samples. Murray and Papell (2005a and 2005b) and Amara and Papell (2004) propose alternative estimation methods.

[2] Many papers explore the issue of sample selection. Grilli and Kaminsky (1989) deal with the historical background of RER through a long set of data and conclude that the RER volatility depends on its historical setting and not on the nominal regime. But, they concede that the post-Bretton Woods period exhibits very high volatility. Diebold, Husted, and Rush (1991) choose to use data from the Gold Standard because those regimes represent the greatest amount of international cooperation, which is necessary for PPP to hold. Frankel and Rose (1996) use only post-World War II data in their panel study because the data exhibits a clear shift before and after the war. Even the choice of countries has an effect on the result of the studies. Cheung and Lai (1998) claim that developed countries are less likely to exhibit stationarity than their developing counterparts.

1-2 years that would be supported by theory through price-stickiness, but the use of long-horizon data assumes a constant underlying data generating process, which is unlikely because of the numerous happenings during the span of the sample. By allowing for multiple states, our regime switching model can use the large sample advantages of long-horizon data while relaxing the constant process assumption. We show that series of RERs are characterized by shifts in the persistence of shocks to the processes, which are not properly picked up in analyses of long horizon data using a single series.

Our model also exposes flaws in the popular method of getting around the problem of multiple regimes in a long horizon dataset by breaking up the data into separate periods. Using arbitrary sample selection methods or running structural breakpoint tests would allow the researcher to keep from mixing regimes. In Taylor (2000), he breaks up his dataset into 4 periods of history: Gold Standard, Interwar, Bretton Woods, and Float. Diebold, Husted, and Rush (1991) use the gold standard periods in their analysis to allow for the greatest amount of cooperation between countries. A problem with specifying break dates *a priori* is that if they are off by a few periods, the estimations might not fully characterize the true, underlying processes. In the regime switching framework, our model endogenously selects the dates for shifting regimes, which takes away the potential for human bias in the analysis.

Using endogenous structural breakpoint tests to find distinct regimes in long horizon data can also allow the model to select the breakdates directly, but we show the tests to be biased in the presence of highly persistent data. For example, Hedgwood and Papell (1998) reduce the half-lives of many RER series by allowing the process to shift whenever a new breakpoint is encountered. They coin their result "Quasi-PPP." We show that the regime switching approach can correctly identify Quasi-PPP, but Quasi-PPP cannot identify our regime switching specification with 2 distinct states.

Our regime switching model also has implications on the alternative way of increasing the sample size to increase the power of Unit Root tests by pooling data across countries. Frankel and Rose (1996) use 150 countries from the International Financial Statistics database to find stationarity with a half-life longer than the acceptable 1-2 years. Alba and Papell (2007) use Feasible GLS (SUR) in their analysis and conclude that one cannot characterize all countries as exhibiting stationary or nonstationary shocks.[3] The results from our estimations caution the use of this technique because there are clear differences between countries; pooling the data would assume a certain degree of homogeneity amongst the countries, which may be unrealistic in the case of RERs.

The proposed model, Switching AR(1) and Unit Root Model, characterizes the RER as a stationary process with occasional permanent shocks.[4] Unlike other regime switching approaches, our model maintains parsimony while allowing for flexibility in its characterization of the RER.[5] The stationary process governs the more common monetary shocks, and the nonstationary process accounts for the less frequent real shocks. The empirical results strongly support our method of representing the RER as these 2 states. The model is robust to variations and encompasses the findings of previous trials using structural breakpoints and univariate Unit Root Tests. Perhaps most importantly, we show that the regimes characterized by our model are

[3] See Wu (1996), Canzoneri, Camby, and Diba (1996), and Papell (2006) for other examples of PPP analysis using panel frameworks.
[4] The model is similar to the Innovation Regime Switching (1; 1, 0) model of Kuan, Huang, and Tsay (2005) used to model Real GDP.
[5] Other Regime Switching models in the RER context include Engel and Hamilton (1990), Engel and Kim (1999), Bergman and Hansson (2000), and Frömmel, MacDonald, and Menkhoff (2002). See Hegwood and Papell (1998), Diebold, Husted, and Rush (1991), Cheung (1993), Cheung and Lai (1993), and Papell and Prodan (2006) for other methods.

not arbitrary but are closely related to historical events, such as wars and nominal currency regime changes.

To further explain shifts between regimes, we tie fundamentals—such as GDP/Capita Differences, Commodity Price Levels and Volatilities, and Trade Openness—into our models. The fundamentals play a definitive role in explaining the RER process, but their effect depends on the country pair in question, which can be explained by different countries having different policies and dependencies on commodities. Furthermore, even though distinct states show up for each RER series, the absolute levels of the parameter estimates vary. These findings support our conclusion that panel analysis may be too restrictive.

Though our methodology does not reduce the half life of the stationary process of every country pair, there are a handful of countries that consistently show quick reversion during the stationary periods. For other countries, the shocks in the stationary periods remain highly persistent, which is consistent with the long half-life findings in the current literature and also with recent theories that suggest traditional theories of PPP reversion to be incomplete.[6]

The following section presents our models and estimation methodology. Section 3 covers the results and discussion for the Switching AR(1) and Unit Root Model and the 2 Unit Root Model; we also include robustness checks for the latter. In Section 4, we directly model fundamentals to explain the behavior of RERs as described by our model. Finally, Section 5 concludes and offers extensions to our project.

[6] See Benigno (2004) on how monetary policy rules may influence persistence of RER shocks. Other reasons for the persistent volatility include MacDonald and Ricci (2002), who argue that the size and competitiveness of the distribution sector of an economy impacts the price adjustment mechanisms of its tradables sector. Obstfeld and Rogoff (2000) and Imbs, Mumtaz, and Ravn (2002) show that transaction costs could impact the price levels of the individual markets as well. Imbs, Mumtaz, Ravn, and Rey (2005a) argues that an aggregation bias in the data yields larger half-lives than the individual sectors would produce. This is questioned in Chen and Engel (2005) but later reiterated in Imbs, Mumtaz, Ravn, and Rey (2005b).

2A. Models

The Switching AR(1) and Unit Root Model allows for both stationary and nonstationary components because shocks could affect RERs in different ways depending on their inherent nature. For example, real shocks may prove to have permanent effects on RERs whereas monetary shocks are merely transitory disturbances. This suggests that the model must allow for only one type of shock each period if we wish to make this distinction. A reduction of the half-life of transitory shocks may even lessen the purchasing power parity puzzle.

In the Switching AR(1) and Unit Root Model, the series is composed of a permanent process and a stationary process, but shocks only affect one process at any given time. For our annual data, a half-life of 1 to 2 years would coincide with an AR coefficient of between 0.5 and 0.7.

Switching AR(1) and Unit Root Model:

$$
\begin{aligned}
y_t &= x_t + z_t \qquad (1)\\
x_t &= x_{t-1} + S_t v_t\\
z_t &= \varphi z_{t-1} + (1-S_t)\, e_t\\
v_t &\sim N(0, \sigma_v^2)\,,\ e_t \sim N(0, \sigma_e^2)
\end{aligned}
$$

For the Switching AR(1) and Unit Root Model, the RER, y_t, is characterized by a Unit Root nonstationary process, x_t, and an AR(1) process with a coefficient of φ. The shocks v_t and e_t, for the Unit Root process and AR(1) process respectively, are distributed normally with mean zero and variances σ_v^2 and σ_e^2. S_t is a state parameter that takes the value of 0 or 1.

The unique feature is that the state variable, S_t, determines the allocation of the shock at time t. If $S_t = 0$ for all t in the Switching AR(1) and Unit Root Model, only the stationary shock, e_t, enters y_t. In other words, for the Switching AR(1) and Unit Root Model, the process becomes $y_t = x_0 + \varphi z_{t-1} + e_t$ where e_t is the transitory shock. This is merely a stationary AR process with a level shift, $x_{0.}$ For the opposite case where $S_t = 1$, only the permanent process comes into effect and yields $y_t = \varphi z_0 + x_{t-1} + v_t$, where v_t is the permanent shock. Here, we have a Unit Root plus a constant, $\varphi z_{0.}$

Unlike many other models that incorporate both permanent and transitory components for RERs, the model above suggests that the shocks are mutually exclusive. Other models, such as Bergman and Hansson (2000) are built upon the single process AR(1) model and only have switching in the intercept and coefficient, φ . The authors cannot attribute the regime shifts to historical events and, thus, fail to interpret the different states afforded by the model. We allow 2 distinct processes that can be interpreted as real and monetary shocks, which coincide with historical events. A similar model that also has switching in variances is presented by Engel and Kim (1999). There are 2 processes, 1 permanent and 1 transitory, which are always on and each have 3 possible variance states. In total, there are potentially 6 different variances. The robustness checks for our model show that any additional processes are superfluous and merely complicate the explanation of the model. So, in our model, there will either be a permanent shock *or* a transitory shock but not both in the same period. Though this may seem restrictive, it keeps the model parsimonious and allows for a clean interpretation of the empirical results. Note that it is entirely possible to express the above model in a general form of an ARMA model with state-dependent coefficients.

2B. Estimation Methodology

To estimate the Switching AR(1) and Unit Root Model, we employ the classical estimation technique for regime switching models. Using the algorithms provided in Kim and Nelson (1998), we first put the models in state-space form:

$$y_t = H_{S_t}\beta_t + A_{S_t} z_t + e_t \qquad (2)$$

$$\beta_t = \tilde{\mu}_{S_t} + F_{S_t}\beta_{t-1} + G_{S_t} v_t$$

$$\begin{pmatrix} e_t \\ v_t \end{pmatrix} \sim N\begin{pmatrix} 0, & \begin{matrix} R_{S_t} & 0 \\ 0 & Q^*_{S_t} \end{matrix} \end{pmatrix}$$

Equation (2) presents an N x 1 observed time-series, y_t , as a function of a J x 1 unobserved series, β_t, and a K x 1 series of weakly exogenous or lagged dependent variables, z_t . β_t is a function of the shock, v_t, which is of dimension L x 1. The dimensions for the remaining variables are as follows: H_{S_t} is N x J, A_{S_t} is N x K, F_{S_t} is J x J, and G_{S_t} is J x L. If the variable is governed by an unobserved Markov-switching state variable, it has the subscript S_t .

Then, we estimate the parameters of interest by numerically maximizing the likelihood functions constructed by their algorithms.

For the Switching AR(1) and Unit Root Model, its state-space representation is as follows:

$$y_t = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} x_t \\ z_t \end{pmatrix} \qquad (3)$$

(Measurement Equation of the form $y_t = H\beta_t$)

$$\begin{pmatrix} x_t \\ z_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \phi \end{pmatrix}\begin{pmatrix} x_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} S_t & 0 \\ 0 & (1-S_t) \end{pmatrix}\begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix}$$

(Transition Equation of the form $\beta_t = F\beta_{t-1} + G_{S_t} v_t$)

Note that $R_{S_t} = 0$ and $Q^*_{S_t} = \begin{pmatrix} \sigma^2_{v_t} & 0 \\ 0 & \sigma^2_{\varepsilon_t} \end{pmatrix}$

The RER is constructed as $q_t = s_t + p_t - p^*_t$, where s_t, p_t, and p^*_t are the logarithms of the nominal exchange rate (foreign price of the US Dollar), domestic and foreign price levels, respectively. The data was obtained from Taylor (2002) and updated through 2004 (if available) using the IFS database and include the following countries in our analysis: Australia, Belgium, Canada, Denmark, Germany, Finland, France, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. We use annual data in our main estimations and quarterly data for a robustness check of the US/UK RER.

3. Results and Discussion

In the Switching AR(1) and Unit Root Model, the shocks are either permanent or transitory each period—there cannot be both types. This allows for the interpretation of the existence of real shocks *or* monetary shocks in each period. The model stems from the trend-cycle literature for economic output such as the work of Kuan, Huang, and Tsay (2005) in which they applied a similar characterization for GDP.

Table 1.1 Program simulations: Switching AR(1) and Unit Root Model

	Pr(UR|UR)	***Pr(St|St)***	φ	$\sigma_{UR}2$	$\sigma_{ST}2$
True Value	0.7	0.8	0.55	0.4	0.8
Estimated (N=150)	0.8420 (0.0816)	0.8799 (0.0605)	0.6606 (0.1026)	0.3663 (0.0492)	0.8065 (0.0827)
Estimated (N=3000)	0.6808 (0.0377)	0.7940 (0.0303)	0.5079 (0.0231)	0.3931 (0.0141)	0.8066 (0.0182)

Table 1.1 shows the validation of the Gauss program itself. Note that it depicts single instances of simulated data series in order to show the potential bias in a single, small sample series such as what we have for the actual RER data.

The probability of a Unit Root state this period given that the previous period was a Unit Root state is Pr(UR|UR). Likewise, the probability of a stationary state this period given that the previous period was stationary is Pr(St|St). $\boldsymbol{\sigma_{UR}}^2$ and $\boldsymbol{\sigma_{ST}}^2$ are the variances of the Unit Root process and of the stationary process respectively. The simulation indicates that the program correctly estimates the true parameters even in our small sample (N=150) example. As expected, the estimates become closer to the true parameter values as the sample size increase to 3000.

Table 1.2 shows the above program's estimations for the log RER data series for our 16 countries with the US Dollar as the base currency. Since our data frequency is annual, an estimate of 0.5 to 0.7 for the AR coefficient, $\boldsymbol{\varphi}$,

would fall in the range of a 1-2 year half-life. As the results indicate, the half-lives varying tremendously based on the country pair in question. In particular, Portugal, Finland, and Belgium now have a transitory process with a half-life that falls within the acceptable range dictated by theory. France, with an AR coefficient of 0.7675, has a half-life of only 2.6 years.

Table 1.3 shows the AR coefficients from our model for all possible country pairs.[7] The large probabilities for remaining in each state demonstrate that both processes come into effect during our long horizon data series. Half-life estimates below 3 years are shown in bold. We see that a few of countries—Belgium, Denmark, Portugal, Spain, and Switzerland—frequently have low half-lives after allowing for the occasional permanent process in our regime switching model. This finding leads us to caution both the use of long horizon and panel data. When working with long horizon data on a country pair that may have shifts in its process, the estimated coefficients may not reflect the true underlying process. If the RER follows a model such as ours, then the single process models become inaccurate. In a subsequent section, we present a robustness check showing our model can correctly identify a single process, but our

[7] Note that exchange rates are two-sided, so the results are the same when running US/UK and UK/US, so only one of the possible pairs are recorded in the table.

regime switching model can be mistaken for a Unit Root process when regime shifts are not permitted.

Table 1.2 Switching AR(1) and Unit Root Model. Standard Errors are in parentheses. US Dollar as base currency.

	Pr(UR|UR)	*Pr(St|St)*	φ	σ_{UR}^2	σ_{ST}^2	LLH Value
Australia	0.8881 (0.0562)	0.8375 (0.1028)	0.8440 (0.0681)	0.0358 (0.0079)	0.1288 (0.0209)	164.0230
Belgium	0.9751 (0.0150)	0.6997 (0.1324)	0.5277 (0.1255)	0.0806 (0.0077)	0.5832 (0.1273)	80.4131
Canada	0.9472 (0.0479)	0.9224 (0.0660)	0.8074 (0.1333)	0.0314 (0.0046)	0.0608 (0.0078)	228.9060
Denmark	0.9834 (0.0138)	0.9673 (0.0202)	0.9267 (0.0457)	0.0350 (0.0038)	0.1340 (0.0113)	125.8168
Germany	0.9398 (0.0299)	0.9075 (0.0493)	0.9580 (0.0190)	0.0270 (0.0024)	0.1280 (0.0145)	169.5601
Spain	0.9493 (0.0314)	0.8634 (0.0667)	0.9997 (0.0065)	0.0530 (0.0087)	0.1592 (0.0194)	95.4154
Finland	0.9125 (0.0353)	0.7205 (0.1141)	0.6673 (0.1390)	0.0587 (0.0072)	0.3224 (0.0508)	85.5425
France	0.8251 (0.0948)	0.8292 (0.0787)	0.7675 (0.1903)	0.0434 (0.0092)	0.1084 (0.0119)	125.6131
Italy	0.9846 (0.0122)	0.9099 (0.0470)	0.9985 (0.0010)	0.0265 (0.0025)	0.2211 (0.0223)	109.3920
Japan	0.9876 (0.0101)	0.8856 (0.0601)	0.9919 (0.0031)	0.1130 (0.0088)	0.0297 (0.0047)	104.8256
Netherlands	0.9379 (0.0308)	0.9483 (0.0320)	0.9442 (0.0257)	0.0275 (0.0026)	0.1241 (0.0117)	166.6399
Norway	0.9505 (0.0275)	0.8880 (0.0606)	0.8208 (0.0445)	0.0327 (0.0028)	0.1633 (0.0195)	163.0848
Portugal	0.9722 (0.0170)	0.6299 (0.1511)	0.5519 (0.0655)	0.0753 (0.0061)	0.3473 (0.0781)	77.8230
Sweden	0.9373 (0.0326)	0.8877 (0.0480)	0.8510 (0.0410)	0.0263 (0.0029)	0.1379 (0.0139)	154.2002
Switzerland	0.8495 (0.0597)	0.8663 (0.0622)	0.9610 (0.0142)	0.0227 (0.0031)	0.1477 (0.0150)	127.9609
United Kingdom	0.9175 (0.0417)	0.8900 (0.0655)	0.9406 (0.0351)	0.0247 (0.0026)	0.1227 (0.0135)	194.3264

Table 1.3 AR Coefficients from Switching AR(1) and Unit Root Model for All Country Pairs

	RAUS	RBEL	RCAN	RDEN	RDEU	RESP	RFIN	RFRA	RITA	RJAP	RNET	RNOR	RPRT	RSWE	RSWI	RUK	RUS
RAUS	X																
RBEL	1.00	X															
RCAN	0.81	1.00	X														
RDEN	1.00	1.00	1.00	X													
RDEU	1.00	**0.78**	1.00	0.88	X												
RESP	1.00	**0.44**	1.00	**0.65**	0.95	X											
RFIN	0.91	0.86	0.82	**0.77**	0.91	0.49	X										
RFRA	0.81	**0.53**	**0.79**	0.99	0.99	1.00	0.87	X									
RITA	0.86	**0.80**	0.99	1.00	1.00	1.00	1.00	1.00	X								
RJAP	0.99	**0.79**	0.99	0.99	0.99	0.99	0.99	0.99	0.88	X							
RNET	**0.61**	**0.69**	1.00	**0.71**	0.85	**0.75**	0.99	1.00	1.00	1.00	X						
RNOR	1.00	**0.46**	0.99	0.82	0.91	**0.44**	1.00	0.92	0.83	**0.53**	0.84	X					
RPRT	0.91	**0.32**	**0.49**	**0.64**	**0.45**	**0.23**	**0.38**	**0.50**	**0.62**	**0.61**	**0.41**	**0.43**	X				
RSWE	0.85	**0.39**	0.92	0.96	0.88	**0.60**	0.94	0.97	1.00	1.00	0.97	1.00	**0.46**	X			
RSWI	0.89	1.00	**0.63**	0.82	0.81	**0.79**	**0.77**	**0.66**	0.97	0.99	**0.79**	0.81	**0.46**	0.86	X		
RUK	0.98	1.00	1.00	**0.49**	1.00	1.00	0.85	1.00	0.99	*0.99*	0.91	1.00	0.86	1.00	1.00	X	
RUS	0.84	**0.53**	0.81	0.93	0.96	1.00	**0.67**	**0.77**	1.00	0.99	0.94	0.82	**0.55**	0.85	0.96	0.94	X

When including countries such as Belgium in a panel framework, a researcher is then pooling together a series of RERs with dissimilar characteristics and regime shifts, which could make the estimates biased. The countries we use are in the OECD and often used in pool analysis, so it is necessary to carefully study the inclusion criteria into a panel data set. For example, Alba and Papell (2007), Murray and Papell (2004), Frankel and Rose (1996), and Wu (1996), include Belgium, Portugal, and Spain in their panel analysis. Canzoneri, Cumby, and Diba (1999) use Belgium and Spain. These countries have much quicker mean reversions when allowing for the distinct regime switching processes, so we believe using them in a panel framework would be mixing regimes and give misleading results.

On the other hand, many of the countries have results like the US/UK RER whose AR coefficient is 0.9406, which shows stationary shocks as dissipating very slowly. In fact, the highly persistent stationary process could be “mistaken” for being a Unit Root itself. For these country pairs, the Switching AR(1) and Unit Root Model can be approximated by 2 Unit Roots switching back and forth, which is when the AR coefficient, $\boldsymbol{\varphi}$, is set to be 1 in our original model:

$$y_t = x_t + z_t \qquad (4)$$

$$x_t = x_{t-1} + S_t v_t$$
$$z_t = z_{t-1} + (1 - S_t)\varepsilon_t$$

$$\Rightarrow y_t = x_{t-1} + z_{t-1} + S_t v_t + (1 - S_t)\varepsilon_t$$

$$= y_{t-1} + S_t v_t + (1 - S_t)\varepsilon_t$$

$$v_t \sim N(0, \sigma_v^2) \text{ and } \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

This Switching 2 Unit Root specification holds for the highly persistent country pairs because it is difficult to distinguish between highly persistent transitory shocks and "quiet" permanent shocks in small samples.[8] Given that the results from the Switching AR(1) and Unit Root Model indicate the shocks to the stationary process last a long time, the characterization imposed by the 2 Unit Root specification is a reasonable statistical approximation for those country pairs with highly persistent stationary processes.[9]

Table 1.4 Program simulations—Switching 2 Unit Root Model

	Pr(LO|LO)	*Pr(HI|HI)*	σ_{LO}^2	σ_{HI}^2
True Value	0.7	0.8	0.4	0.8
Estimated (N=150)	0.6366 (0.3352)	0.4309 (0.2802)	0.3165 (0.1300)	0.6860 (0.1016)
Estimated (N=3000)	0.7732 (0.0515)	0.8210 (0.0536)	0.3858 (0.0219)	0.6784 (0.0296)

Simulations show that the program can correctly identify the different variances. Table 1.4 shows the estimated parameters of a generated series for which we defined the parameter values. Pr(LO|LO) (Pr(HI|HI)) is the probability that the current state is quiet given the previous state was also quiet. Likewise, Pr(HI|HI) is the probability that the current state is noisy given the previous state was also noisy. σ_{LO}^2 and σ_{HI}^2 are the variances of the quiet state and noisy states respectively. Again, this is just a single simulation of a small sample size and of a larger sample size. The reason behind this is that we wish to show the potential variation in estimating a single, small-sample series. Monte Carlo simulations show that as the simulations increase, the

[8] See Hamilton (1994), page 444, for a detailed explanation of this equivalence.
[9] The new model can be interpreted as having "quiet" and "noisy" periods.

average parameter estimates become increasing close to the true parameters. Nevertheless, from the above tables, we see that, for the small sample size, the true variance of the quiet process, 0.4000, is estimated to be 0.3165 with a standard error of 0.1300 by the program. As the sample size increases to 3000, the estimated parameter is 0.3858 with a standard error of 0.0219, which is not significantly different at the 95% confidence interval from the true parameter of 0.4.

Next, we check to make sure the program does not falsely break the processes into quiet and noisy states. We generate a single unit root process with normally distributed error terms to see how the program reacts. The results are in Table 1.5. The program is attempting to characterize the single series into 2 series with different variances. Regardless of the sample size, the large standard errors surrounding the probabilities of staying in their respective states indicate it does not know in which state the process resides. The variances are all around 1,

Table 1.5 Program simulations—Single Unit Root Process run on Switching 2 Unit Root Model

	Pr(LO|LO)	***Pr(HI|HI)***	σ_{LO}^2	σ_{HI}^2
Estimated (N=150)	0.6505 (4.2793)	0.6184 (1.8384)	1.1172 (0.1422)	1.1173 (0.1328)
Estimated (N=3000)	0.6514 (0.3781)	0.6171 (0.3792)	0.1011 (0.0545)	0.1011 (0.0182)

which is the true variance. So, given that the 2 identified processes are identical, it makes sense that the program does not know in which state it belongs.

Table 1.6 shows the Switching 2 Unit Root program run for our 16 countries and that this new characterization is indeed a good approximation for many country pairs. We see that the different variances are indeed present. For the US/UK RER, the quiet and noisy periods have

Table 1.6 Switching 2 Unit Roots Model. Notes: Standard Errors are in parentheses. RERs are in terms of USD.

	Pr(LO|LO)	*Pr(HI|HI)*	σ_{LO}^2	σ_{HI}^2	**LLH Value**
Australia	0.8911 (0.0570)	0.8966 (0.0463)	0.0311 (0.0043)	0.1227 (0.0131)	164.0491
Belgium	0.7660 (0.1703)	0.9916 (0.0091)	0.0951 (0.0070)	0.8250 (0.2683)	80.0729
Canada	0.8748 (0.0665)	0.7150 (0.1178)	0.0136 (0.0033)	0.0551 (0.0049)	231.6840
Denmark	0.9811 (0.0154)	0.9401 (0.0339)	0.0360 (0.0039)	0.1360 (0.0114)	121.6769
Germany	0.9102 (0.0465)	0.9406 (0.0309)	0.0271 (0.0028)	0.1257 (0.0142)	166.8682
Spain	0.8565 (0.0681)	0.9650 (0.0231)	0.0540 (0.0073)	0.1604 (0.0790)	87.9914
Finland	0.7297 (0.1071)	0.9005 (0.0451)	0.0561 (0.0113)	0.3302 (0.0565)	81.8098
France	0.8733 (0.0541)	0.7301 (0.1141)	0.0217 (0.0042)	0.1053 (0.0093)	130.1611
Italy	0.6247 (0.2397)	0.9671 (0.0191)	0.0829 (0.0060)	2.4071 (0.6779)	80.7979
Japan	0.1053 (3.9427)	0.9914 (0.0088)	0.1000 (0.0065)	6.2768 (4.4975)	96.0614
Netherlands	0.9519 (0.0292)	0.9416 (0.0289)	0.0291 (0.0027)	0.1264 (0.0120)	164.1804
Norway	0.8946 (0.0531)	0.9550 (0.0270)	0.0349 (0.0033)	0.1564 (0.0166)	154.3472
Portugal	0.7786 (0.1624)	0.9894 (0.0146)	0.0894 (0.0089)	0.3603 (0.1000)	73.4821
Sweden	0.9039 (0.0447)	0.9500 (0.0281)	0.0291 (0.0037)	0.1409 (0.0144)	147.4977
Switzerland	0.9112 (0.0500)	0.8803 (0.0512)	0.0258 (0.0034)	0.1443 (0.0146)	125.2769
United Kingdom	0.8770 (0.0702)	0.9098 (0.0427)	0.0252 (0.0027)	0.1258 (0.0138)	191.4932

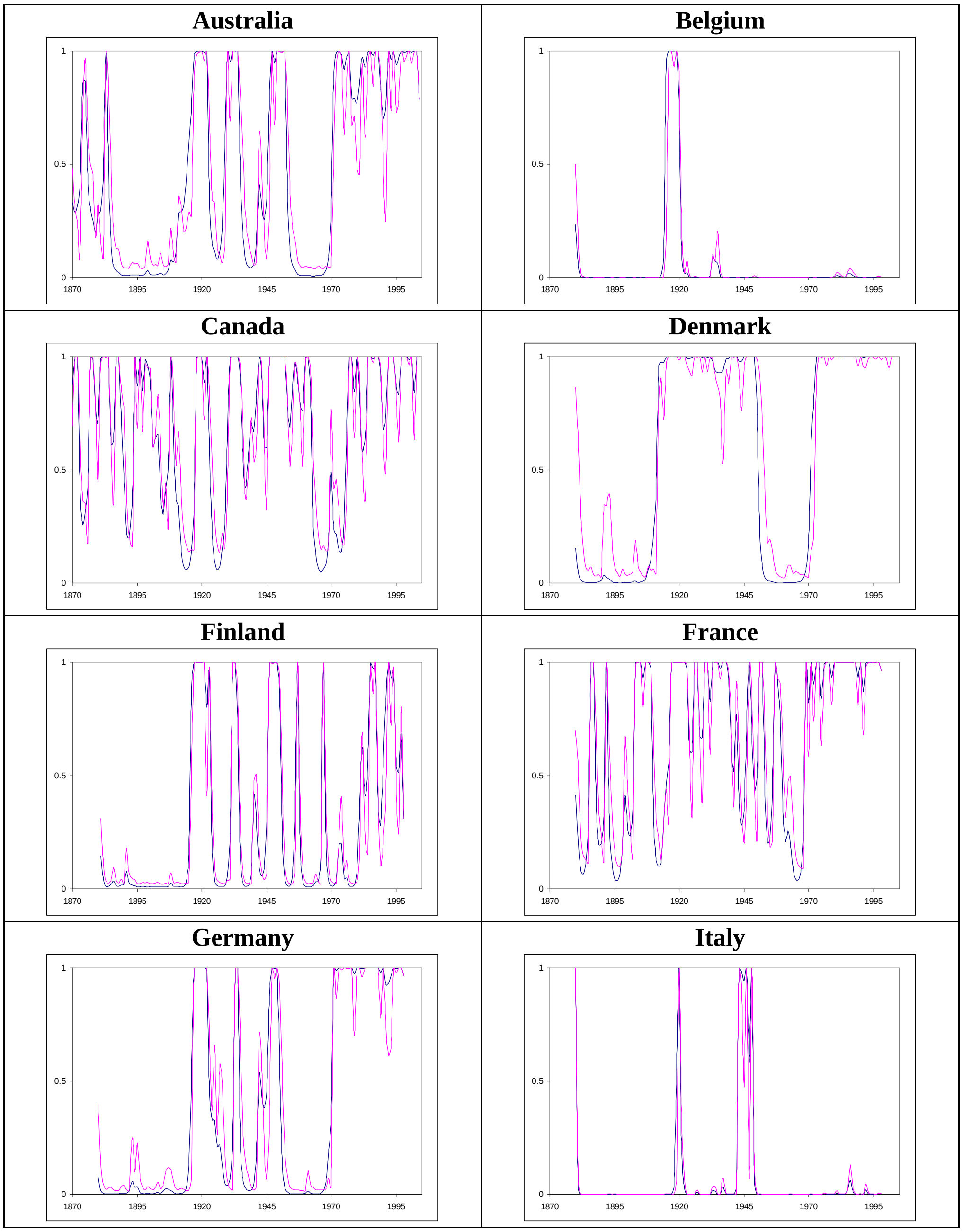

Figure 1.1 Estimated Probabilities of High Variance Regime for Switching AR(1) and Unit Root Model. The solid lines are the smoothed estimates and the dotted lines are the filtered estimates.

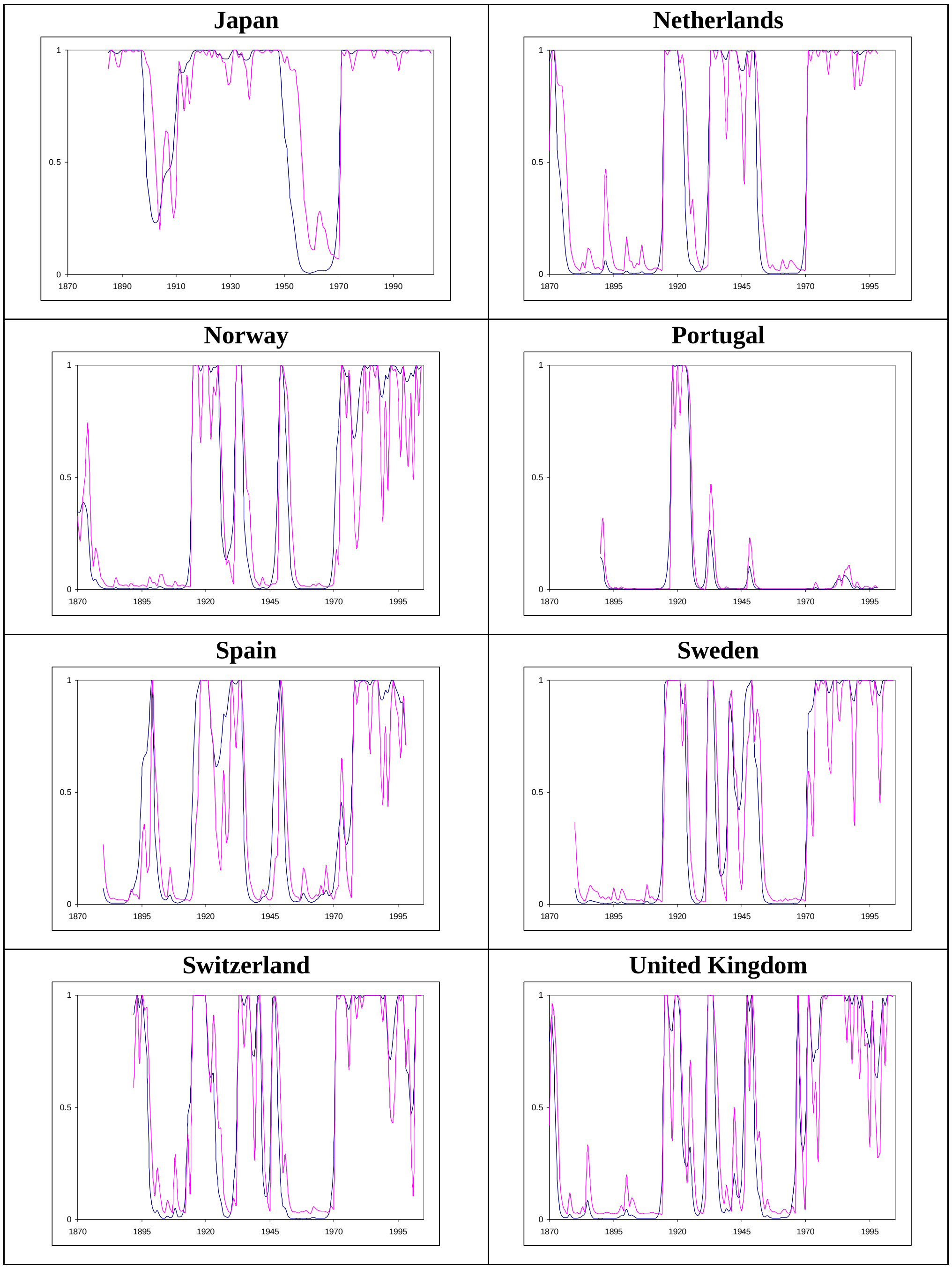

Figure 1.2 Estimated Probabilities of High Variance Regime for Switching AR(1) and Unit Root Model. The solid lines are the smoothed estimates and the dotted lines are the filtered estimates.

variances of 0.0252 and 0.1258 respectively, which is identical to the Unit Root and stationary variances from the Switching AR(1) and Unit Root Model.[10]

Returning to our original Switching AR(1) and Unit Root Model, the plots in Figure 1.1 show the probability of being in a transitory state (vertical axis) during a given year (horizontal axis). For most of the countries, the plots show shocks are transitory most of the time and only becomes permanent on a few occasions. For example, Italy is shown to have transitory shocks with a couple of exceptions such as during the period around 1945 when it was involved in World War II. In the next section, we will show in more detail how the spikes indicating noisy periods coincide directly with historical events.

3. Robustness Checks

We run a series of robustness checks. For instance, we add an I(2) process to simulate a double-drift, but those results do not differ much from our current model. The addition of time trends to these models does not reduce the half-lives, which is consistent with the findings in the current literature. The model is also robust for data of other frequencies too; using Post-Bretton Woods monthly data, we observe the distinct regimes in our RERs. The unifying result in all of these variations implies a model exhibiting a (sometimes highly persistent) transitory process and a Unit Root process: the Switching AR(1) and Unit Root Model. The robustness checks against structural breaks and Unit Root tests are presented below. Then, we show how the regime shifts coincide with actual events by using the US/UK transitions as an example.

[10] For the highly persistent countries, this characterization of the RER as having quiet and noisy permanent shocks is consistent with the results from the current literature showing very persistent shocks to the RER. The shocks during the quiet periods are small, permanent deviations from the mean. This suggests that there is no arbitrage possibility unless the deviations from the mean exceed a certain threshold. Transportation costs, transaction costs, and aggregation bias could explain the lack of reversion within a certain limit.

3A. Robustness Check: Quasi-PPP

In Hedgwood and Papell (1998), the authors find a short half-life for the shocks on the RER and call their result "Quasi-PPP." They use endogenous structural breakpoint tests on series of RERs and then run simple AR(1) regressions on the data while allowing for structural shifts for the dates indicated by the breakpoint tests. Their resulting AR coefficient is low enough to fall into the 1-2 year range for PPP to hold in the short-run. If their result holds true, the persistence exhibited by the Switching AR(1) and Unit Root Model for some country pairs is not consistent with their findings. In this exercise, we attempt to replicate their results given a data generating process from the 2 Unit Root specification.[11] Our method is as follows:

1) Generate the 2 Unit Root Model data with persistent states and different variances
2) Confirm high ϕ in a simple AR1 model:

$$q_t = \phi q_{t-1} + v_t ,\ v_t \sim N(0, \sigma_v^2) \quad (5)$$

3) Run Bai and Perron (1998) programs to find multiple breakpoints
4) Estimate AR(1) model allowing for different levels for each regime: $q_t = \phi q_{t-1} +$

$$D_1 t_1 + \ldots + D_n t_n + v_t ,\ v_t \sim N(0, \sigma_v^2) , \quad (6)$$

where D_i is a dummy variable that can take the value of 0 or 1 and

t_i is the level shift for the period without breaks.

In our generated series of 135 observations, which is the same number we have for US/UK RERs, we find a high coefficient (0.9983) in the simple autoregressive model. Then, the multiple breakpoint test program finds 3 breaks that would coincide with the years 1881, 1901, and

[11] If the transitory process is not persistent, then the switching AR(1) and Unit Root Model and the Quasi-PPP results are identical because the latter will always reduce the half-life down to an "acceptable" level. So, we use the extreme case of highly persistent shocks, which exist quite frequently as indicated in our earlier results (See Tables 1.2 and 1.3).

1970.[12] This is somewhat disturbing because we know our true data generating process is merely 2 Unit Roots switching back and forth and not a process with 3 breaks.[13] Nevertheless, we proceed and run the AR(1) regression allowing for level shifts in our 4 regimes. Using Eviews 5.1, our regression yields a φ of 0.63, which falls into the range of 1-2 years for the half-life on a shock to the system.

We have shown that the 2 Unit Root specification can be mistaken as Quasi-PPP, but can Quasi-PPP be mistaken as the 2 Unit Root specification? If the models are all equivalent in small samples, it would be impossible to identify the "true" model. We generate a series of data following the Quasi-PPP method. We assume 1) 3 breaks, 2) 4 levels that are 0.7, -0.5, 0.6, and -1 in that respective order, 3) AR(1) coefficient is 0.6, and 4) errors are *iid* (0,1). Then, we generate data for sample sizes of 150 and 1000 for both the Switching AR(1) and Unit Root Model and the 2 Unit Root specification. The break dates occur on observations 30, 60, 90 for the small sample and on 100, 500, and 750 for the large sample.

Table 1.7 Program simulations: Quasi-PPP (Switching AR(1) and Unit Root Model)

	Pr(UR|UR)	***Pr(St|St)***	$\boldsymbol{\varphi}$	$\boldsymbol{\sigma_{UR}^2}$	$\boldsymbol{\sigma_{ST}^2}$
N = 150	0.8625	0.9595	0.6053	1.4928	0.8975
	(0.1343)	(0.0279)	(0.1184)	(0.2744)	(0.0859)
N = 1000	0.0001	0.9853	0.5984	1.4062	0.9798
	0.0000	(0.0073)	(0.0299)	(0.3251)	(0.0215)

[12] We use the tests from Bai and Perron (1998). See http://people.bu.edu/perron/code.html for code.
[13] Nunes, Kuan, and Newbold (1995) use simulations to show that unit root processes can generate "spurious breaks." Bai (1998) provides a note in which he presents a mathematical proof for this. Another issue for testing structural breaks in RERs is the low power in the Bai and Perron (1998) tests; this issue is addressed in Prodan (2005).

As Table 1.7 indicates, the autoregressive coefficients are estimated to be close to 0.6, which is the true parameter from the data generating process. As the sample size, N, increases, the unit root process fades into the background, which is shown by the estimate of Pr(UR|UR) as 0.0001.

In Table 1.8, the program for the 2 Unit Root specification only identifies a single Unit Root process for both the small sample and the large sample. Furthermore, the variances are not very different. The results we get from using RER data for these models are different from what we just simulated above. Our RER data yield both low and high AR coefficients, 2 processes, and distinct variances. It does not appear that a Quasi-PPP data generating process can correctly replicate the empirical results brought forth by our models.[14]

Table 1.8 Program simulations: Quasi-PPP (Switching 2 Unit Root Model)

	P(UR1|UR1)	***P(UR2|UR2)***	σ_{UR1}^2	σ_{UR2}^2
N = 150	1.0000	0.0208	1.1955	1.2239
	(0.0043)	0.0000	(0.0700)	(10.1152)
N = 1000	0.5671	0.0001	1.0126	1.3354
	(0.1441)	0.0000	0.0000	(0.0805)

3B. Robustness Check: Unit Root Tests

Next, we test the robustness of the Switching AR(1) and Unit Root Model against standard Unit Root tests. Many studies using Long Horizon data, such as Taylor (2002), run series of Unit Root tests to determine the stationarity of RERs. If a Unit Root is present, then the shocks are permanent and PPP does not hold. We first show that 2 commonly used Unit Root Tests (Augmented Dickey-Fuller and Dickey-Fuller GLS) cannot account for the switching

[14] This phenomenon can also be interpreted as spurious structural breaks due to heteroskedasticity. On the flipside, structural breaks are not mistakenly attributed to be heteroskedastic processes.

processes in our model and, hence, incorrectly conclude that the process is actually a Unit Root series.[15] Then, we demonstrate the robustness of our model in that it can correctly characterize single process series.

The parameter assumptions used to generate the simulated data follow the estimates from the Belgium/US RER, which exhibits quick reversion in the transitory process. See Table 1.9 for the parameter assumptions.[16]

Table 1.9 Parameters used to generate simulated data

	Pr(UR|UR)	***Pr(St|St)***	φ	σ_{UR}^2	σ_{ST}^2
True Parameter	0.95	0.7	0.5	0.08	0.55

We double-check that a single process AR(1) with constant regression would yield persistent shocks under this data generating process. The output from a least squares regression done in Eviews 5.1 is in Table 1.10.

Table 1.10 Output from AR(1) with constant regression for generated data

Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
Constant	-0.049155	0.029627	-1.659108	0.0995
Lagged Value	0.965463	0.020188	47.82455	0.0000

The large coefficient of the lagged variable, the AR(1) parameter, is indicative of persistence and is consistent with the findings in the current literature using long horizon data. Next, we run the

[15] Taylor (2002) shows the evidence for stationarity to be inconclusive in univariate settings.
[16] Sample size is 135.

same series of data through 2 separate Unit Root tests: Augmented Dickey-Fuller and Dickey-Fuller GLS. The null hypothesis of the tests is that the series has a Unit Root.

Table 1.11 Unit Root test results on generated data

		ADF	DF-GLS
Test statistic		-1.7108	-0.8986
Test critical values:	1% level	-3.4797	-2.5823
	5% level	-2.8831	-1.9432
	10% level	-2.5783	-1.6151

In Table 1.11, we see that neither Unit Root test can reject the null hypothesis. So, one may incorrectly conclude that our generated series is a Unit Root process.

Next, we show that a single process series is correctly picked up by our Switching 2 Unit Root Model. We generate data from the following series:

$y_t = \varphi\, y_{t-1} + e_t$, where e_t is *iid* (0,1) shock.[17]

Table 1.12 Single processes run on Switching AR(1) and UR Model

	Pr(UR|UR)	*Pr(St|St)*	φ	σ_{UR}^2	σ_{ST}^2
True φ = 0.5	0.0013	1.0000	0.5631	0.0003	0.0720
	(0.1615)	(0.0001)	(0.0717)	(0.0570)	(0.0044)
True φ = 1	0.9681	0.1044	0.5715	0.0827	0.0094
	(0.0145)	(0.0040)	(0.0026)	(0.0057)	(0.0302)

In the first trial, the AR coefficient is set to equal 0.5 in order to produce a half-life of 1 year and in the second trial, it is merely a Unit Root process. When φ = 0.5, only the stationary process exists because the probability of entering a Unit Root state is effectively zero. Then when the series is truly a Unit Root, φ = 1, the Switching AR(1) and Unit Root Model is again correct in showing the Unit Root process to be dominant. So, if the process was indeed simply a single

[17] Sample size is 135.

series without any regime shifts, then our model would characterize it as so. However, the results run on real data (see Table 1.2) clearly indicate that this is not the case and there are 2 distinct processes that govern the RER. This robustness check is evidence that the use of long horizon data without accounting for regime shifts may produce spurious results.

3C. Historical Implications

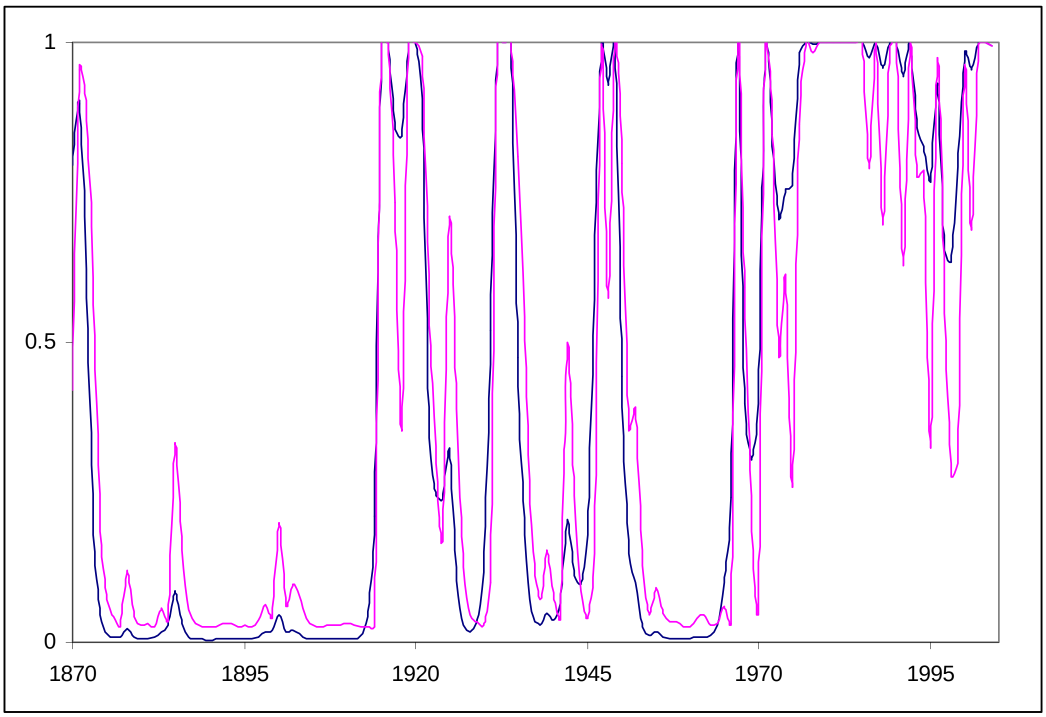

Figure 1.3 Estimated Probabilities of High Variance Regime for Switching AR(1) and Unit Root Model (US/UK). The blue/solid lines are the smoothed estimates and the pink/dotted lines are the filtered estimates.

Figure 1.2 is of the US/UK in which the vertical axis represents the probability of being in a permanent, Unit Root state at the date indicated on the horizontal axis.[18] Our goal is to explain the permanent states indicated by the model. Using the smoothed estimates, we define a period to be in the permanent state should its probability exceed 0.5. Otherwise, it is in a transitory state. The following are the resulting permanent states with potential explanations.

1870-3: The United States government passes the "Fourth Coinage Act" in 1973 as a response to newly discovered Silver in the American West. The US leaves the bimetallism currency system where the dollar could be expressed in both Silver and Gold out of fear that the increased Silver supply would cause inflation.

1915-1921: This is a period of great instability for both the United States and the United Kingdom because of World War I. Furthermore, the world shifts towards an Anchored Dollar Standard in which the other currencies base themselves on the American Dollar.

1931-4: During the early 1930s, many governments change their currency away from being based on Gold. In 1931, the United Kingdom leaves the Gold Standard and the United States follows suit in 1933. The following year, the United States raises the price of gold from $20/oz to $35/oz. Another reason for the noise during this period is the Great Depression, where much of the stability is lost in the financial markets and the economy as a whole.

1946-9: After World War II, the United Nations held a conference and established the Bretton Woods institutions. The participants agreed to use Gold as the common currency standard.

[18] Figure 1.1 shows plots for the other countries; again, the variations in the figures are another indication that panel analysis may be too restrictive.

1967: This spike does not have as clear of an explanation as the others. It could be a result from the American War in Vietnam or the formation of OPEC a few years earlier.

1971-present: The Bretton Woods agreement collapses after the US abandons it. Ever since, both the United States and the United Kingdom have been in a world of flexible exchange rates.

The continued Unit Root state since the end of Bretton Woods is consistent with current findings dubbed as the Exchange Rate Disconnect Puzzle. Economic fundamentals should, in theory, be closely related to exchange rates. As the previous figure indicates, the RER has experienced large, highly volatile shocks since 1971, but this volatility is not reflected in the fundamentals. There is seemingly a "disconnect" between the exchange rate and the underlying, economic variables.[19]

4. Modeling Fundamentals

In the previous section, we established the robustness of the models and showed how the different states coincide with historical events. Now, we will apply rigorous, econometric techniques to explain the behavior of the switching. First, we apply OLS and regress the filtered probabilities on fundamentals. Then, we include the fundamentals directly within the measurement equation to see if it results in reduced residual volatility. Lastly, the model is allowed to have Time-Varying Transition Probabilities dependent on fundamentals. These exercises show that while fundamentals do influence the switching behavior of our model, the effect varies depending on the choice of bilateral exchange rates. This is more evidence that pooling the data may lead to spurious results.

[19] Flood and Rose (1995) have a thorough discussion of this disconnect, but recent work –such as Cheung and Lai (1997), which uses two efficient unit root tests to find PPP holding—challenge this finding.

4A. OLS Regressions

The filtered probability of being in a transitory state or a permanent state is the dependent variable. Even though there is little difference in the 2 series for our models, we use the filtered as opposed to the smoothed probabilities because the latter depicts an overall "trend" by utilizing the entire data sample whereas the former only uses the data up to the point in question to derive its estimate. We include both prices and volatilities of Gold, Silver, Oil, and The Economist Commodity Index. Then, we construct 3 series of dummy variables:

1) When the US is on the Gold Standard, DUM_GOLDUS takes a value of 1
2) When the UK is on the Gold Standard, DUM_GOLDUK takes a value of 1
3) If the US is involved in a war, the WAR dummy takes a value of 1.[20]

Other variables in the analysis include Inflation Volatility Differences, GDP/Capita Differences, and Average Openness, which is the average Total Trade / GDP between the 2 countries.[21] The data is obtained from Global Financial Services. We log the prices and construct volatilities by taking the standard deviation of monthly data over the year in question.

Table 1.13 shows the OLS results for selected countries.[22] The variable inclusion criterion is based on Bayesian Model Averaging.[23] For the US/UK, Openness, US on the Gold Standard, Silver Volatility, and Commodity Index Volatility have significant effects on the probability of being in a low or high variance state. We note that each variable (except for Inflation Volatility Differences, which is not listed in the table) is significant under some country

[20] We include DUM_GOLDUS (DUM_GOLDUK) when the US (UK) is one of the countries in the bilateral RER. For US/UK, we use DUM_GOLDUS but both dummies yield similar results because both countries were on the Gold Standard during nearly identical periods.
[21] For our selected countries, Inflation Volatility Differences are available for only US/UK, and Openness is constructed for only US/UK and Switzerland-US.
[22] Results for other country pairs are available upon request.
[23] See Raftery (1995).

pair.[24] Perhaps different countries have different dependencies on fundamentals due to different industry structures, commodity endowments, monetary policies, and inflation expectations. The results point out that fundamentals do have explanatory power on the behavior of shocks in our model, but which fundamentals depends on the bilateral RER being analyzed. This is another outcome that suggests pooling countries for analysis is not suitable. We do not find any systematic differences between the country pairs exhibiting quick reversion and those that do not.

[24] Inflation Volatility Differences is only available for US/UK from 1914 onwards.

Table 1.13 OLS Regressions on Fundamentals selected by BMA grouped by quick reverting (top) and persistent (bottom) country pairs.

	Gold	Gold Volatility	Silver	Silver Volatility	Oil	Oil Volatility	GDP/Capita Difference
Belgium-US	--	--	0.3035 (0.0401)	-1.3353 (0.7009)	0.2850 (0.0873)	--	--
Finland-US	--	-1.9730 (0.1337)	--	--	--	1.0260 (0.3711)	0.5469 (0.1768)
France-US	--	--	--	1.0538 (0.5011)	--	--	--
Portugal-US	--	--	0.2836 (0.0634)	--	--	--	--
Denmark-UK	0.0010 (0.0005)	--	-0.2170 (0.1059)	--	--	1.5970 (0.3504)	--
Canada- Swi.	--	--	0.2000 (0.0453)	--	0.1727 (0.0844)	--	--
Canada – Ger.	--	--	0.1241 (0.0405)	--	0.2360 (0.1335)	--	--
Canada - US	0.0009 (0.0002)	--	--	(1.2943) (0.3668)	--	--	1.4498 (0.2414)
France - UK	--	--	--	--	--	--	0.2959 (0.1119)
Ger. - UK	--	--	0.1244 (0.0552)	--	0.3575 (0.0624)	1.1101 (0.2967)	--
Swi. - US	--	2.6854 (0.9287)	--	0.3085 (0.0743)	0.3251 (0.1144)	--	--
US - UK	--	--	--	1.4389 (0.3782)	--	--	-0.3197 (0.1383)

Table 1.13 (continued) OLS Regressions on Fundamentals selected by BMA grouped by quick reverting (top) and persistent (bottom) country pairs.

	Commodity Index	Commodity Volatility	Openness	War	Gold Standard	Intercept	R^2
Belgium-US	--	--	--	--	--	-0.8039 (0.2063)	0.4190
Finland-US	0.1716 (0.0705)	--	--	--	-0.2113 (0.1043)	-0.6127 (0.2892)	0.3220
France-US	--	--	--	-0.3325 (0.0725)	-0.2068 (0.1065)	0.6346 (0.2968)	0.2990
Portugal-US	-0.1116 (0.0759)	--	--	--	--	0.1961 (0.2659)	0.4060
Denmark-UK	--	--	--	--	--	0.6333 (0.2076)	0.3910
Canada- Swi.	--	--	--	-0.0937 (0.0556)	-0.2943 (0.0603)	-0.3651 (0.2207)	0.4590
Canada – Ger.	0.1335 (0.0407)	3.7104 (0.9894)	--	--	--	(0.7850) (0.1747)	0.5158
Canada - US	--	2.2652 (0.7954)	--	(0.1495) (0.0546)	(0.1927) (0.0530)	(0.0479) (0.0985)	0.4193
France - UK	--	3.9612 (0.2959)	--	(0.2091) (0.0667)	--	0.6772 (0.0625)	0.2730
Ger. - UK	--	4.2346 (0.9753)	--	--	(0.3464) (0.0861)	(0.6456) (0.1703)	0.6151
Swi. - US	(0.1173) (0.0574)	--	--	--	(0.4912) (0.1226)	(0.2448) (0.2307)	0.3100
US - UK	-- (0.8532)	3.5897 (0.3772)	1.9174	-- (0.0590)	(0.3231) (0.0543)	0.2558	0.6439

4B. Parameters in the Measurement Equation

In this exercise, we incorporate the fundamentals directly in the measurement equation. We use the Switching 2 Unit Root specification because the country pair analyzed is US/UK, which has a highly persistent transitory process in the original Switching AR(1) and Unit Root model.[25] If the fundamentals affect our RER series, the model should pick up a significant coefficient in β and reduce the weight placed on the original switching UR processes.

$$y_t = x_t + z_t + \beta * Fundamental_t \qquad (7)$$
$$x_t = x_{t-1} + S_t v_t$$
$$z_t = z_{t-1} + (1-S_t)\, e_t$$
$$v_t \sim N(0, \sigma_v^2),\ e_t \sim N(0, \sigma_e^2)$$

The results for the US/UK RER series are in Table 1.14.

[25] Using the Switching AR(1) and Unit Root Model yields the same results, which is further evidence that this alternative specification holds for the country pairs with slow reversion.

Table 1.14 Fundamentals in Measurement Equation (US/UK)

	Pr(LO|LO)	*Pr(HI|HI)*	σ_{LO}^2	σ_{HI}^2	β	**LLH Value**
Gold	0.9035	0.8752	0.0252	0.1146	0.1586	194.5409
	(0.0493)	(0.0801)	(0.0028)	(0.0144)	(0.1044)	
Gold Volatility	0.8890	0.8055	0.0253	0.1368	(0.4467)	193.1937
	(0.0468)	(0.0834)	(0.0033)	(0.0182)	(0.3426)	
Silver	0.9128	0.8858	0.0243	0.1241	0.0364	192.6870
	(0.0419)	(0.0632)	(0.0027)	(0.0132)	(0.0202)	
Silver Volatility	0.9106	0.8774	0.0252	0.1259	(0.0099)	191.2102
	(0.0425)	(0.0699)	(0.0027)	(0.0139)	(0.0776)	
Oil	0.9081	0.8719	0.0252	0.1263	0.0058	191.2342
	(0.0439)	(0.0750)	(0.0026)	(0.0141)	(0.0221)	
Oil Volatility	0.9098	0.8762	0.0252	0.1259	(0.0017)	191.2028
	(0.0429)	(0.0715)	(0.0027)	(0.0141)	(0.0498)	
Commodity Index	0.8994	0.8579	0.0244	0.1256	0.0669	192.6646
	(0.0494)	(0.0842)	(0.0026)	(0.0140)	(0.0383)	
Commodity Volatility	0.9097	0.8763	0.0252	0.1257	(0.0113)	191.2050
	(0.0429)	(0.0706)	(0.0027)	(0.0139)	(0.1184)	
Openness	0.8803	0.7568	0.0241	0.1379	2.6612	184.0305
	(0.0501)	(0.1088)	(0.0025)	(0.0176)	(0.4497)	
War	0.9113	0.8855	0.0231	0.1226	0.0282	195.6347
	(0.0429)	(0.0615)	(0.0025)	(0.0127)	(0.0087)	
Gold Standard	0.9122	0.8778	0.0241	0.1290	0.0432	193.7624
	(0.0406)	(0.0670)	(0.0024)	(0.0139)	(0.0177)	
GDP/Capita Difference	0.9008	0.8329	0.0257	0.1312	0.0166	189.5344
	(0.0474)	(0.0950)	(0.0033)	(0.0164)	(0.0627)	

The coefficient on the fundamental, β, is not significant for any of the series, and the other parameter estimates are very close to the original estimates.[26] The sample size is too small for the confidence intervals to be tight enough to yield significant regressors.

4C. Time Varying Transition Probabilities

In our current specification, the transition probabilities are constant for the duration of our data sample. It is quite conceivable that the behavior of the RER is linked directly to the performance of underlying, economic fundamentals. Here, we attempt to explain the switching states in terms of these fundamentals. Again, we choose to analyze the US/UK series, so we run this program on the Switching 2 Unit Root specification. If these extra parameters have a significant impact on our model, it hints that there are other processes that must be accounted for in addition to the two permanent processes. The basic methodology is to enhance the original program to allow the transition probabilities to vary based on other variables. The basic probit probabilities are:

$$\text{PPr} = \Pr\,(\text{St=Hi} \mid \text{St=Hi}) = 1 - \frac{1}{1+\exp(p_0 + p_1 Z_t)} \qquad (8)$$

$$\text{QPr} = \Pr\,(\text{St=Lo} \mid \text{St=Lo}) = 1 - \frac{1}{1+\exp(q_0 + q_1 Z_t)}$$

p_0, p_1, q_0, and q_1 are unconstrained parameters in the optimization and Z_t is the value of the fundamental at time *t*. Figures 1.3 and 1.4 show the plots PPr and QPr with their 95%

[26] We ran the Commodity Index as a fundamental for the US-Australia RER because Australia is a commodity economy. Again, the coefficient for the fundamental is not significant. Then, we ran this exercise for the AR(1) with Unit Root Model and the other models with AR(1) processes.

confidence bands for Oil.[27] Oil Prices show significant movement through time, the horizontal-axis.

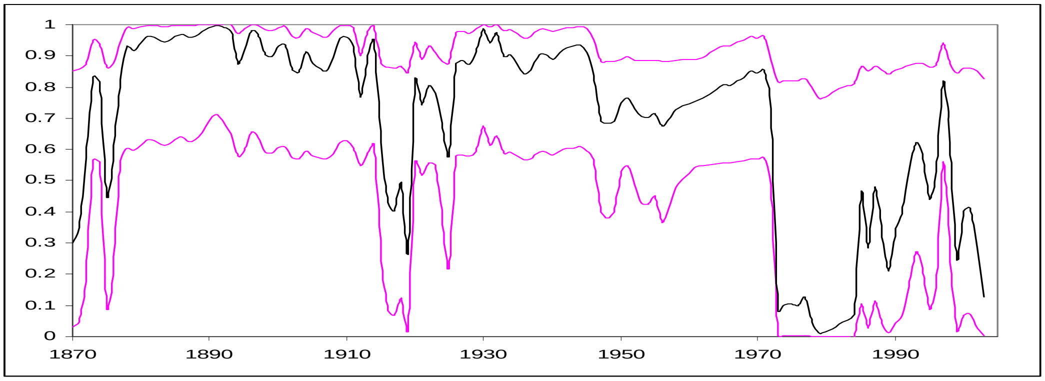

Figure 1.4 Time-Varying Transition Probabilities (PPr)

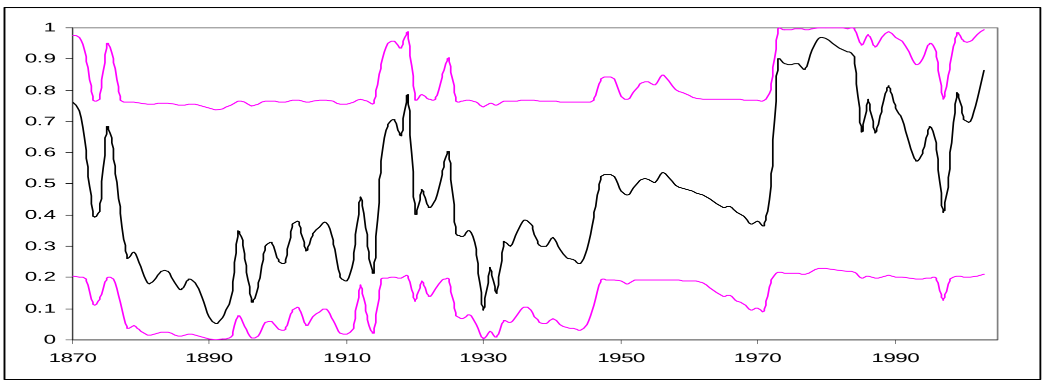

Figure 1.5 Time-Varying Transition Probabilities (QPr)

Even so, the confidence intervals are wide enough that a straight line can be drawn through the entire timeframe, so a constant probability could fit the 2 Unit Root Model. This is even more apparent for the other commodities, so the

[27] Figures for other fundamentals are available upon request.

Table 1.15 Time-varying Transition Probabilities (US/UK)

	p_0	q_0	σ_{v2}	σ_{e2}	p_1	q_1	LLH Value
Gold	2.4745 (0.6136)	1.7331 (0.7046)	0.0252 (0.0028)	0.1293 (0.0147)	2.4317 (1.1866)	2.9870 (1.3996)	195.321
Gold Vol.	0.4406 (1.0719)	0.7762 (0.7685)	0.0245 (0.0027)	0.1297 (0.0144)	(91.2717) (43.2762)	21.6333 (24.4934)	196.581
Silver	2.0829 (0.5613)	1.5488 (0.6453)	0.0246 (0.0031)	0.1284 (0.0145)	1.4715 (0.7861)	0.3384 (0.8208)	193.722
Silver Vol.	2.5899 (0.8714)	14.6758 (7.1335)	0.0224 (0.0022)	0.1139 (0.0100)	(72.7028) (36.1170)	355.5128 (197.909)	201.585
Oil	1.1290 (0.4707)	(0.1143) (0.6389)	0.0245 (0.0024)	0.1358 (0.0158)	(3.7771) (0.5661)	2.4490 (1.1434)	198.008
Oil Vol.	2.7608 (0.7493)	2.2264 (0.6426)	0.0249 (0.0027)	0.1236 (0.0128)	(9.8680) (5.1328)	3.0652 (0.8255)	193.320
Comm. Ind.	2.6422 (0.6693)	1.9619 (0.6190)	0.0250 (0.0029)	0.1248 (0.0133)	3.3886 (3.1249)	2.0477 (1.8179)	192.188
Comm. Vol.	1.0920 (0.5184)	1.1860 (0.8175)	0.0215 (0.0031)	0.1216 (0.0125)	(58.9334) (31.7486)	89.7928 (51.6711)	197.482
Openness	0.5839 (0.9521)	0.7913 (0.6960)	0.0248 (0.0026)	0.1435 (0.0176)	(58.1809) (35.8827)	19.5544 (14.695)	186.0224
War	2.4331 (0.6458)	2.0640 (0.6344)	0.0243 (0.0027)	0.1240 (0.0136)	(1.8569) (1.2003)	(1.9195) (1.1515)	193.533
Gold Std.	1.8747 (0.5694)	0.7391 (0.9110)	0.0240 (0.0027)	0.1266 (0.0145)	2.2930 (1.2689)	(2.4198) (2.1376)	195.831
GDP Diff	1.6572 (0.6876)	2.1023 (0.5312)	0.1273 (0.0145)	0.0254 (0.0029)	(1.3025) (3.0261)	2.9605 (2.5029)	192.590

probabilities do not necessarily vary through time in our finite sample. Table 1.15 shows the numerical estimation produced by the program. We see that Oil yields significant coefficients for both probabilities. The coefficients and the probabilities move in opposite directions, so as Oil increases, PPr (probability of remaining in a Unit Root state) decreases and QPr (the probability of remaining in a stationary state) increases. In other words, as the log of Real Oil Prices increases, the probability of staying in (or moving to) a more volatile RER regime increases. Nonetheless, the smoothed and filtered plots of the states barely change after allowing for the additional parameters. In general, the confidence intervals for the probabilities are very wide, so there is not enough power in our sample size to determine whether or not the transition probabilities are indeed time-varying.

From our 3 ways of incorporating fundamentals into the regime switching model, we find it difficult to pinpoint the exact fundamentals that determine the RER process. The OLS approach shows the fundamentals to play a role in determining the state of the RER (transitory of permanent), but there is much collinearity among the regressors, and fundamentals are significant for different country pairs; perhaps, due to different industry structures, commodity endowments, monetary policies, and inflation expectations, different countries have different dependencies on fundamentals. Nevertheless, certain series—such as GDP/Capita Differences—show up frequently. This suggests the Balassa-Samuelson effect may explain some of the variation in the RER. In general, the estimations reveal parameters to be of different values and significance for different country pairs, so the data should not be pooled.

5. Conclusion

Using over 100 years of data from 16 OECD countries, we find that the RER is best described as having both transitory and permanent shocks, a framework that overcomes many issues arising from typical long horizon, structural break, and pooled data analyses. The

Switching AR(1) and Unit Root Model allows the RER to possess either stationary or Unit Root nonstationary shocks in any given period. The majority of the shocks to the RER are transitory ones though the degree of persistence varies depending on the country pair in question. Certain countries, such as Belgium, Portugal, and Spain, consistently have half-lives below 2 years during the transitory states.

This leads us to caution the common practices of both using long horizon data analysis and panel analysis as means of circumventing the testing power issues associated with the search for the existence of PPP. Our model clearly shows distinct regimes that would become mixed in simple long horizon and panel frameworks, which in turn could lead to biased estimations. One of the robustness checks for the model shows that using structural breaks to account for regime shifts is also misleading. If the stationary process in our model is highly persistent, then the endogenous structural breakpoint tests show spurious breaks.

Our specification presents regimes that are consistent with historical events. The model implies that shocks due to wars and currency standard shifts are permanent, whereas the other shocks are transitory and mean reverting an a stationary regime. After the collapse of Bretton Woods, our model consistently shows noisy periods, which supports the current literature exploring the Exchange Rate Disconnect Puzzle, where the large volatility observed in exchange rates is not reflected in economic fundamentals.

In a more rigorous analysis of the regime shifts, we find that the inclusion of fundamentals diminishes the puzzle by providing explanatory power for the changes of regimes. The fundamentals we use are commodity prices and volatilities, GDP/Capita differences, inflation differentials, war periods, openness, and gold standard periods. In addition to OLS regressions selected by BMA, we model the fundamentals as time-varying transition probabilities and also directly in the measurement equation; the significance of the variable depends on the bilateral RER used in the analysis, which could be because different countries have different policies and dependencies on fundamentals. The variation in the role of each fundamental further supports the potential problems in pooling data to increase sample size.

Though a handful of countries have reduced half-lives, the PPP Puzzle still remains for many others. Our model is in agreement with that literature that finds a long half-life for the shocks to RERs. One possible extension would be to pool highly persistent country pairs could and use the Switching 2 Unit Root framework to increase the power of the tests. Assuming proper correction for heteroskedasticity, this would allow the researcher to exploit the increased sample size while maintaining a homogenous sample. However, countries, such as Belgium, that clearly possess a transitory process must be omitted from such panels. Lastly, we could modify the model to include more states, but that would reduce the parsimony of our current framework.

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