A Macro-Finance Approach to Exchange Rate Determination*

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Abstract. The nominal exchange rate is both a macroeconomic variable equilibrating international markets and a financial asset that embodies expectations and prices risks associated with cross border currency holdings. Recognizing this, we adopt a joint macro-finance strategy to model the exchange rate. We incorporate into a monetary exchange rate model macroeconomic stabilization through Taylor-rule monetary policy on one hand, and on the other, market expectations and perceived risks embodied in the cross-country yield curves. Using monthly data between 1985 and 2005 for Canada, Japan, the UK and the US, we summarize information in the relative yield curves between country-pairs using the Nelson and Siegel (1987) latent factors, and combine them with monetary policy targets (output gap and inflation) into a vector autoregression (VAR) for bilateral exchange rate changes. We find strong evidence that both the financial and macro variables are important for explaining exchange rate dynamics and excess currency returns, especially for the yen and the pound rates relative to the dollar. Moreover, by decomposing the yield curves into expected future yields and bond market term premiums, we show that both expectations and perceived risks are priced into the currencies. These findings provide support for the view that the nominal exchange rate is determined by both macroeconomic as well as financial forces.

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1 Introduction

This paper proposes to model nominal exchange rates by incorporating both macroeconomic determinants and latent financial factors, bridging the gap between two important strands of recent research. First, against decades of negative findings in testing exchange rate models, recent work by Engel, Mark, and West (2007), Molodstova and Papell (2009) among others, show that exchange rate models in which monetary policy follows an explicit Taylor (1983) interest rate rule deliver improved empirical performance, both in in-sample estimations and in out-of-sample forecasts. These papers emphasize the importance of expectations, and argue that the nominal exchange rate should be viewed as an asset price embodying the net present value of its expected future fundamentals. Since the Taylor-rule fundamentals – measures of inflation and output gap – affect expectations about future monetary policy actions, changes in these variables induce nominal exchange rate responses. This literature further recognizes the presence of risk, but in its empirical testing, risk is largely ignored and rendered an "unobservable". 1 This is an obvious shortcoming in modeling short-run exchange rate dynamics. 2 On the finance side, recent research efforts show that systematic sources of financial risk, as captured by latent factors, drive excess currency returns both across currency portfolios and over time (see Inci and Lu 2004, Lustig et al 2009, and Farhi et al 2009, among others). 3 Some, such as Bekaert et al (2007), further advocate that risk factors driving the premiums in the term structure of interest rates may also drive the risk premium in currency returns. 4 These papers, however, are silent on the role of macroeconomic conditions, including monetary policy actions, in determining risk. They thus fall short on capturing the potential feedback between macroeconomic forces, expectation formation, and perceived risk in determining exchange rate dynamics. Our paper shows that the macro and finance approaches should be combined. We propose a framework that captures intuition from both bodies of literature, by incorporating into a monetary exchange rate model explicit interest rate feedback rules, and by using latent

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1 Engel, Mark, and West (2007), for example, establish a link between exchange rates and fundamentals in a present value framework. After explicitly recognizing the possibility that risk premia may be important in explaining exchange rates, they "do not explore that avenue in this paper, but treat it as an 'unobserved fundamental." Molodstova and Papell (2009), show that Taylor rule fundamentals (interest rates, inflation rates, output gaps and the real exchange rate) forecasts better than the commonly used interest rate fundamentals, monetary fundamentals and PPP fundamentals. Again, they explain exchange rate using only observed fundamentals and do not account for risk premium.

2 Faust and Rogers (2003) for instance argue that monetary policy accounts for very little of the exchange rate volatility.


4 Clarida and Taylor (1997) uses the term structure of forward exchange premiums to forecast spot rates. de los Rios (2009) and Krippner (2006) connect the interest rate term structure factors and exchange rate behavior. Again, these papers do not examine the role of macroeconomic fundamentals or monetary policy.
factors extracted from the cross-country yield curves to capture market expectations and perceived risks in the currency markets. Specifically, using monthly data between 1985 and 2005 for Canada, Japan, the UK and the US, we summarize information in the relative yield curves between country-pairs using the Nelson and Siegel (1987) latent factors, and combine them with monetary policy targets (output gap and inflation) into a vector autoregression (VAR) for bilateral exchange rate changes.

The joint macro-finance strategy has proven fruitful in modeling other financial assets. For example, Ang and Piazzesi (2003) and Diebold, Rudebusch and Aruoba (2006) among others, illustrate that a joint macro-finance modeling strategy provides the most comprehensive description of the term structure of interest rates. As stated in Diebold, Piazzesi, and Rudebusch (2005), the joint approach captures both the macroeconomic perspective that the short rate is a monetary policy instrument used to stabilize the economy, as well as the financial perspective that yields of all maturities are risk-adjusted averages of expected future short rates. Our exchange rate model is a natural extension of this idea into the international context. The nominal exchange rate is a macroeconomic variable that links prices across borders and equilibrates purchasing parity; it is also an asset that prices expectations and risk in the currency markets. The no-arbitrage condition in the international asset markets further connects exchange rate dynamics to the cross-country yield differences at the corresponding maturities. These characteristics suggest that exchange rates, just like yield curves, should incorporate both macroeconomic and financial elements.

We present an open economy model where central banks follow a Taylor-type interest rate rule that stabilizes the expected inflation, the output gap, and the deviation of the real exchange rate from its steady state value. Evoking the international asset market efficiency condition -the risk-adjusted uncovered interest parity (UIP) -we can then express the nominal exchange rate as the net present value of the expected future paths of interest rate differentials and risk premiums between the country pair. This framework establishes a direct link between the exchange rate with its current and expected future macroeconomic fundamentals, and allows country-specific risk premiums to play an explicit role as well. As emphasized in the earlier literature, this exchange rate formulation puts a much higher weight on the expectations of future fundamentals than on the current ones, so properly measuring these expectations, as well as risk, is important for empirical testing. Previous papers largely fail to address this appropriately, however, as they often ignore risk or make

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5 This is the risk-adjusted uncovered interest parity (UIP) condition.

6 Note that following Clarida, Gali, and Gertler (1998), the incorporation of the exchange rate term to an otherwise standard Taylor rule has become commonplace in recent literature, especially for modeling monetary policy in non-US countries. See, for example, Engel and West (2006), Benigno (1999), and Molodtsova and Papell (2009).

7 See Engel and West (2005), Engel et al (2007) for a more detailed presentation and discussion.
overly simplistic assumptions about these expectations, such by using simple VAR forecasts of macro fundamentals as proxies.\footnote{Engel and West (2006) and Mark (2007) fit VARs to construct forecasts of the present value expression. Engel et al (2007) note that the VAR forecasts may be a poor measure of actual market expectations and use surveyed expectations of market forecasters as an alternative, so did several other papers have done. The surveyed data has its own problems as discussed in the literature.} Following the discussion in Chen and Tsang (2009), we use information extracted from the yield curves to capture expectations about future macroeconomic conditions and systematic risks.

The yield curves are of particular interest for several reasons. First, traditional models of the yield curve posit that the shape of the yield curve is determined by expected future paths of short rates and perceived future uncertainty, or the bond term premiums.\footnote{According to the expectations hypothesis (EH) of the term structure of interest rates, at time $t$, the long yield of maturity $m$ can be decomposed into: 1) the average of the current time $t$ one-period yield and the expected one-period yields for the upcoming $m-1$ periods, and 2) the term risk premium perceived at $t$ associated with holding the long bond until $t+m$.} Given that short rates are monetary policy instruments that react to macroeconomic fundamentals, longer yields, containing averaged expected future short rates, thus embody information about market expectations for future macroeconomic conditions. The additional bond market term premiums in the yield curve further provide a measure for the market pricing of risk, of various origins, over different future horizons.\footnote{Kim and Orphanides (2007) and Wright (2009), for example, provide a comprehensive discussion of the bond market term premium, covering both systematic risks associated with macroeconomic conditions, variations in investors' risk-aversion over time, as well as liquidity considerations and geopolitical risky events.} Complementing theories, from a practical standpoint, the shape and movements of the yield curves have long been used to provide continuous readings of market expectations; they are a common indicator for central banks to receive timely feedback to their policy actions. Recent literature, such as the aforementioned Deibold et al (2006), also demonstrates strong dynamic interactions between the macroeconomy and the yield curves. By incorporating the relative yield curves between countries into our model, we thus have an empirically robust proxy that captures the two "asset price" attributes of nominal exchange rates: expectations about future macroeconomic conditions and perceived time-varying risks. This formulation allows us to examine the dynamic interactions among exchange rates, macroeconomic fundamental, and expectations and risk.

We extract the Nelson-Siegel (1987) latent factors from the cross-country yield curves as they are well-known to provide excellent empirical fit, giving us a succinct summary of the small number of systematic sources of risk that may underlie the pricing of various financial assets. By assuming that the same country-specific time-varying latent risks are priced into both the bond and the currency markets, we model the currency risk premium as a linear function of the bond term premiums as in Bekaert et al (2007).\footnote{Bekaert et al (2007) examines the relationship between deviations from uncovered interest parity condition in the currency markets and deviations from the expectations hypothesis in the bond markets at
yields from the expected ones, we can separate out a measure of time-varying risk, from expectations (about future macro conditions), and assess their individual impact on currency returns.

Empirically, we look at monthly exchange rate changes for three currency pairs - the Canada dollar, the British pound, and the Japan yen relative to the US dollar - over the period from August 1985 to July 2005.\textsuperscript{12} For each country pair, we extract the three Nelson-Siegel factors from the zero-coupon yield differences between them, using yield data of 17 maturities ranging from one month to ten years. These three factors, which we refer to as the relative level, relative slope, and relative curvature, capture movements at the long, short, and medium part of the relative yield curves between the two countries. We then estimate a 6-variable VAR by combining the three relative factors with the one-month exchange rate change, and the relative output gap and inflation rates between the two countries. The model is set up as state-space system and estimated using maximum likelihood.

To evaluate the success of our model, and the relative importance of macroeconomic variable versus yield curve factors, we evaluate the longer-horizon exchange rate prediction outcome using the our model, a model with only the macro variables, and a model with only the yield curve factors, and that of a random walk. Here, rather than using the out-of-sample comparisons or longer-horizon regressions, we iterate the full-sample (?) VAR estimates to generate exchange rate predictions, following the procedure of the recent finance literature for evaluating long horizon predictability. We choose this approach for several reasons. First, while out-of-sample forecasts in the tradition of the Meese and Rogoff (1983) literature are commonly used in the macro-exchange rate literature, it has been pointed out that it is an unnecessarily stringent test to impose upon a model. That is, for both theoretical and econometric reasons, it is not the most appropriate test for the validity of a model. In addition, our short data sample prevents accurate estimates of long-horizon regressions, and the approach advocated by Patelis (1997), Lettau and (200X), of using iterated VAR give us more meaningful comparisons. That is, We then use the estimated model to generate

Using the VAR structure, we obtain expected relative yields at different maturities for a given country-pair and use them to construct the term premiums in the relative yield

\textsuperscript{12}We present results based on the dollar cross rates, though the qualitative conclusions extend to other pair-wise combinations of currencies.
curves at each time for different future maturities.\(^{13}\) We demonstrate that: 1) the macro-
fundamental based empirical exchange rate equations miss out on two crucial elements that
drive currency dynamics: expectation and risk; 2) both elements are captured in the latent
factors extracted from the cross-country yield curves, which are found to be empirically
very important; 3) decomposing the yield curves into expectations of future rates and term
premiums, we show that both contain information on future exchange rate changes and
excess currency returns. These findings support the view that exchange rates should be
modeled using a joint macro-finance framework.

In the subsequent sections, we present our theoretical framework, our estimation strategy,
and our empirical results.

2 Theoretical Framework

2.1 Taylor Rule and the Exchange Rate

Recent literature advocates incorporating a Taylor-type interest rate rule in modeling ex-
change rates in countries that have credible inflation control policies\(^{14}\). This approach models
central banks as setting short-term interest rates in response to target variables such as the
output gap and inflation. Together with the uncovered interest rate parity condition, this
approach delivers a set of macroeconomic fundamentals and their expectations that deter-
mine the current level of nominal exchange rate. These models with endogenous monetary
policy have been shown to work better empirically than the traditional monetary models.
Below we present the basic framework.

Consider a two-country model where the home country sets its interest rate, \(i_t\), and the
foreign country sets a corresponding \(i^*_t\). Since our main results in the empirical section below
are based on exchange rates relative to the dollar, one can view the foreign country here as
the United States. The respective monetary policy rules are then described as follows:

\[
\begin{align*}
    i_t &= \mu + \beta_y y_{t, gap} + \beta_{\pi^e} \pi^e_t - \delta q_t + u_t \\
    i^*_t &= \mu^* + \beta_y y^*_{t, gap} + \beta_{\pi^e} \pi^e_t + u^*_t
\end{align*}
\]  

where \(y_{t, gap}\) is the output gap, \(\pi^e_t\) is the expected inflation in the home country, \(q_t = s_t - p_t + p^*_t\)
is the real exchange rate, defined as the nominal exchange rate, \(s_t\), adjusted by the CPI-price

\(^{13}\) The term premium at time \(t\) for maturity \(m\) is just the difference between the actual maturity-
m yield and the predicted yield. See Diebold, Rudebusch and Aruoba (2006) and Cochrane and Piazzesi (2006),
for more discussions.

\(^{14}\) See Engel and West (2006), Engel, Mark and West (2007), Molodtsova and Papell (2008), and Wang
and Wu (2009), among others.
level differences between home and abroad, \( p_t - p_t^* \) (all variables are in log form except for the rates in these equations). \( \mu_t \) absorbs the inflation and output targets and the equilibrium real interest rate, and the stochastic shock \( u_t \) represents policy errors. We assume \( \beta_y, \beta_\pi > 0 \). All the corresponding foreign variables are denoted with a "*". The U.S. is assumed to follow a standard Taylor rule, reacting to inflation and output deviations from their target levels. Note that we assume that the home central bank also targets the real exchange rate, or the purchasing power parity, in addition. This captures the notion that central banks would raise interest rates when their currency depreciates, as supported the empirical findings in Clarida, Gali, and Gertler (1998) and previous work in the literature.\(^{15}\) For notation simplicity, we assume the home and foreign central banks to have the same policy weights \( \beta_y \) and \( \beta_\pi \).\(^{16}\)

Under the rational expectations assumption, the efficient market condition for the foreign exchange markets equates cross-border differentials in interest rates of maturity \( m \), with the expected rate of home currency depreciation and the risk premium over the same horizon:

\[
\tilde{r}_t^m - \tilde{r}_t^{m,*} = E_t \Delta s_{t+m} + \tilde{\rho}_t^m, \forall m
\]

Here \( \tilde{\rho}_t^m \) denotes the risk premium of holding home relative to foreign currency investment between time \( t \) and \( t + m \). We assume that it depends linearly on the general bond-holding risks within each country over the same period:

\[
\tilde{\rho}_t^m = a_0 + a_{m,H} \rho_t^{m,H} - a_{m,F} \rho_t^{m,F} + \epsilon_t
\]

In subsequent sections, we will more explicitly relate these county-specific risks to the bond term premiums within each country’s yield curves.

Combining the above equations, we can express the exchange rate in the following differenced expectation equation for the case where \( m = 1 \):

\[
s_t = \gamma f_t^{TR} + \kappa \tilde{\rho}_t^1 + \psi E_t s_{t+1} + v_t
\]

where \( f_t^{TR} = [p_t - p_t^*, y_t - y_t^*, \pi_t^y - \pi_t^{y*}] \), \( v_t \) is a function of policy error shocks \( u_t \) and \( u_t^* \), and coefficient vectors, \( \gamma, \kappa, \) and \( \psi \) are functions of structural parameters defined above.\(^{17}\)

Iterating the equation forward, we show that the Taylor-rule based model can deliver a net

\(^{15}\)It is common in the literature to assume that the Fed reacts only to inflation and output gap, yet other central banks put a small weight on the real exchange rate. See Clarida, Gali, and Gertler (1998), Engel, West, and Mark (2007), and Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), among many others.

\(^{16}\)Our setup is what Papell et al (2008) term “asymmetric homogenrous” in their comparisons of several variations of the Taylor-rule based forecasting equations.

\(^{17}\)Since these derivations are by now standard, we do not provide detailed expressions here but refer readers to e.g. Engel and West (2005), Chen and Tsang (2009) for more details.
present value equation where exchange rate is determined by the current and the expected future values of cross-country differences in macro fundamentals and risks:

\[ s_t = \lambda \sum_{j=0}^{\infty} \psi^j E_t(f^T_{t+j} | I_t) + \zeta \sum_{j=0}^{\infty} \psi^j \tilde{\rho}_t^j + \varepsilon_t \] (5)

where \( \varepsilon_t \) incorporates shocks and is assumed to be uncorrelated with the macro and risk variables. We can express the above equation in first-differences:

\[ \Delta s_{t+1} = \lambda \sum_{j=1}^{\infty} \psi^j E_t(\Delta f^T_{t+j} | I_t) + \zeta \sum_{j=0}^{\infty} \psi^j \tilde{\rho}_t^j + \varepsilon_t \]

XXX do we need policy shocks? how else to link

In the next section, we discuss that the Taylor-rule fundamentals are exactly the macroeconomic indicators for which the yield curves appear to embody information. Empirically, nominal exchange rate is best approximated by a unit root process, so we express equation (2) in a first-differenced form. From here, rather than following the common approach in the literature and imposing additional assumptions about the statistical processes driving the fundamentals, we discuss in the next section how to use the information in the yield curves to proxy the expected discounted sum on the right-hand side of equation.

### 2.2 The Yield Curve, the Macroeconomy, and Risk

We model the yield curve as a VAR system of the unobserved components and macroeconomic variables. Our approach is similar to the macro-finance literature of Diebold, Rudebusch and Aruoba (2006) and Ang, Piazzesi and Wei (2006). Below discuss the yield curve literature and how the yield curve is connected to the macroeconomy (see Chen and Tsang (2009a) for a more detailed discussion).

### 2.3 The Nelson-Siegel Factors

The yield curve or the term structure of interest rates describes the relationship between yields and their time to maturity. Traditional models of the yield curve posit that the shape of the yield curve is determined by expected future paths of interest rates and perceived

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18 We note that just as in Engel and West (2006) and others, we do not structurally estimate a Taylor rule or impose any structural restrictions in our VAR. Our aim is to explore the relationship between \( s \) with Taylor rule fundamentals \( y \) and \( \pi \), as well as with measures of expectations and risks. Engel and West (2006) and others.

19 See Chen and Tsang (2009a) for a more detailed discussions of the standard estimation techniques that impose a joint statistical process for the fundamentals.
future uncertainty (a time-varying term premiums). While the classic expectations hypothesis is rejected frequently, research on the term structure of interest rates has convincingly demonstrated that the yield curve contains information about expected future economic conditions, such as output growth and inflation.\textsuperscript{20}

We use the Nelson-Siegel factor model to capture price information contained in the yield curves, without imposing the no-arbitrage condition.\textsuperscript{21} This is because the no-arbitrage factor models no dlessness well in describing the dynamics of the yield curve over time (e.g. Diebold et al 2006, Duffee 2002), and our focus is to connect the dynamics of the yield curves to the evolution of the macroeconomic variables and the exchange rates.

As discussed in Diebold, Piazzasi and Rudebusch (2005), factor models can succinctly summarize the small number of sources of systematic risks that underlie the pricing of various tradable financial assets. The Nelson-Seigel factors are well-known to produce excellent empirical fit of the term structure. By allowing long rates to deviate from the average expected future short rates, we can then link the time-varying term premium with the risk premium that separates expected exchange rate changes from the interest differentials, as in Bekaert et al (2007), for example. To this framework we further add in macroeconomy fundamentals.

As argued in Diebold, Rudebusch and Aruoba (2006), the Nelson-Siegel is flexible enough to avoid arbitrage opportunities in the data, and, if arbitrage opportunities do exist, our model avoids the misspecification problem. We model the three factors, exchange rate change and two macroeconomic fundamentals as a reduced-form VAR, and we do not attempt to impose any structural relationships among the variables.

The Nelson-Siegel (1987) model succinctly summarizes the shape of the yield curve using three factors. To derive the factors, they first approximate the forward rate curve at a given time \( t \) with a Laguerre function that is the product between a polynomial and an exponential decay term. This forward rate is the (equal-root) solution to the second order differential equation for the spot rates. A parsimonious approximation of the yield curve can then be obtained by averaging over the forward rates, with the resulting function capable of capturing the relevant shapes of the empirically observed yield curves: monotonic, humped, or S-shaped. It takes the following form:

\[
i_t^m = L_t + S_t \left( \frac{1 - \exp(-\lambda m)}{\lambda m} \right) + C_t \left( \frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right)
\]

\textsuperscript{20}Briefly, the expectations hypothesis says that a long yield of maturity \( m \) can be written as the average of the current one-period yield and the expected one-period yields for the coming \( m - 1 \) periods, plus a term premium. See Thornton (2006) for a recent example on the empirical failure of the expectations hypothesis.

\textsuperscript{21}Ang and Chen (2010) explores the relationship between exchange rate XXXX
where \( i^m_t \) is the continuously-compounded zero-coupon nominal yield on a \( m \)-month bond. The parameter \( \lambda \) controls the speed of exponential decay, and instead of imposing the usual value of 0.0609 we estimate the parameter directly in this paper. One of the main advantages of the Nelson-Siegel approach, compared to the popular no-arbitrage affine or quadratic factor models, is that the three factors, \( L_t \), \( S_t \), and \( C_t \), are easy to estimate and have simple intuitive interpretations. The level factor \( L_t \), with its loading of 1, has the same impact on the whole yield curve. The loading on the slope factor \( S_t \) starts at 1 when \( m = 0 \) and decreases down to zero as maturity \( m \) increases. This factor captures short-term movements that mainly affect yields on the short end of the curve, and an increase in the slope factor means the yield curve becomes flatter, holding the long end of the yield curves fixed. The curvature factor \( C_t \) is a “medium” term factor, as its loading is zero at the short end, increases in the middle maturity range, and finally decays back to zero. It captures how curvy the yield curve is at the medium maturities. These three factors typically capture most of the information in a yield curve. The \( R^2 \) of the cross-section fit is usually close to 0.99.

### 2.4 The Macro-Finance Connection

The recent macro-finance literature connects the observation that the short rate is a monetary policy instrument with the idea that yields of all maturities are risk-adjusted averages of expected short rates. This more structural approach offers deeper insight into the relationship between the yield curve factors and macroeconomic dynamics. The macro-finance literature can be divided into two types. The first type does not model the macroeconomic fundamentals structurally and instead capture their dynamics using a general VAR. For example, Ang, Piazzesi and Wei (2006) estimate a VAR model for the US yield curve and GDP growth. By imposing non-arbitrage condition on the yields, they show that the yield curve predicts GDP growth better than a simple unconstrained OLS of GDP growth on the term spread. More specifically, they find that the term spread (the slope factor) and the short rate (the sum of level and slope factor) outperform a simple AR(1) model in forecasting GDP growth 4 to 12 quarters ahead. Diebold, Rudebusch and Aruoba (2006) is similar to Ang, Piazzesi and Wei, but they adopt the Nelson-Siegel model instead of a no-arbitrage affine model. The second type of studies models the macroeconomic variables structurally, using some version of the standard New Keynesian model. Bekaert, Cho and Moreno (2006) demonstrate that the level factor is mainly moved by changes in the central bank’s inflation target, and monetary policy shocks dominate the movements in the slope and curvature factors. Dewachter and Lyrio (2006) estimate an affine model for the yield curve with macroeconomic variables. They find that the level factor reflects agents’ long run inflation expectation, the slope factor captures the business cycle, and the curvature repre-
sents the monetary stance of the central bank. Rudebusch and Wu (2007, 2008) contend that the level factor incorporates long-term inflation expectations, and the slope factor captures the central bank’s dual mandate of stabilizing the real economy and keeping inflation close to its target. They provide macroeconomic underpinnings for the factors, and show that when agents perceive an increase in the long-run inflation target, the level factor will rise and the whole yield curve will shift up. They model the slope factor as behaving like a Taylor-rule, reacting to the output gap and inflation. When the central bank tightens monetary policy, the slope factor rises, forecasting lower growth in the future.

As we jointly estimate a model of the yield curve and a VAR system of the unobserved components and macroeconomic variables, our paper is similar to the macro-finance literature of Diebold, Rudebusch and Aruoba (2006) and Ang, Piazzesi and Wei (2006). We use the Nelson-Siegel curve without imposing no-arbitrage condition. As argued in Diebold, Rudebusch and Aruoba (2006), the Nelson-Siegel is flexible enough to avoid arbitrage opportunities in the data, and, if arbitrage opportunities do exist, our model avoids the misspecification problem. We model the three factors, exchange rate change and two macroeconomic fundamentals as a reduced-form VAR, and we do not attempt to impose any structural relationships among the variables.

2.5 Bond Term Premium and Currency Risk Premium

The typically upward-sloping yield curves reflect the positive risk premium - or bond term premia - required to compensate investors for holding bonds of longer maturity. As mentioned above, these risks may include inflation as well as consumption risk over the maturity of the bond, and previous research has documented them to be substantial and volatile (Campbell and Shiller 1991; Wright 2009).

Coming out of the Nelson-Siegel model is the concept of term premium, which we will try to tie to excess return in the currency market. The term premium of maturity $m$ is defined as the difference between the current $m$-period yield and the average of the current 1-period yield and its expected value in the coming $m − 1$ periods. Different measures of the term premium come from different methods of forecasting the short rates. As the short rate is a highly persistent and predictable variable, the term premium can be understood as the compensation for bearing the risk from holding long-term instead of short-term bond. Despite a long history of interest in the term premium, there is no consensus among economists on its sources and its effects on the macroeconomy. According to the "common sense" interpretation of the term premium among practitioners, a drop in term premium, which reduces the spread between short and long rates, is expansionary and predicts an increase in real activity. Bernanke (2006) agrees with such a view. According to the canonical
New Keynesian framework, the term premium has no such implication. As pointed out by Rudebusch, Sack and Swanson (2007), only the expected path of short rate matters in the dynamic output Euler equation and term premium does not predict more real activity in the future. For the purpose of this paper, we use the difference between the term premium between two countries to measure the difference in interest rate risk, and we do not attempt to explain the movements of the term premium.

3 Data and Estimation Strategy

This section discusses the empirical implementation of the framework discussed above. We note that just as in Engel and West (2006) and others, we do not structurally estimate a Taylor rule or impose any structural restrictions in our VAR estimations. Our aim is to explore the dynamic interaction among the nominal exchange rate, the macroeconomy captured by the Taylor rule fundamentals, as well as with measures of expectations and risks embodied in the yield curves. We use the atheoretical forecasting equations to capture endogenous feedback among these variables.

3.1 Data Description

The data we examine consists of monthly observations from August 1985 to July 2005 for the US, Canada and Japan, and October 1992 to July 2005 for the United Kingdom.\textsuperscript{22} We look at zero-coupon bond yields for maturities 3, 6, 9, 12, 24, 36, 48 and 60 months, where the yields are computed using the Fama-Bliss (1987) methodology.\textsuperscript{23} To match with the timing of the macroeconomic variables, the yields are measured at the second trading day of next month (i.e. the yields for May 2001 are yields quoted on the second trading day of June 2001). Data for inflation and industrial production are from the IMF’s International Financial Statistics database. Inflation is defined as the annualized 3-month percentage change of the log of seasonally adjusted CPI, and the log industrial production index is fitted to a quadratic trend and use the residual to calculate the relative output gap $\ddot{y}_t$. Monthly exchange rate (last observation of the month) is from the FRED dataset. One-month forward rates (last observation of the month) are obtained from Global Financial Data.

\textsuperscript{22}For the period October 1990 - September 1992, the UK was a participant of the Exchange Rate Mechanism (ERM). During that period, the UK pound was effectively pegged within a small margin to countries in the European Community. Since the UK pound was a "semi-flexible" currency for that period, we dropped that period when doing the RMSE comparison in the previous section. Since the sample size is too small (24 observations), it is infeasible to fit our model and investigate the relationship between the UK pound and the term structure just for that period.

Also, we have done the same estimation for the non-US pairs of Canada-Japan, UK-Japan and Canada-UK and find similar forecasting results.

\textsuperscript{23}For details on the data, please see Diebold, Li and Yue (2007).
Additional yields data for the UK over the period October 1992 - July 2009 is from the Bank of England.

3.2 A Dynamic Latent Factor Macro-Yield Model of Nominal Exchange Rate

To connect the exchange rate with the term structure, we estimate a model that describes the dynamics among the exchange rate, yield curve factors and the macroeconomic fundamentals. We begin with a discussion of the latent-factor representation of the Nelson-Siegel (1987) yield curve. Given a panel of yields, we can estimate the level, slope and curvature factors as latent variables that follow a first-order vector autoregression. Next, we explain how to include exchange rate and other macroeconomic fundamentals in the model. We note that the empirical work below does not impose any structural parameters or restrictions to close the model. Instead, we use atheoretical forecasting equations obtained from the VVV.\(^{24}\)

Noting that the exchange rate fundamentals discussed in previous section are in cross-country differences, we propose to measure the discounted present value with the cross-country differences in their yield curves. Assuming symmetry and exploiting the linearity in the factor-loadings, we fit three Nelson-Siegel factors of the relative level \(L^R \) , the relative slope \(S^R\) , and the relative curvature \(C^R\) . The interpretation of the relative factors is straightforward. For example, an increase in the relative level factor means the vertical difference of the home yield curve to the foreign one is more positive or less negative. We now proceed to estimate the yields-only model for relative yields. At each point of time \(t\), we can fit the cross-section of the difference of yields \(i^{m*}_t - i^m_t\), where \(m\) denotes maturity, with the Nelson-Siegel curve:

\[
i^{m*}_t - i^m_t = L^R_t + S^R_t \left( \frac{1 - \exp(-\lambda m)}{\lambda m} \right) + C^R_t \left( \frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right) + \epsilon^m_t
\]

Each yield of maturity \(m\) has a loading of 1 on the level factor, a loading of \(\frac{1-\exp(-\lambda m)}{\lambda m}\) on the slope factor, and a loading of \(\frac{1-\exp(-\lambda m)}{\lambda m} - \exp(-\lambda m)\) on the curvature factor. The parameter \(\lambda\), which will be estimated, controls the at which maturity the loading on the curvature is maximized. As the number of yields is larger than the number of factors, the factors cannot give a perfect fit to all the yields. As a result, an error term \(\epsilon^m_t\) is appended to each yield as a measure of the goodness of fit. The typical application of the Nelson-Siegel curve involves

\(^{24}\)This approach follows from Engel and West (2006), Molodstova and Papell (2009) and so forth on the exchange rate side, and Diebold et al (2006), among others, on the finance side in employing a nonstructural VAR.
estimating (1) period by period, and does not model how the yield curve evolves over time. We model the three factors together as a VAR(1) system:\(^{25}\)

\[
f_t - \mu = A(f_{t-1} - \mu) + \eta_t \tag{7}
\]

where

\[
f_t - \mu = \begin{pmatrix} L_t^R - \mu_L \\ S_t^R - \mu_S \\ C_t^R - \mu_C \end{pmatrix}.
\]

The term \(\eta_t\) is a 3 by 1 vector of disturbances, and the term \(A\) is a 3 by 3 matrix of VAR coefficients describing the dynamics of the three factors. With (2), we can write the Nelson-Siegel curve (1) more succinctly as vectors:

\[
y_t = \Lambda f_t + \epsilon_t
\]

Since Equation (2) and (3) together form a state-space system, which can be estimated by the Kalman filter. For the estimation to be feasible, the two sets of error terms are uncorrelated:

\[
\begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \sim i.i.d.N \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right]
\]

We can add in the exchange rate and two macroeconomic variables into the VAR system to have \(f_t = (\tilde{y}_t^R, \pi_t^R, \Delta s_t, L_t^R, S_t^R, C_t^R)\) and explain jointly the interaction between the relative term structure and the macroeconomy. We call it the \textit{macro-yields model}. If we drop the two macroeconomic fundamentals, we are only explaining the dynamics between the term structure and exchange rate. We call it the \textit{yields-only model}. If we drop the term structure factors, we are only estimating a simple VAR with the macroeconomic fundamentals and exchange rate. We call it the \textit{macro-only model}. Finally, we measure exchange rate \(s\) as the units of foreign currency per USD, which is measured at the end of each month. A higher \(s\) means an appreciation of the foreign currency, the USD. For all horizons, we define exchange rate change as the change of the log exchange rate \(s\). We denote the 1-month exchange rate, from the end of last month \(t - 1\) to the end of this month \(t\), as \(\Delta s_t\).

We take inflation and output gap as the macroeconomic fundamentals. We define relative inflation \(\pi_t^R\) as the difference of the 12-month percentage change of the CPI between the foreign country and the US, and relative output gap \(\tilde{y}_t^R\) as the difference of the cyclical part of the industrial production index between the foreign country and the US.

\(^{25}\)While we do not have a rigorous justification for this specification, Borak, Härdle, Mammen and Park (2007) argue that, for a high-dimensional system, it is acceptable to describe its dynamics with a VAR for a few factors.
Through the Kalman filter, we can estimate the model using maximum likelihood (See Nelson and Kim (1998) or Harvey (1981) for a discussion of estimating a state-space model by maximum likelihood). To ensure that the variances in the model are positive, we estimate log variances and obtain standard errors by the delta method. Since we have a large number of parameters, choosing the initial values for the optimization problem becomes an important issue. We try two sets of initial values. First, we set the variances to 1, $\lambda$ to 0.0609 (the value commonly imposed for the Nelson-Siegel curve) and all other parameters to 0. The model takes long to converge with these initial values. Second, we use the Diebold-Li (2005) two-step method and obtain the factors with OLS. We then estimate a VAR and use the coefficient estimates to initialize the Kalman filter. The model converges faster with these initial values but the final results are almost identical to those using the first set. The Marquart algorithm is used for the optimization, and the convergence criterion is set to $10^{-6}$.

3.3 Implications for Longer Horizons

To assess longer horizon predictability, we follow Patelis (1997). Using the VAR to make long-horizon forecasts avoids the small-sample bias in long-horizon regressions with overlapping data, and it also allows feedback from the exchange rate change to the macroeconomic fundamentals and yield factors. See Patelis (1997) and Lettau and Ludvigson (2005) for recent applications of the method on stock return. Based on the coefficient estimates of the VAR, we can calculate in-sample forecasts of exchange rate change of any horizon in each month. According to the model, (ex ante) exchange rate change from time $t$ to any future period $t+m$ is a function of the time $t$ values of the six VAR variables. To evaluate the performance of the model, we compare the exchange rate change as implied by the model with its ex post value. Given $\mu$, $A$ and $f_t$, we can calculate the forecast for $f_{t+m}$ as $A^m(f_t - \mu)$. In particular, the third row of the vector $A^m(f_t - \mu)$ gives you a forecast of $\Delta s_{t+m}$. We denote the third row of the vector as $A^m(f_t - \mu)_3$. To forecast exchange rate change from period $t$ to period $t + m$, $s_{t+m} - s_t$, we simply need to calculate (we also annualize the forecast in our calculation):

$$A(f_t - \mu)_3 + A^2(f_{t-1} - \mu)_3 + \ldots + A^m(f_{t-m} - \mu)_3$$

With $A$ and $\mu$ estimated using the whole sample, the forecasts are in-sample.\(^{26}\) Notice that the overlapping variable $s_{t+m} - s_t$ is not used directly in the VAR, and in the model we only have the 1-month exchange rate change $\Delta s_t$. Given the parameter estimates, we explain future exchange rate change using only current exchange rate change $\Delta s_t$, current macro-

\(^{26}\)Our short sample and the large number of parameters keep us from forecasting out of sample. Despite the practical attractiveness of out-sample forecasting, Engel, Mark and West (2007) argue that it is not a reliable criterion of measuring a model.
economic fundamentals $\bar{y}_t^R, \pi_t^R$, and the current term structure $L_t^R, S_t^R, C_t^R$. To compare the long-horizon predictive power of the models (macro-yields, macro-only and yields-only), we use the method proposed by Hodrick (1992) to calculate the contribution of each variable in the VAR system in predicting future exchange rate change. Appendix A describes how to calculate the $R^2$ for each variable and for all variables together.

3.4 Connecting Risk premiums in the Bond and Currency Markets

As discussed above, empirically, both the currency market and the bond market exhibit significant deviations from the respective risk-neutral efficient market conditions. Fama (1984) and subsequent literature documented significant deviations from uncovered interest parity, with the presence of a time-varying currency risk premium a leading explanation. In the bond markets, the failure of the expectation hypothesis is well-established. Wright (2009) and Rudebusch and Swanson (2009) provide recent examples of the research that studies how the bond term premiums may capture crucial market information about future real and nominal risks.

To construct the yield term premiums for a future maturity, $X_{t,t+m}$, we make use of our VAR model to break the term structure factors into an expectations part and a term premiums part. We then investigate their separate role in explaining future exchange rate movements and excess returns.

In each month we can use the VAR model to forecast future relative short rates (i.e. 1-month rate) or rates of any maturity. The procedure is similar to the one in the previous section, but instead of forecasting the exchange rate we forecast the three yield curve factors in the VAR. With the forecasts of the three factors, we can compute the forecasts of the relative short rate based on the Nelson-Siegel curve. Consider some horizon $m$. Take the average of the time $t$ short rate, and the $t+1, ..., t+m-1$ short rate forecasts, we can subtract it from the time $t$ yield of $m$-period maturity to obtain the term premium of maturity $m$. The VAR approach is adopted by Diebold, Rudebusch and Aruoba (2006) and Cochrane and Piazzesi (2006), among others. While this approach may suffer from inconsistency between the yield curve at time $t$ and forecasts of the yields, Rudebusch, Sack and Swanson (2007) shows that the VAR measure of the term premium behaves similarly as other measures that impose no-arbitrage conditions.27

We calculate the relative term premium using the relative yield curve factors, and we

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27For example, see Rudebusch and Wu (2008) and Kim and Wright (2005). The first paper combines a no-arbitrage affine term structure model with a New Keynesian model, while the second paper estimates a three-factor no-arbitrage model without connection to macroeconomic variables.
denote it as:

\[ \rho_t^{(m)} = i_t^{m*} - i_t^m - \frac{1}{m} \sum_{j=0}^{m-1} E_t \left[ i_{t+j}^{m*} - i_{t+j}^m \right] \]

The expectation operator \( E_t \) refers to using the VAR model based on variables known at time \( t \). The relative term premium of maturity \( m \) can be interpreted as the difference in the amount of risk in the foreign and US bond markets at horizon \( m \). More specifically, it measures the difference of the amount of compensation required for bearing the interest rate risk from holding long-term (maturity \( m \)) instead of short-term (maturity 1) debt between the foreign country and the US. An increase in the premium can be interpreted as an increase in the interest risk of maturity \( m \) in the foreign country relative to the US. As the horizon increases, the average of the short rate forecast will approach the sample mean of the short rate, and the relative term premium of maturity \( m \) is roughly equal to the relative yield of maturity \( m \) minus a constant.

4 Results and Discussion

4.1 Explaining Long-Horizon Exchange Rate Change

We report the RMSE and Diebold-Mariano test results in Table 1, and we calculate the \( R^2 \) of each variable in explaining future exchange rate change using Hodrick’s method in Table 2. The macro-yields model has higher predictive power than a simple random walk for all three currencies. Using a quadratic loss function (using the absolute loss function gives similar results), the Diebold-Mariano test concludes that the macro-yields model beats random walk for all horizons, and it also beats the models with only yields and only macroeconomic fundamentals for Japan and the UK. For Canada the predictive power of the macro-yields model and the macro-only model is similar. The predictions based on the macro-yields model are plotted with the actual exchange rate changes in Figures 1 to 3. The model is successful in capturing the dynamics of the exchange rate, and the performance improves as we increase the horizon. Table 2 gives us a better sense of the performance of the model. The 6 variables together can explain more than 10% of the exchange rate change of most horizons, and the results for Japan are the most impressive. At the 12-month horizon, the six variables together can explain 40% of the movement of the exchange rate. For Canada, the two macroeconomic fundamentals are contributing more to the prediction than the term structure factors. The slope factor is the most important explanatory variable for Japan. For the UK the macroeconomic fundamentals have virtually zero predictive power, and it is mainly the level and slope factors that are contributing to the prediction. Estimates for the VAR system, which we use to calculate the long-horizon predictions, and plots of the term
structure factors are reported in Appendix C.

Table 3 shows OLS regression results for exchange rate change and excess return using non-overlapping data. The term structure factors are the smoothed estimates from the state space model, but using the period-by-period OLS factors gives similar results. The 1-month and 3-month (using the last month of every 3 months) exchange rate change results confirm the conclusions from the state space model. For Canada the factors contributing little compared to the macroeconomic fundamentals, but for Japan and the UK the factors are important explanatory variables. We define excess return of horizon \( m \) as \( i_t^{m*} - i_t^m - (s_{t+m} - s_t) \), with the exchange rate change annualized. The macroeconomic fundamentals and term structure factors also explain 4% to 7% of the movement in 1-month excess return, and 14% to 28% of the movement in 3-month excess return.

We also use more recent data for the UK from the Bank of England, over the period October 1992 - July 2009. The data are produced using the Svensson (1994) model, which is essentially a four-factor Nelson-Siegel model, and we find that the state space model is unidentified with the data: the variances of the measurement errors of yields are poorly estimated. We find that the model is identified if we drop the measurement errors of three of the yields (i.e. three yields can be reproduced from the factors perfectly). Instead of arbitrarily modifying the model, we instead estimate the factors by period-by-period OLS. The factors, the exchange rate change and the two macroeconomic fundamentals are then estimated as a six-variable VAR. In Table 4 we have the results using the Hodrick’s method, with the predicted value plotted in Figure 4 with the actual exchange rate change. We are still able to explain around 10% of the exchange rate movement for all horizons, and the slope factor is contributing the most.

### 4.2 Linking Excess Currency Return to Term Premium

In Table 5 we regress the 9-month and 12-month excess returns on the macroeconomic fundamentals and the corresponding term premium (see the Section 3.3 for the calculation). In Figures 5 and 6 we plot the excess return with the term premium for 9 and 12-month horizons. We do not consider excess returns of shorter horizons as some of the 3 and 6-month yields are missing. We use Newey-West standard errors to correct for serial correlation due to overlapping data, and results using non-overlapping data are similar. We find that the term premium, which is calculated only using the current yields, is positively related to the future realized excess currency return. When there is an increase in the \( m \)-period relative

---

28 The \( R^2 \) for the regressions in Table 3 are lower than those in Table 2 for three reasons: 1) the 3-month results in Table 3 discard data while the VAR does not, 2) the VAR allows of feedback from exchange rate change to the explanatory variables, and 3) the results in Table 3 preclude the (tiny) predictive power of lagged exchange rate.
term premium, we can interpret it as an increase in the compensation for risk taken in the foreign bond market at the $m$-period horizon, relative to the US. Results in Table 5 show that a rise in the term premium at horizon $m$ predicts a rise in excess currency return at the same horizon, conditional on the current macroeconomic fundamentals.

5 Conclusion

This paper is the first step of combining the monetary policy approach and the finance approach to modeling exchange rate. The first strand of literature argues that macroeconomic fundamentals usually included in the Taylor rule forecast exchange rate well, but it ignores the presence of risk in the analysis. The second strand of literature explains exchange rate movements or excess return using return-based latent factors, but it does not link the factors to monetary policy directly. We connect the two by estimating a model that jointly describes the dynamics of exchange rate, yield curve factors, inflation and output gap. The model fits the data well, especially at long horizons. Based on the term premiums estimated from the VAR model, we show that both the expected path of relative short rate and term premiums explain future exchange rate movements and excess return. Investors’ view on the future path of monetary policy (which is driven by current and future fundamentals) and their risk appetite are both factors that move future exchange rate.

While this is the first step of bridging the two approaches, this is certainly not the last. Our results call for a model that jointly accounts for forward premium and term premium, tracing both back to preference.
References


Table 1. Predicting Exchange Rates: Models vs. the Random Walk

\[ f_t - \mu = A(f_{t-1} - \mu) + \eta_t \]
\[ [i^m_t - \bar{i}^m_t] = \Lambda \left( L^R_t, S^R_t, C^R_t \right) + \epsilon_t \]

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<th>Macro Yields</th>
<th>RandomWalk</th>
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<td>Δs(_{t+k})</td>
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<td>12</td>
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<td>4.92 (0.03)</td>
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<td>24</td>
<td>4.22 (0.04)</td>
<td>3.85 (0.00)</td>
<td>5.18 (0.53)</td>
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US-Canada

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US-Japan

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<td>4.54 (0.04)</td>
<td>4.87 (0.64)</td>
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US-UK

Note: We estimated the state space model using Kalman filter. The state equation \( f_t - \mu = A(f_{t-1} - \mu) + \eta_t \), is a VAR(1) with a model-dependent vector \( f_t \), as defined in the table. In the measurement equation, \( [i^m_t - \bar{i}^m_t] \) is the vector of relative yields of maturities \( m = 3, 6, 12, 24, 36, 48 \) and 60 months at time \( t \), and matrix \( \Lambda \) is the Nelson-Siegel factor loadings. We iterate the estimated VARs forward to generate predicted exchange rate changes \( \Delta s_{t+k} \) for future horizons from 3 to 24 months and calculate the root mean square prediction errors (RMSEs). The p-values for the Diebold-Mariano test comparing the model’s prediction and that of the random walk are reported in the parentheses. Note that the sample for the UK starts after the ERM crisis (1992M10).
Table 2. Explaining Exchange Rate Changes $\Delta s_{t+k}$ with Macroeconomic Fundamentals and Yield Curve Factors

$$f_t - \mu = A(f_{t-1} - \mu) + \eta_t, \text{ where } f_t = (\hat{y}_t, \pi_t, \Delta s_t, L_t^R, S_t^R, C_t^R)$$

$$[i_t^m - i_t^{ms}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$$

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Note: We iterate the estimated $\hat{A}$ forward to generate forecasts for $k$-period exchange rate changes, $\Delta s_{t+k}$. The partial $R^2$ reports the contribution of each variable in explaining $\Delta s_{t+k}$. It is calculated using $\hat{A}$ and the estimated covariance matrix of the VAR, $\hat{Q}$, based on the Hodrick (1992) method. Please refer to Appendix B for details.
Table 3: Explaining Exchange Rate Changes and Excess Returns
Macroeconomic Fundamentals, Yield Factors, or Both?

\[
\Delta s_{t+k} = a_0 + a_1 y_t^R + a_2 \pi_t^R + a_3 L_t^R + a_4 S_t^R + a_5 C_t^R + \epsilon_t
\]

\[
XR_{t+k} = a_0 + a_1 y_t^R + a_2 \pi_t^R + a_3 L_t^R + a_4 S_t^R + a_5 C_t^R + \epsilon_t
\]

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**Canada**

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</table>

**UK**

Note: We use the Newey-West standard errors in the \(\Delta s_{t+k}\) and \(XR_{t+k}\) OLS regressions. The "No Macro" column reports the p-values of the Wald tests for the null hypothesis that macroeconomic fundamentals have no explanatory power \((a_1 = a_2 = 0)\), and the "No Factors" column tests the null hypothesis that the relative factors do not matter \((a_3 = a_4 = a_5 = 0)\). We use the last month of each quarter to create non-overlapping samples for the 3-month regressions. One-month excess return, \(XR_{t+1}\), is calculated using the forward premium. The sample for the UK starts after the ERM crisis (1992M10). For Japan, the \(XR_{t+1}\) regression starts on October 1998 due to the limited availability of 1-month forward rate data.
Table 4: Explaining Exchange Rate Changes $\Delta s_{t+k}$
with Macroeconomic Fundamentals and Yield Curve Factors

$$f_t - \mu = A(f_{t-1} - \mu) + \eta_t;$$
where $f_t = (\bar{y}_t^R, \bar{\pi}_t^R, \Delta s_t, L_t^R, S_t^R, C_t^R)$

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Output Gap</th>
<th>Inflation</th>
<th>Ex. Rate</th>
<th>Level</th>
<th>Slope</th>
<th>Curvature</th>
<th>Total $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.00</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.11</td>
</tr>
<tr>
<td>6</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.03</td>
<td>0.00</td>
<td>0.11</td>
</tr>
<tr>
<td>12</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.05</td>
<td>0.01</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Note: The yield curve factors are obtained by running the Nelson-Siegel model: $[i_t^m - i_t^{m*}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$ period by period. We then estimate the VAR(1) above and iterate the estimated $\hat{A}$ forward to generate forecasts for the $k$-period exchange rate changes, $\Delta s_{t+k}$. The partial $R^2$ reports the contribution of each variable in explaining $\Delta s_{t+k}$. It is calculated using $\hat{A}$ and the estimated covariance matrix of the VAR, $\hat{Q}$, based on the Hodrick (1992) method. Please refer to Appendix B for details.
Table 5: Predicting 9-Month and 12-Month Excess-Returns with Macro Fundamentals and Relative Term Premium

\[ XR_{t+k} = a_0 + a_1 y_t^R + a_2 \pi_t^R + a_3 \rho_t^{(m)} + \varepsilon_t, \quad k = 9, 12 \]

<table>
<thead>
<tr>
<th>Country</th>
<th>Output Gap</th>
<th>Inflation</th>
<th>Term Premium</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-Month Excess Return</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>1.13 (0.29***)</td>
<td>5.87 (1.35***)</td>
<td>3.70 (1.35***)</td>
<td>0.40</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.15 (0.15)</td>
<td>9.93 (3.01***)</td>
<td>27.35 (3.60***)</td>
<td>0.51</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.20 (0.32***)</td>
<td>10.16 (3.38**)</td>
<td>11.82 (3.65**)</td>
<td>0.28</td>
</tr>
</tbody>
</table>

| 12-Month Excess Return |
| Canada          | 1.05 (0.26***)| 5.55 (1.22***)| 4.15 (1.19***)| 0.47    |
| Japan           | -0.12 (0.13)  | 10.94 (2.34)  | 22.63 (2.74***)| 0.58    |
| United Kingdom  | 1.04 (0.24***)| 9.65 (2.80**)| 11.72 (2.93**)| 0.40    |

Note: The regressions are estimated with Newey-West standard errors. Refer to the text for the calculation of the term premium \(\rho_t^{(m)}\). We have also estimated the same regression with non-overlapping 9-month and 12-month data and obtain similar results.
Note: Predicted exchange rate changes $\Delta s_{t+k}$ are generated as follows: We first estimate a state space model with a VAR (1) state equation: $f_t - \mu = A(f_{t-1} - \mu) + \eta_t$, where $f_t = (y_t^R, \pi_t^R, \Delta s_t, L_t^R, S_t^R, C_t^R)$, and a measurement equation: $[i_t^m - i_t^{m*}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$ where matrix $\Lambda$ is defined by the Nelson-Siegel factor loadings. The estimated VAR(1) is then iterated forward $k$-periods to generate predicted exchange rate changes for $k = 3$ and 12 months ahead. The model-generated predictions are plotted against the actual exchange rate changes over the corresponding horizons.
Figure 2: Exchange Rate Predictions from the Macro+Yields Model

US-Japan

Note: Predicted exchange rate changes $\Delta s_{t+k}$ are generated as follows: We first estimate a state space model with a VAR (1) state equation: $f_t - \mu = A(f_{t-1} - \mu) + \eta_t$, where $f_t = (\bar{y}_t, \pi_t^R, \Delta s_t, L_t^R, S_t^R, C_t^R)$, and a measurement equation: $[i_t^m - i_t^{m*}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$ where matrix $\Lambda$ is defined by the Nelson-Siegel factor loadings. The estimated VAR(1) is then iterated forward $k$-periods to generate predicted exchange rate changes for $k = 3$ and 12 months ahead. The model-generated predictions are plotted against the actual exchange rate changes over the corresponding horizons.
Note: Predicted exchange rate changes $\Delta s_{t+k}$ are generated as follows: We first estimate a state space model with a VAR (1) state equation: $f_t - \mu = A(f_{t-1} - \mu) + \eta_t$, where $f_t = (\tilde{y}_{t}^R, \pi_t^R, \Delta s_t, L_t^R, S_t^R, C_t^R)$, and a measurement equation: $[i_t^m - i_t^{m*}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$ where matrix $\Lambda$ is defined by the Nelson-Siegel factor loadings. The estimated VAR(1) is then iterated forward $k$-periods to generate predicted exchange rate changes for $k = 3$ and 12 months ahead. The model-generated predictions are plotted against the actual exchange rate changes over the corresponding horizons.
Note: Using data provided by the Bank of England, we first obtain the relative yield curve factors by running period-by-period OLS regressions of the Nelson-Siegel model. We then estimate a VAR(1) model: \( f_t - \mu = A(f_{t-1} - \mu) + \eta_t \) where \( f_t = (\bar{y}_t^R, \pi_t^R, \Delta s_t, L_t^R, S_t^R, C_t^R) \), and iterate it forward to generate predicted exchange rate changes for different future horizons. The model-generated predictions are plotted against the actual exchange rate changes over the corresponding horizons.
Fig. 5: 9-Month Excess Currency Return and the Relative Term Premium

US-Canada

US-Japan

US-UK
Fig. 6: 12-Month Excess Currency Return and the Relative Term Premium

US-Canada

US-Japan

US-UK
6 Appendix

6.1 Appendix A: VAR Multi-Period Predictions

We follow the procedure as described in Hodrick (1992), but the method is also adopted in Campbell and Shiller (1988), Kandel and Stambaugh (1988) and Campbell (1991), among others. Our six-variable VAR can be written as (the constant term does not matter and we drop it for now):

\[ f_t = A f_{t-1} + \eta_t \]

We drop the constant term for convenience. Denote the information set at time \( t \) as \( I_t \), which includes current and past values of \( f_t \), a forecast of horizon \( m \) can be written as \( E(f_{t+m} | I_t) = A^m f_t \). By repeated substitution, the first-order VAR has an MA(\( \infty \)) representation:

\[ f_t = \sum_{j=0}^{\infty} A^j \eta_{t+j} \]

From the above representation, we can calculate the unconditional variance of \( f_t \) as:

\[ C(0) = \sum_{j=0}^{\infty} A^j Q A^j \]

What is the variance of \( Z_{t+1} + Z_{t+2} + \ldots + Z_{t+m} \)? Denoting \( C(j) \) as the \( j \)th-order covariance of \( f_t \), which is calculated as \( C(j) = A^j C(0) \), the variance of the sum, denoted as \( V_m \), is then:

\[ V_m = m C(0) + \sum_{j=1}^{m-1} (k-j) [C(j) + C(j')] \]

We are not interested in the variance of the whole vector, and we only care about the variance of the long-horizon exchange rate change. As exchange rate change \( \Delta s_t \) is the third element in the vector \( f_t \), we can define \( e_3' = (0, 0, 1, 0, 0, 0) \), and obtain the variance of the \( m \)-period exchange rate change as \( e_3' V_m e_3 \).

To answer the question of whether a variable in time \( f_t \), the level factor \( L_t^R \) say, explains exchange rate change \( s_{t+m} - s_t \), we can run a long-horizon regression for \( s_{t+m} - s_t \) on \( L_t^R \). The VAR model for \( f_t \) allows us to calculate the slope from this regression, based on only the VAR coefficient estimates. Since the level factor is the fourth element of \( f_t \), the slope is defined as:

\[ \beta_4(m) = \frac{e_3' [C(1) + \ldots + C(m)] e_4}{e_4' C(0) e_4} \]

The vector \( e_4 \) is defined as \( e_4 = (0, 0, 0, 1, 0, 0) \). The numerator is the covariance between \( s_{t+m} - s_t \) and \( L_t^R \), and the denominator is the variance of \( L_t^R \). Finally, the \( R^2 \) as reported in the paper is calculated as:
\[ R^2_4(m) = \beta_4(m)^2 \frac{e'_4 C(0) e_4}{e'_4 V_m e_3} \]

The \( R^2 \) for other variables in the vector \( f_t \) is obtained by replacing \( e_4 \) with \( e_1, e_2, e_3, e_5, e_6 \).

To calculate the total \( R^2 \) for the explanatory variables, we calculate the innovation variance of the exchange rate change as \( e'_1 W_m e_1 \), where

\[ W_m = \sum_{j=1}^{m} (I - A)^{-1} (I - A^j) Q (I - A^j)' (I - A)^{-1} \]

We can then calculate the total \( R^2 \) as:

\[ R^2(m) = 1 - \frac{e'_1 W_m e_1}{e'_m V_m e_m} \]

### 6.2 Appendix B: VAR with Quarterly Data

We pick the last month of each quarter over our monthly sample to create a quarterly sample, and we have 80 observations. Since the original model as described has more parameters than the observations, we cannot estimate the model using the state-space model using maximum likelihood. As a compromise (with some loss of efficiency), we first obtain the level, slope and curvature factors by an OLS regression for the Nelson-Siegel curve in every period, as in Chen and Tsang (2009a). We then estimate a VAR for the extracted factors, output gap, inflation and 3-month exchange rate change. Only the estimated equation for the 3-month exchange rate is reported below.

**Table A1: VAR Estimates with Quarterly Data for \( s_{t+3} - s_t \)**

<table>
<thead>
<tr>
<th>Country</th>
<th>( \tilde{y}_t^R )</th>
<th>( \pi_t^R )</th>
<th>( s_t - s_{t-3} )</th>
<th>( L_t^R )</th>
<th>( S_t^R )</th>
<th>( C_t^R )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>1.400</td>
<td>7.176</td>
<td>-0.119</td>
<td>-0.015</td>
<td>-0.128</td>
<td>0.224</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>(0.554)</td>
<td>(2.753)</td>
<td>(0.117)</td>
<td>(1.704)</td>
<td>(0.653)</td>
<td>(0.482)</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>-1.276</td>
<td>-1.837</td>
<td>-0.084</td>
<td>8.942</td>
<td>5.127</td>
<td>-0.209</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td>(0.494)</td>
<td>(7.055)</td>
<td>(0.116)</td>
<td>(4.573)</td>
<td>(1.858)</td>
<td>(1.144)</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>2.238</td>
<td>2.688</td>
<td>0.009</td>
<td>-9.027</td>
<td>-1.762</td>
<td>-1.963</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(1.345)</td>
<td>(6.152)</td>
<td>(0.137)</td>
<td>(3.803)</td>
<td>(1.114)</td>
<td>(0.769)</td>
<td></td>
</tr>
</tbody>
</table>

The sample for the UK is again after the ERM crisis (1992Q3-2005Q2), and the VAR is of order one as in the main text.
6.3 Appendix C: Estimates for the 6-Variable VAR in the Full Model

The model we are estimating is:

\[ y_t = \Lambda f_t + \epsilon_t \]
\[ f_t - \mu = A(f_{t-1} - \mu) + \eta_t \]
\[ \begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \sim i.i.d.N \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right] \]

We use yields of maturities 3, 6, 12, 24, 36, 48 and 60 months. Each yield difference is modeled by the Nelson-Siegel functional form:

\[ i_{t}^{m*} - i_{t}^{m} = L_t^R + S_t^R \left( \frac{1 - \exp(-\lambda m)}{\lambda m} \right) + C_t^R \left( \frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right) + \epsilon_{t}^{m} \]

Here we only report the estimates for the "macro and yields" model, where the vector of variables is defined as \( f_t = (\tilde{y}_t^R, \pi_t^R, \Delta s_t, L_t^R, S_t^R, C_t^R) \), we can write down a state space model with the measurement equation \( y_t = \Lambda f_t + \epsilon_t \), where \( y_t \) is the vector of yields of all maturities at time \( t \), and the state equation \( f_t - \mu = A(f_{t-1} - \mu) + \eta_t \), which is a VAR(1) for the vector \( f_t \). The matrix \( \Lambda \) is defined by the Nelson-Siegel model. We report the VAR estimates for \( A \) and \( Q \) below. We also plot the estimated factors with the factors estimated by period-by-period OLS (the coefficient \( \lambda \) is fixed at the value estimated by the state-space model).
US-Canada - VAR Coefficient Estimates

\[
A = \begin{pmatrix}
0.975 & -0.040 & 0.022 & -0.026 & 0.022 & 0.040 \\
(0.035) & (0.140) & (0.043) & (0.119) & (0.032) & (0.026) \\
0.000 & 0.932 & 0.005 & 0.004 & -0.004 & 0.001 \\
(0.007) & (0.031) & (0.011) & (0.027) & (0.008) & (0.006) \\
0.0822 & 0.475 & 0.014 & 0.090 & 0.015 & 0.039 \\
(0.076) & (0.347) & (0.082) & (0.281) & (0.093) & (0.064) \\
0.065 & -0.065 & 0.003 & \textbf{0.745} & 0.015 & \textbf{0.069} \\
(0.036) & (0.145) & (0.053) & (0.112) & (0.029) & (0.026) \\
-0.038 & 0.097 & -0.032 & 0.157 & \textbf{0.817} & 0.053 \\
(0.075) & (0.250) & (0.095) & (0.210) & (0.067) & (0.044) \\
-0.189 & 0.371 & 0.114 & 0.570 & 0.168 & \textbf{0.577} \\
(0.161) & (0.648) & (0.236) & (0.524) & (0.146) & (0.120)
\end{pmatrix}
\]

\[
Q = \begin{pmatrix}
0.487 & -0.003 & -0.133 & \textbf{0.104} & -0.005 & -0.369 \\
(0.055) & (0.010) & (0.090) & (0.058) & (0.082) & (0.227) \\
\textbf{0.025} & -0.017 & -0.007 & 0.002 & 0.040 \\
(0.002) & (0.021) & (0.014) & (0.021) & (0.063) \\
\textbf{2.263} & -0.033 & 0.115 & 0.464 \\
(0.250) & (0.136) & (0.208) & (0.604) \\
\textbf{0.291} & 0.054 & -\textbf{1.215} \\
(0.129) & (0.082) & (0.447) \\
\textbf{1.208} & -1.390 \\
(0.226) & (0.300) \\
\textbf{7.391} \\
(1.584)
\end{pmatrix}
\]
Figure A1: Smoothed State-Space Factors vs. OLS Factors (US-Canada)
US-Japan - VAR Coefficient Estimates

\[
A = \begin{pmatrix}
0.972 & -0.648 & -0.041 & 0.217 & 0.056 & -0.028 \\
0.004 & 0.919 & -0.004 & -0.008 & -0.001 & 0.002 \\
0.113 & 0.303 & -0.052 & -0.582 & -0.399 & -0.052 \\
0.011 & -0.011 & -0.004 & 0.857 & -0.023 & 0.035 \\
0.005 & 0.145 & -0.014 & 0.001 & 0.773 & 0.104 \\
-0.016 & -0.146 & 0.038 & 0.158 & 0.278 & 0.805 \\
\end{pmatrix}
\]

\[
Q = \begin{pmatrix}
1.829 & 0.002 & -0.181 & 0.065 & -0.073 & 0.093 \\
0.033 & 0.024 & 0.003 & -0.004 & 0.032 \\
11.171 & -0.113 & 0.096 & 0.426 \\
0.179 & -0.131 & -0.123 \\
0.508 & -0.428 \\
1.540 \\
\end{pmatrix}
\]
Figure A2: Smoothed State-Space Factors vs. OLS Factors (US-Japan)
### US-UK - VAR Coefficient Estimates

\[
A = \begin{pmatrix}
0.874 & -0.397 & -0.087 & 0.379 & 0.160 & 0.082 \\
0.004 & 0.928 & -0.001 & -0.009 & -0.008 & 0.000 \\
0.004 & 0.052 & 0.000 & 0.038 & 0.015 & 0.011 \\
-0.273 & 0.456 & -0.078 & 1.207 & 0.355 & 0.275 \\
0.021 & 0.021 & -0.016 & 0.999 & 0.031 & 0.031 \\
0.032 & 0.198 & -0.016 & 0.099 & 0.938 & 0.047 \\
0.026 & -0.182 & 0.063 & -0.572 & -0.197 & 0.720 \\
0.384 & 1.615 & 0.369 & 1.806 & 0.626 & 0.445 \\
\end{pmatrix}
\]

\[
Q = \begin{pmatrix}
0.862 & 0.002 & 0.083 & -0.038 & -0.017 & 0.236 \\
0.122 & 0.020 & 0.373 & 0.262 & 0.236 & 0.758 \\
0.022 & 0.013 & -0.006 & 0.005 & 0.005 \\
0.003 & 0.040 & 0.037 & 0.036 & 0.118 \\
5.237 & -0.042 & 0.006 & -0.573 \\
0.747 & 0.624 & 0.632 & 2.385 \\
1.187 & -0.627 & -3.493 \\
1.263 & 0.619 \\
1.263 & 0.457 & 1.035 \\
12.864 \\
\end{pmatrix}
\]
Figure A3: Smoothed State-Space Factors vs. OLS Factors (US-UK)