Growth and Inequality in a Small Open Economy*

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Abstract

This paper analyzes the growth and inequality relation for a small open economy where agents differ in their initial endowments of capital stock and international bond-holdings. Using an endogenous growth model with elastic labor supply, we analyze the aggregate growth and the distributional impacts of different structural shocks through their effects on agents' relative wealth and their labor supply decisions. Both theoretical analysis and numerical simulations demonstrate that openness – access to an international capital market – enriches the growth-inequality relations from those of the corresponding closed economy. Specifically, we show that the growth and distributional consequences of structural shocks depend crucially on whether the underlying heterogeneity originates with the initial endowment of domestic capital or foreign bonds. While this result contrasts sharply with the positive growth-inequality relation found in an analogous closed economy, it is more consistent with the ambiguous relationships that characterize the existing empirical evidence.

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1. Introduction

The relation between growth and inequality is one of the fundamental questions in economics, dating back to the seminal work of Kuznets (1955). Despite the intensive research activity that this issue has generated, the nature of the relationship remains unresolved, with the empirical evidence being inconclusive. Early growth regressions by Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996), and others, yield a negative growth-inequality relationship.[1] But more recent studies obtain a positive, or at least more ambiguous, relationship; see for example, Li and Zou (1998), Forbes (2000), and Barro (2000).[2] From a theoretical perspective, this empirical controversy should not be surprising. Because an economy's growth rate and its income distribution are both endogenous equilibrium outcomes of the economic system, the income inequality-growth relationship – whether positive or negative – will reflect the underlying set of forces to which both are reacting. To understand these linkages, it is necessary to adopt a structural, consistently-specified general equilibrium approach.

Following this approach, recent literature explores various economic factors and modeling strategies in analyzing the growth-inequality relationship, emphasizing issues such as the roles of endogenous labor supply, increasing returns, and the homogeneity of the utility function.[3] However, virtually the entire growth-inequality literature is restricted to a closed economy, which is a severe shortcoming given the increasing openness characterizing most economies. Accordingly, the goal of this paper is to examine the linkage between growth and inequality in the context of a small open economy and to explore the additional dimensions that international transactions bring to the relation,

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[1] The various explanations for this include: the political economy consequences of inequality (Alesina and Rodrik, 1994), the potential harm inequality may cause for investment in physical or human capital, (Galor and Zeira, 1993; Aghion and Bolton, 1997), and the unequal distribution of natural resources (Gylfason and Zoega, 2003).

[2] In particular, Forbes finds a positive relationship when the short-term impact is considered. Barro finds a negative relationship between inequality and growth for poorer countries, but a positive relationship for richer countries. Explanations for the positive relationship include: a positive relationship between inequality and higher tax rates to finance public education (Saint-Paul and Verdier, 1993), socio-economic stratification (Bénabou, 1996a), and the nature of technological progress (Galor and Tsiddon, 1997).

[3] Aghion, Caroli, and García-Peñalosa (1999) provide an overview. More recent examples include Caselli and Ventura (2000), Turnovsky and García-Peñalosa (2008), and Sorger (2000), which all adopted a Ramsey framework where agents have heterogeneous capital wealth endowments. The former two emphasize the tractability of utility functions which are homogeneous in consumption and leisure in, and the latter two consider endogenous labor supply. García-Peñalosa and Turnovsky (2006), on the other hand, employs an endogenous growth model, and again emphasizes the joint determination of growth and distribution as well as elastic labor supply.

Specifically, we focus on openness in the form of access to an international bond market and employ an open economy adaptation of Romer's (1986) canonical endogenous growth model.

In a completely general setup, in which the equilibrium growth rate and income distribution are mutually dependent, their joint determination and the analysis of their relationship become intractable [see for example, Sorger, 2000)]. Following García-Peñalosa and Turnovsky (2006), our sructure exploits the fact that if the utility function is homogeneous in its relevant arguments, the aggregate economy can be summarized by a representative agent.[4] As a result, aggregate behavior becomes independent of the economy's distributional characteristics, and the implied recursive structure enables one to address distributional issues in a tractable way. The class of constant elasticity utility function that dominates contemporary growth theory possesses this property, and we adopt it in our analysis.

While in general inequality may reflect diverse sources of underlying heterogeneity, several recent studies focus on agents' initial endowments of assets as being a key element. For example, Alesina and Rodrik (1994), Bertola (1993), García-Peñalosa and Turnovsky (2006), among others, develop closed-economy growth models where agents differ in their initial stocks of capital. By extending the analysis to an open economy with a competitive international bond market, we introduce an addition source of inequality, namely agents' initial endowment of the internationally-traded bonds. This extension enriches the growth-inequality relation in several dimensions. For example, to the extent that agents' relative endowment of domestic capital differs from that of international bonds, structural shocks, through their effects on the relative price of the two assets, may not only impact income inequality and its correlation with aggregate growth, but also wealth and welfare distribution as well.

The key mechanism generating the endogenous distribution of income is the positive equilibrium relationship we derive between agents' relative wealth and their relative allocation of time to leisure. This relationship has a simple intuition. Wealthier agents have a lower marginal utility of wealth. They therefore choose to increase consumption of all goods including leisure, and

[4] This strategy is also adopted by Caselli and Ventura (2000) and García-Peñalosa and Turnovsky (2007, 2008), among others. While knowledge of this feature dates back to Gorman (1953), it assumes particular importance in the present context.

reduce their labor supply. Given their relative capital endowments, this translates to an endogenously determined distribution of income. The negative relationship between wealth and labor supply also has substantial empirical support. For example, Holtz-Eakin, Joulfaian, and Rosen (1993) find evidence to support the view that large inheritances decrease labor participation. Cheng and French (2000) and Coronado and Perozek (2003) use data from the stock market boom of the 1990s to study the effects of wealth on labor supply and retirement, finding a substantial negative effect on labor participation. Algan, Chéron, Hairault, and Langot (2003) use French data to analyze the effect of wealth on labor market transitions, and find a significant wealth effect on the extensive margin of labor supply. Overall, these studies and others provide compelling evidence in support of the wealth-leisure mechanism being emphasized in this paper.

Using this framework, we analyze the joint determination of the growth rate and inequality and consider how they respond to various structural changes, including an increase in productivity, an increase in savings (decrease in rate of time preference), and an increase in the foreign interest rate. The structural approach allows us to consider not just wealth and income inequality, but also welfare inequality. In doing so, we show how the impacts of these structural changes on the growth-inequality tradeoff depend upon the underlying origin of the heterogeneity, i.e. whether it originates with the endowment of capital or bonds. We also demonstrate that the presence of adjustment costs to capital accumulation may drive a wedge between an agent's relative wealth standing and her relative income, depending again on the relative endowments of domestic capital and foreign bonds. These findings highlight the relevance of international asset markets in understanding the growth-inequality relationship, and how this tradeoff facing a small open economy may be dramatically different from that confronting a closed economy. While access to an international riskless bond market captures only a limited aspect of openness, our analyses already illustrates the importance of considering such additional channels.

We should note, however, that by adapting the Romer model, we are ignoring other important elements relevant to the growth-inequality relationship, most notably human capital and education. This aspect is emphasized by Galor and Zeira (1993), Bénabou (1996b), and Viaene and Zilcha (2003), among others. By identifying agents' heterogeneity with their initial physical asset endowments, we are embedding distributional issues within a more traditional growth-theoretic framework. Indeed, the role of the return to capital, which is essential in that literature, has largely been ignored in the recent discussions of income inequality. The argument that the return to capital is essential to understanding distributional differences has, however, been addressed by Atkinson (2003), and is supported by recent empirical evidence for the OECD (see Checchi and García-Peñalosa (2005). Like the Romer model, our model has the feature that the economy always lies on its balanced growth path. While this rules out the dynamics of income distribution, which are clearly important, it has the pedagogic advantage of highlighting the growth-inequality relation in a lucid way.[5]

The rest of the paper is organized as follows. Sections 2 presents the structure of the model. Sections 3 and 4 derive the macroeconomic equilibrium and examine the determinants of the distributions of income and welfare. Section 5 analyzes the relationship between growth and inequality in response to specified structural changes. Section 6 supplements our theoretical analysis with some numerical simulations. Section 7 concludes, while technical details are provided in the Appendix.

2. Small open economy with heterogeneous agents and endogenous labor supply

This section sets up the structure of our small open economy, which is an endogenous growth model with elastic labor supply. We assume private and convex adjustment cost to capital, and that agents have heterogeneous income stemming from initial distributions of endowments of capital and of international bonds. The homogeneity of the utility function implies that the macroeconomic equilibrium is independent of any distributional characteristics. We show that the economy's growth rate is determined by the world interest rate and the preference parameters (the discount rate and the inter-temporal elasticity of substitution), and that this growth rate pins down the equilibrium relative price of capital and allocation of time to labor. In response to any structural shocks, leisure choice and the relative price of capital adjust so that the economy is always on its balanced-growth path.

[5] For the analysis of distributional dynamics in a closed economy see Caselli and Ventura (2000), Turnovsky and García-Peñalosa (2008).

2.1 Technology and factor payments

The economy consists of a fixed number of firms indexed by *j*. The representative firm produces output using the production function[6]

$$Y_j = F\left(L_j K, K_j\right) \tag{1a}$$

where K_j denotes the individual firm's capital stock, L_j denotes the individual firm's employment of labor, K is the average stock of capital in the economy, a proxy for the economy-wide stock of knowledge, so that $L_j K$ measures the efficiency units of labor. Production has the usual neoclassical properties of positive, but diminishing, marginal physical products and constant returns to scale in K_j and $L_j K$. This means that the production function has constant returns to scale both in the accumulating factors, K_j and K, necessary for endogenous growth, and in the private factors K_j and L_j, necessary for marginal product factor pricing in a competitive equilibrium.

As all firms face identical production conditions, they choose the same level of employment and capital stock implying that $K_j = K$ and $L_j = L$, for all *j*, where L is the average economy-wide level of employment. In equilibrium the economy-wide aggregate (average) production function can then be expressed as a linear function of the aggregate capital stock, as in Romer (1986), namely:[7]

$$Y = F\left(LK, K\right) = f(L)K \qquad f'(L) > 0, f'' < 0 \tag{1b}$$

Assuming competitive factor markets, the wage rate and the return to capital are determined by their respective marginal physical products. Since the overall labor supply is assumed to be constant but the capital stock grows along with output, we see that the equilibrium wage rate increases with the average stock of capital, while the return to capital does not:

$$\left.\frac{\partial F}{\partial L_j}\right|_{K_j=K, L_j=L} = f'(L)K \equiv \omega(L)K \tag{2a}$$

[6] To simplify notation, we normalize to unity here the productivity parameter A, which enters the production function multiplicatively with function F. In Section 5 we will examine the effects of a change in A.

[7] We assume that the production function satisfies the Inada conditions $f(0) = 0, f'(0) \to \infty, f'(\infty) \to 0$

$$\left.\frac{\partial F}{\partial K_j}\right|_{K_j=K, L_j=L} = f(L) - Lf'(L) \equiv r_k(L) \tag{2b}$$

2.2 Consumers

The economy is populated by a mass 1 of infinitely-lived consumers, indexed by *i*. Agents are identical in all respects except for their initial endowments of capital, $K_{i,0}$, and net international bond holdings, $B_{i,0}$. Since the economy is growing we focus on the relative shares of individual *i*'s holding of capital and bonds, $k_i(t) \equiv K_i(t)/K(t),\ b_i(t) \equiv B_i(t)/B(t)$, where $K(t), B(t)$ denote the corresponding economy-wide average quantities. The initial relative endowments have mean 1 and standard deviations $\sigma_{k,0}, \sigma_{b,0}$ across agents.

Consumers have a unit of time that can be allocated to either leisure, l_i or labor $L_i \equiv 1 - l_i$. Agents maximize lifetime utility, which depends on complementary consumption C_i and leisure in the following iso-elastic form:[8]

$$\max \int_0^\infty \frac{1}{\gamma}(C_i(t) l_i(t)^\theta)^\gamma e^{-\beta t} dt, \qquad \theta > 0,\ -\infty < \gamma \leq 1,\ \gamma\theta < 1,\ \gamma(1+\theta) < 1 \tag{3}$$

The parameter γ is related to the agent's inter-temporal elasticity of substitution, $\kappa = 1/(1-\gamma)$; θ captures the relative importance of leisure to consumption; and β is the instantaneous subjective discount rate. The last two parameter restrictions in (3) ensure the concavity of the utility function with respect to *C* and *l*.

Agents accumulate capital subject to convex adjustment (installation) costs for any given change, *I,* of the capital stock. Specifically, we assume the adjustment costs to be proportional to the rate of investment per unit of installed capital, *I/K*, specified by the following quadratic function:[9]

$$\Phi(I_i, K_i) = I_i\left(1 + \frac{h}{2}\frac{I_i}{K_i}\right) \tag{4}$$

We assume that capital does not depreciate, so that agent *i* accumulates capital at the rate

[8] The complementarity instead of additive separability of consumption and leisure in the utility function preserves homogeneity and consistency with a balanced-growth equilibrium [see Turnovsky, 1999, and Ladron-de-Guevara, Ortigueira, and Santos, 1999].

[9] The linear homogeneity of this cost function is necessary to sustain steady-state growth.

$$\dot{K}_i = I_i . \tag{5}$$

Individuals also accumulate net foreign bonds, B_i, which pay an exogenously given world interest rate, r, subject to the accumulation equation:

$$\dot{B}_i = \omega K(1-l_i) + r_k K_i + rB_i - C_i - I_i(1+\frac{h}{2}\frac{I_i}{K_i}) \tag{6}$$

Note that agents earn gross income from production and foreign interest: $Y_i = r_k K_i + \omega K(1-l_i) + rB_i$.

2.3 Consumer optimality conditions

The consumer chooses consumption, leisure, investment, and rates of capital and foreign bond accumulation to maximize (3) subject to the accumulation equations, (5) and (6). The corresponding first-order optimality conditions with respect to the first three decisions are:

$$C_i^{\gamma-1} l_i^{\theta\gamma} = \lambda_i \tag{7a}$$

$$\theta C_i^{\gamma} l_i^{\theta\gamma-1} = \lambda_i \omega(L) K \tag{7b}$$

$$\frac{I_i}{K_i} = \frac{q_i - 1}{h} \tag{7c}$$

where q_i is agent i's shadow price of capital, normalized by her marginal utility of wealth, λ_i.

The optimality conditions with respect to B_i and K_i yield the following arbitrage relationships:

$$r = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{8a}$$

$$\frac{r_k}{q_i} + \frac{\dot{q}_i}{q_i} + \frac{(q_i-1)^2}{2hq_i} = r \tag{8b}$$

Equation (8a) is the standard Keynes-Ramsey consumption rule equating the rate of return on consumption to the exogenous rate of return on foreign bonds. Since both r and β are constant, the marginal utility λ_i grows at a constant rate. Equation (8b) equates the net rate of return on domestic capital – the sum of the flow return, capital gains, and benefits from reduced installation costs associated with the new investment and higher capital stock – to the return of the traded bond. In addition, the following transversality conditions must hold:

$$\lim_{t\to\infty} q_i \lambda_i K_i e^{-\beta t} = 0 \tag{9a}$$

$$\lim_{t\to\infty} \lambda_i B_i e^{-\beta t} = 0 \tag{9b}$$

Aggregate labor market clearing condition implies:

$$\sum_j L_j = L = 1 - l = \int_0^1 (1 - l_i) di \tag{10}$$

Using this condition, asset returns, which we have expressed in terms of L, can equally well be written as functions of $(1-l)$, namely

$$\omega(l) \equiv f'(L) \equiv f'(1-l),\ r_k(l) \equiv f(L) - Lf'(L) \equiv f(1-l) - (1-l)f'(1-l)$$

implying $(1-l)\omega(l) + r_k(l) = f(1-l) \equiv f(L)$.

2.4 Macroeconomic equilibrium

From the optimality conditions, together with the individual's accumulation equation, and the corresponding conditions for the aggregate economy, we can derive the macroeconomic equilibrium, showing that the economy is in fact always on its balanced growth path. Details of these derivations are provided in the Appendix A.1, where we show that the macroeconomic equilibrium is summarized by the pair of equations

$$\dot{l} = \frac{l}{\Sigma(l)}\left[r - \beta - (1-\gamma)\frac{(q-1)}{h}\right] \tag{11a}$$

$$\frac{\dot{q}}{q} = r - \frac{r_k(l)}{q} - \frac{(q-1)^2}{2hq} \tag{11b}$$

where $\Sigma(l) \equiv \dfrac{1-\gamma(1+\theta)}{l} + (1-\gamma)\dfrac{(1-l)f'}{(1-l)f} + \dfrac{1-\gamma}{1-l} > 0$

As a consequence of the homogeneity of the utility function, the structure of this equilibrium is *independent* of any distributional characteristics. It comprises an autonomous pair of differential equations in the aggregate variables, q, l, and is similar to Turnovsky (1999). Linearizing these equations around steady state we can show that the two eigenvalues are negative. Thus, in response to any structural change, q and l always jump so that the aggregate economy is always in steady state. Setting $\dot{l} = \dot{q} = 0$ in (11a) and (11b), the aggregate equilibrium values of $\tilde{q}, \tilde{l}$ as well as growth rate $\tilde{\psi}$, are determined by

$$\tilde{\psi} = \frac{r-\beta}{1-\gamma} = \frac{\tilde{q}-1}{h} \tag{12a}$$

$$\frac{r_k(\tilde{l})}{\tilde{q}} + \frac{(\tilde{q}-1)^2}{2h\tilde{q}} = r \tag{12b}$$

With $l(t)$ being constant at all points of time, it then follows from (A.4) that $l_i(t)$ is also constant over time. In addition, the transversality condition (9a) implies $r > \tilde{\psi}$. Using (12a) and combining this the requirement that the equilibrium price be non-negative, imposes the constraints[10]

$$0 < \tilde{q} < 1 + hr \text{ or equivalently } \frac{1-\gamma}{h} + r > \beta > \gamma r$$

Equation (12a) implies that the equilibrium growth rate is determined by the difference between the world interest rate and the rate of time preference, along with the intertemporal elasticity of substitution. Given this growth rate, this equation also determines the price of capital that ensures that the domestic capital stock will grow at this equilibrium rate. Having obtained $\tilde{q}$ (12b) then determines the allocation of time to labor such that the rate of return on capital equals the given world interest rate.

3. Distributions of wealth, income, and welfare

Through defining three measures of inequality, we derive in this section several key results and provide the intuition behind them. We first show that given initial bond and capital endowments, an agent's relative wealth is endogenously determined by the shadow price of capital, and remains constant over time, beyond any upon-impact adjustments to shocks. Structural changes in the

[10] These restrictions hold under plausible conditions. They certainly hold if $\gamma < 0$, i.e. if the intertemporal elasticity of substitution is less than one, a condition that virtually all empirical studies confirm; see e.g. Guvenen (2006).

economy, through affecting the equilibrium ratio of bonds to capital, may thus affect the agent's relative wealth. The exact impact on overall inequality, however, will depend on the relative distribution of bond versus capital endowments. We also demonstrate that wealthier agents with larger capital endowment supply less labor, leading to an exact offsetting effect on overall growth rate, which is thus decoupled from any distributional characteristics. Examining the correlation between wealth and income inequality more fully, we show that wealthier individuals do not necessarily have above average income; the relationship depends on the cost of adjustment for capital as it affects the rate of return for capital investment relative to labor. Overall, the initial distributions of bond and capital endowments matter crucially for determining how the wealth and income distributions change after structural shocks and how they relate to aggregate growth, a message we illustrate more fully in Sections 5 and 6. Finally, this structural approach also allows us to consider welfare inequality, which, due to the concavity of the utility function we show is always less than wealth inequality, and also smaller than income inequality when the growth rate is positive.

3.1 Distribution of wealth and its decoupling with the growth rate

We first turn to the distribution of wealth. As established in Appendix A.1, we see that q is constant over time and identical across agents. The wealth of agent i at time t can be defined by

$$V_i(t) \equiv B_i(t) + qK_i(t) \tag{13}$$

and the initial wealth is

$$V_i(0) \equiv B_{i,0} + qK_{i,0} \tag{13'}$$

This definition shows that given asset endowments, $K_{i,0}, B_{i,0}$, initial wealth is endogenously determined in response to a structural change, through the response of q. Summing (13) and (13') over all agents, the corresponding aggregate quantities are, respectively

$$V(t) \equiv B(t) + qK(t) \tag{14}$$

$$V(0) \equiv B_0 + qK_0 \tag{14'}$$

Denoting individual i's share of aggregate wealth as: $v_i \equiv V_i / V$, we focus on two key results (detailed derivations are in Appendix A.2.) First, we see that $\dot{v}_i(t) \equiv 0$, implying that

$$\frac{B_i + qK_i}{B + qK} = \frac{B_{i,0} + qK_{i,0}}{B_0 + qK_0}$$

This condition tells us that following a structural change and the associated initial jump in q, there is no further evolution of the distribution of wealth.

Second, an agent's labor supply and her relative wealth position obey the following condition:

$$l_i - l = \left(l - \frac{\theta}{1+\theta} \right)(v_i - 1) \tag{15a}$$

where the transversality condition implies

$$l > \frac{\theta}{1+\theta} \tag{15b}$$

Equations (15) demonstrate a positive relationship between relative wealth and leisure, such that the relative wealth position of agents is unchanging over time. Since wealth of all agents grows at the same rate, the share of agent i, $v_i(t)$, at any point in time remains equal to her initial share, $v_{i,0}$. Equation (15a) illustrates the key mechanism whereby the agent's initial relative wealth impacts on the distribution of income. Wealthier agents have a lower marginal utility of wealth. They therefore choose to supply less labor and to "buy" more leisure. In effect, they compensate for their larger capital endowment, and the higher growth rate it would support, by providing less labor, thereby having an exactly offsetting effect on the growth rate, which is therefore independent of the distribution of capital.

3.2 Intertemporal viability

Before characterizing relative wealth and income further, we look at the intertemporal viability condition that each agent also needs to satisfy. In Appendix A.3 we show that the transversality condition (9b) reduces to

$$V_i(0) = \frac{\omega(l)\left[1 - l_i - \dfrac{l_i}{\theta}\right]K_0}{\psi - r} \tag{16}$$

Combining it with (15a), we obtain the following equivalent to the aggregate viability condition

$$V(0) = \frac{\omega(l)\left[1 - l - \dfrac{l}{\theta}\right]K_0}{\psi - r} \tag{16'}$$

which shows that the aggregate economy is viable if and only if each individual is intertemporally viable. Combining (15b) and (16'), we can see that the transversality condition, together with aggregate solvency, implies that the economy has net positive wealth, $B_0 + qK_0 > 0$.

The intertemporal viability condition (16') can be expressed as follows:

$$V(0) + \frac{\omega(l)(1-l)K_0}{r - \psi} = \frac{\omega(l)\left(l/\theta\right)K_0}{r - \psi}.$$

This condition asserts that initial wealth plus the capitalized value of labor income must equal the capitalized value of consumption. Combining it with (14'), we obtain a key condition regarding the initial relative asset composition for viability:

$$\frac{B_0}{K_0} = \frac{\omega(l)\left(\dfrac{l}{\theta} + l - 1\right)}{r - \psi} - q \tag{17}$$

Once the equilibrium values of f $\tilde{l}, \tilde{q}$ are determined as discussed in section 2.4, equation (17) determines the combination of the initial capital stock, K_0, and the initial stock of foreign bonds, B_0, necessary for the equilibrium to be inter-temporally viable.[11] If the inherited stocks of these assets violate (17), we assume that the government engages in an initial trade, described by $dB_0 + \tilde{q}dK_0 = 0$, to bring about the correct ratio.[12]

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[11] Note that for a creditor country ($B_0 > 0$), (17) implies $f'(l)\left(l/\theta\right) + f'(l)(l-1) > qr - q(q-1)/h$, while (2b) and (8b) yield $f - f'(l)(l-1) = qr - (q-1)^2/2h$. Combining these relationships we find $f(l) < f'(l)\left(l/\theta\right) + \left(q^2 - 1\right)/2h$. That is, production must be less than the sum of consumption and the investment cost.

[12] This is a manifestation of the "knife-edge condition" that plagues all small open economy having access to a perfectly competitive world financial market; see Turnovsky (2002). In a stationary model it reduces to the condition $r = \beta$,

3.3 Initial asset compositions and the relative wealth and income

An important message we emphasize in this paper is that unlike the closed-economy finding that growth and inequality are positively related, the distributions of agents' initial endowment composition plays a key role in the open economy in affecting the growth-inequality relation. Below we define a few variables to help illustrate this result, which we also discuss further in Section 6.

Using (13) and (14), the relative wealth of individual *i*, $v_i \equiv V_i / V$, can be expressed as a weighted average of her relative ownership of capital and bonds, $k_i \equiv K_i / K$ and $b_i \equiv B_i / B$, namely

$$v_i - 1 = \frac{B}{B + qK}(b_i - 1) + \frac{qK}{B + qK}(k_i - 1) \tag{18}$$

Since individual and aggregate variables all share the same growth rates, we can express (18) in terms of the initial endowments:

$$v_i - 1 = \Gamma_b (b_{i,0} - 1) + \Gamma_k (k_{i,0} - 1) \tag{18'}$$

where

$$\Gamma_b \equiv \frac{B_0 / K_0}{B_0 / K_0 + q} = 1 - \Gamma_k$$

That is, Γ_b measures the relative importance of the initial distribution of bond holdings in determining the agent’s relative wealth, and B_0 / K_0 is determined by the intertemporal viability condition (17). If $B_0 > 0$, and the country is a net creditor, $0 < \Gamma_b < 1,\ 0 < \Gamma_k < 1$.

From (18’) we see that a structural change that leads to a change in the equilibrium ratio of bonds to capital will in general have an effect on agent *i*’s relative wealth. The direction of the effect will depend upon the agent’s relative endowment of capital and bonds. For example, an increase in the ratio of bonds to capital, Γ_b, will raise v_i if and only if $b_i > k_i$; that is, if the agent holds relatively more bonds than capital her relative wealth will rise. This in turn means that if the agent’s

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which implies complete consumption smoothing. In a growth model with inelastic labor supply it implies that production and consumption grow at different constant rates, which are reconciled by international capital movements. With endogenous labor supply, adjustment in the allocation of leisure drives the two growth rates to equality, but in order for the equilibrium to be sustainable, B_0, K_0 must be adjusted to satisfy the intertemporal budget constraint (17).

wealth is above the average, wealth inequality will rise, while if it is below, it will fall.

We now consider the relative income of an individual having capital stock, K_i, and bond holdings, B_i. As noted in Section 2, her gross income from production and foreign interest is $Y_i = r_k K_i + wK(1-l_i) + rB_i$, and the average economy-wide income is $Y = r_k K + wK(1-l) + rB$. Using (15a) to substitute for the individual's labor supply, we can write the relative income of agent i, $y_i \equiv Y_i/Y$, as

$$y_i - 1 \equiv \frac{Y_i}{Y} - 1 = \frac{r_k(l)K}{f(1-l)K + rB}(k_i - 1) - \frac{\omega(l)\left(l - \frac{\theta}{1+\theta}\right)K}{f(1-l)K + rB}(v_i - 1) + \frac{rB}{f(1-l)K + rB}(b_i - 1) \qquad (19)$$

Using (18') and the equilibrium growth rate, this can be expressed in terms of the initial endowments

$$y_i - 1 = \Omega_b(b_{i,0} - 1) + \Omega_k(k_{i,0} - 1) \qquad (19')$$

where the respective contributions to relative income from bonds versus capital endowment are

$$\Omega_b \equiv \frac{B_0/K_0}{f + r\,B_0/K_0}\left[r - (r-\psi)\frac{\theta}{1+\theta}\right]; \quad \Omega_k \equiv \frac{1}{f + r\,B_0/K_0}\left[r_K - q(r-\psi)\frac{\theta}{1+\theta}\right]$$

and r_k, ψ, q are determined from the aggregate equilibrium. In addition we impose the weak condition, $fK_0 + rB_0 > 0$, implying that the country has positive gross income. For a net creditor economy, $B_0 > 0$, and for positive equilibrium growth rate, we can show $\Omega_b > 0, \Omega_k > 0$.

3.4. The role of capital adjustment cost

Looking at equations in the sub-section above, we see that in the presence of adjustment costs ($h > 0$),

$$\frac{\Omega_k}{\Omega_b} < \frac{q}{B_0/K_0} = \frac{\Gamma_k}{\Gamma_b} \qquad (20)$$

while in the absence of such costs ($h = 0$),

$$\frac{\Omega_k}{\Omega_b} = \frac{1}{B_0/K_0} = \frac{\Gamma_k}{\Gamma_b} \qquad (20')$$

These relationships have implications for the comparative positions of individuals in the relative wealth and income distributions. Specifically, with no adjustment costs and a net creditor economy, (18'), (19') and (20') imply

$$y_i - 1 = \frac{\Omega_b}{\Gamma_b}(v_i - 1)$$

This shows that an agent having above (below) average wealth also has above (below) average income. And this will be the case if the agent has above (below) average endowments of both assets. However, in the presence of adjustment costs, and if, $k_{i,0} - 1$, $b_{i,0} - 1$ lie in the range

$$-\frac{\Gamma_k}{\Gamma_b} < \frac{b_{i,0} - 1}{k_{i,0} - 1} < -\frac{\Omega_k}{\Omega_b} < 0 \tag{21}$$

individual i will have above average wealth, but below average income. This is because with high adjustment costs ($h >> 0$), return from capital investment r_k is driven down by arbitrage, which disadvantages wealth owners relative to labor.

3.5 Measuring wealth and income inequality

We can measure wealth inequality by the standard deviation of its distribution, σ_v, which, given the linearity of (18'), can be conveniently related to the distributions of the initial asset endowments, $\sigma_{k,0}, \sigma_{b,0}$ across the agents as well as their covariance, $\sigma_{bk,0}$, as follows:

$$\sigma_v = \left[\Gamma_b^2 \sigma_{b,0}^2 + 2\Gamma_b \Gamma_k \sigma_{bk,0} + \Gamma_k^2 \sigma_{k,0}^2\right]^{1/2} \tag{22}$$

Likewise, income inequality is given by an analogous measure

$$\sigma_y = \left[\Omega_b^2 \sigma_{b,0}^2 + 2\Omega_b \Omega_k \sigma_{bk,0} + \Omega_k^2 \sigma_{k,0}^2\right]^{1/2} \tag{23}$$

Expressed in this way we see that Γ_b, Ω_b are the contributions to wealth and income inequality arising from the initial heterogeneity of endowments in bonds, while the same apply to Γ_k, Ω_k with respect to the capital endowment. In general, these components will respond to structural changes.

Of particular interest is the case where each agent's relative holding of bonds coincides with her relative holdings of capital ($b_{i,0} = k_{i,0}$), so that the heterogeneity of the asset holdings are uniform across agents (i.e. $\sigma_{b,0}^2 = \sigma_{k,0}^2 = \sigma_{bk,0}$). In this case (22) and (23) simplify to

$$\sigma_v = \sigma_{k,0} = \sigma_{b,0} \tag{22'}$$

$$\sigma_y = (\Omega_b + \Omega_k)\sigma_{k,0} = \left(1 - \frac{f'}{(f + r B_0/K_0)(1+\theta)}\right)\sigma_{k,0} \tag{23'}$$

In this case σ_v is independent of any structural changes, while income inequality is less than wealth inequality. Substituting (17) into (23'), we can obtain the following

$$\frac{\sigma_y}{\sigma_v} = \frac{y_i - 1}{v_i - 1} = \frac{f'(r+\theta\psi)(l - (\theta/(1+\theta))) - (\theta h/2)(r-\psi)\psi^2}{\{f'[rl - \theta(1-l)\psi] - (\theta h/2)(r-\psi)\psi^2\}} \tag{24}$$

While in general, $0 < (y_i - 1)/(v_i - 1) < 1$, we cannot rule out $(y_i - 1)/(v_i - 1) < 0$. From (24) this will occur for l lying in the range

$$l^* > l > \frac{\theta}{1+\theta} \tag{25a}$$

where

$$f'\frac{(r+\theta\psi)}{\theta}\left(l^* - \frac{\theta}{1+\theta}\right) = \frac{h}{2}(r-\psi)\psi^2 > 0 \tag{25b}$$

Thus, inequality (25a) may occur if the adjustment cost term in (25b) is sufficiently large. In this case, we obtain a reversal in ranking for income from wealth, i.e. an individual having *above* average wealth will have *below* average income. With the agent having above (below) endowments in both assets this can be reconciled with (21) by observing that (25b) implies that the country is a net debtor. Finally, if there no adjustment costs, $h = 0$, and (25b) reduces to the transversality condition (15b).

3.5 Welfare Inequality

The structural approach allows us to compute individual welfare and to analyze its distribution. By definition, an agent's welfare equals the value of the inter-temporal utility function (3) evaluated along the equilibrium growth path. Substituting (A.1) into (3) and evaluating, the optimized utility for an agent starting with asset endowments, $K_{i,0}, B_{i,0}$ can be expressed as

$$X(K_{i,0}) = \frac{1}{\gamma}\int_0^\infty (C_i l_i^\theta)^\gamma e^{-\beta t} dt = \frac{1}{\gamma}\int_0^\infty \left(\frac{\omega(L) l_i K}{\theta} l_i^\theta\right)^\gamma e^{-\beta t} dt = \frac{\omega(L) l_i^{\gamma(1+\theta)}}{\gamma \theta^\gamma (\beta - \gamma\psi)} K_0^\gamma \tag{26}$$

The welfare of agent *i* relative to that of the individual with average wealth and therefore supplying average leisure, l, is

$$x(v_i) = \left(\frac{l_i}{l}\right)^{\gamma(1+\theta)} \tag{27}$$

Now using (15a), we can express relative welfare in the form

$$x(v_i) = \left[1 + \left(1 - \frac{\theta}{1+\theta}\frac{1}{l}\right)(v_i - 1)\right]^{\gamma(1+\theta)} \tag{28}$$

Consider now two individuals having relative wealths $v_2 > v_1$. Individual 2 will have a higher mean income. The transversality condition (15b) implies that if $\gamma > 0$, then their relative welfares satisfy $x(v_2) > x(v_1) > 0$, while if $\gamma < 0$, $x(v_1) > x(v_2) > 0$. However, in the latter case absolute welfare, as expressed by (26) is negative. Thus in either case, the better endowed agent will have the higher absolute level of welfare.

We can now compute a measure of welfare inequality. A natural metric for this is obtained by applying the following monotonic transformation of relative lifetime utility, enabling us to express the relative utility of individual *i* as

$$x(v_i)^{1/\gamma(1+\eta)} = u(v_i) = 1 + \varphi(l)(v_i - 1) \quad \text{where} \quad \varphi(l) \equiv 1 - \frac{\theta}{1+\theta}\frac{1}{l} \tag{29}$$

From (15a), $0 < \varphi(l) < 1$, and is an increasing, concave function in *l*. Welfare inequality, expressed in terms of equivalent units of wealth, can then be measured by the standard deviation of relative utility

$$\sigma_u = \varphi(l)\sigma_v \tag{30}$$

It is straightforward to show that if growth is positive then $\sigma_v > \sigma_y > \sigma_u$, so that income inequality exceeds welfare inequality. The reason is the concavity of the utility function so that increases in income yield proportionately smaller increases in welfare.

4. Summary of equilibrium

As noted, the economy is always on its balanced growth path, which for convenience we may summarize as follows, highlighting its recursive structure:

Equilibrium growth rate

$$\psi = \frac{r-\beta}{1-\gamma} \tag{31a}$$

Equilibrium rates of return

$$r_k = r(1+h\psi) - \frac{h}{2}\psi^2 \tag{31b}$$

Factor returns[13]

$$r_k(l) = A[f(1-l)-(1-l)f'(1-l)]; \qquad w(l) = Af'(1-l) \tag{31c}$$

Intertemporal solvency

$$\frac{B_0}{K_0} = \frac{\omega(L)\left[l\left(1+\frac{1}{\theta}\right)-1\right]}{r-\psi} - q \tag{31d}$$

Wealth inequality

$$\sigma_v = \left[\Gamma_b^2\sigma_{b,0}^2 + 2\Gamma_b\Gamma_k\sigma_{bk,0} + \Gamma_k^2\sigma_{k,0}^2\right]^{1/2} \tag{31e}$$

where

$$\Gamma_k = \frac{q}{q+B_0/K_0}, \quad \Gamma_b = \frac{B_0/K_0}{q+B_0/K_0} = 1-\Gamma_k$$

Income inequality

[13] Here we added in the productivity parameter A, which we set to unity in Section 2.1.

$$\sigma_y = \left[\Omega_b^2\sigma_{b,0}^2 + 2\Omega_b\Omega_k\sigma_{bk,0} + \Omega_k^2\sigma_{k,0}^2\right]^{1/2} \tag{31f}$$

where

$$\Omega_k = \frac{1}{f + rB_0/K_0}\left[r_k - q(r-\psi)\frac{\theta}{1+\theta}\right],\ \ \Omega_b = \frac{B_0/K_0}{f + rB_0/K_0}\left[r - (r-\psi)\frac{\theta}{1+\theta}\right]$$

Welfare inequality

$$\sigma_u = \left(1 - \frac{\theta}{1+\theta}\frac{1}{l}\right)\sigma_v \tag{31g}$$

In addition the following restrictions apply:

1. The transversality condition implies $r > \psi$, and in turn is equivalent to (15a) and $B_0 + qK_0 > 0$ (country has positive net wealth).
2. $f + rB_0/K_0 > 0$ (country has positive net income).

5. Structural changes and growth-inequality correlation

In this section, we examine the growth-inequality relation in response to three structural shocks: (i) an increase in productivity, *A*; (ii) an increase in savings generated by a reduction in the rate of time discount, β; and (iii) an increase in the world interest rate, *r*. The three examples are chosen to illustrate our main message: the correlation between growth and income inequality can be positive, negative or even zero in our small open economy, unlike the positive relationship between found under a closed-economy setting [García-Peñalosa and Turnovsky, 2006]. As discussed earlier, the growth and distributional consequences of structural shocks depend crucially on the source of the underlying heterogeneity: the initial distributions of the allocations of international bonds and capital. In addition, the relation between wealth inequality and income inequality depends on the size of the adjustment cost, as discussed in Section 3.4. Below we consider two cases: (i) no adjustment costs, $h = 0$, and (ii) positive adjustment costs. We also restrict ourselves to the case of a uniform initial asset endowment, $b_{i,0} = k_{i.0}$ for each *i*.

5.1 Equilibrium under no adjustment costs

Setting $h = 0$, involves two modifications to the equilibrium laid out in (31). First, the rate of return to capital, $r_k(l) = r$, is constant, implying that employment/leisure is determined by this equilibrium condition as well. Second, the absence of adjustment costs implies $q = 1$.

The PP and RR lines in Fig.1 plot (31a) and (31b), respectively, with $h = 0$ in the latter. Their point of intersection, A, determines the initial equilibrium growth rate. The locus FF relates the rate of return to capital to *l*, in accordance with (31c). Its concavity is a direct property of the production function. Given $r_K = r$, the corresponding allocation of time to leisure is denoted by *B*. The lower right quadrant plots income and welfare inequality by WY and WU, respectively, in both cases the (fixed) wealth inequality being normalized to $\sigma_v = 1$. These represent (31f) [after substituting for (31e)] and (31g) respectively and can be easily shown to have the indicated concavity properties.[14] The equilibrium income inequality depicted by *D* exceeds the welfare inequality, *C*, consistent with the formal properties derived earlier.

5.1.1 Increase in productivity *A*

A positive productivity shock is represented by an outward rotation in the FF locus to FF'. The equilibrium growth rate remains unchanged and the enhanced productivity is enjoyed in the form of increased average leisure illustrated by the move from B to B'. The increase in leisure (decrease in labor supply) raises the wage rate, in response to which wealthier people take relatively more leisure [see (15a)]. Their relative labor income therefore declines. However, an increase in their relative interest income tends to offset this drop. As dictated by the intertemporal viability condition [see (17) or (31d)], the increase in the wage rate and the average consumption flow it generates requires an increase in the relative foreign bond holding. This is met by selling capital, and the resulting interest income tends to increase income inequality. Overall, this latter effect dominates, and income inequality increases from D to D'. Correspondingly, welfare inequality

[14] WU can be immediately shown to be concave. WY will be concave if and only if $r + \theta\psi \geq 0$, which will hold except in extremes cases of contraction.

increases from C to C'. Accordingly, a rise in productivity results in no growth effect, yet it increases both income and welfare inequality. We emphasize that this result contrasts from the closed economy outcome, in which a higher productivity raises both the growth rate and income inequality (see García-Peñalosa and Turnovsky 2006.)

5.1.2 Decrease in rate of time preference β

In Figure 1, a decline in β is represented by a leftward shift in the PP curve to P'P' and a downward rotation in the WY curve to WY'. As agents become more patient, savings increase, booting the equilibrium growth rate to A". The labor-leisure choice is unaffected as it is pinned down by factor returns and the foreign interest rate, which remain unchanged.[15] With leisure unchanged at B, welfare inequality remains at C. The higher growth rate raises the capitalized value of the trade deficit - the difference between consumption and labor income - which requires more bonds to finance according to the intertemporal viability condition (31d). This leads to an increased income inequality from D to D". Under this scenario, a positive growth is accompanied with higher income inequality but no change in the distribution of welfare.

5.1.3 Increase in foreign interest rate, *r*

A higher world interest rate raises labor supply and the growth rates domestically. In Figure 1, this corresponds to a shift in the RR curve upward to R'R' and the PP leftward to P'P'. As the equilibrium shifts from from A to A"', both the return to capital and the growth rate increase. The higher return to capital is the result of the increase in labor supply, and the corresponding reduction in leisure from B to B"' reduces welfare inequality from C to C"'. Graphically, this corresponds to a rotation of curve WY downward to WY'. The new equilibrium level of income inequality is at D"'. While this is drawn in Fig. 1 as a reduction in inequality, whether D"' lies above or below D depends, in part, upon the elasticity of substitution in production, as we demonstrate Section 6. In short, an exogenous rise in the foreign interest rate enhances growth domestically, has an ambiguous

[15]This result again contrasts with the closed-economy outcome, where a drop in the discount rate reduces leisure and raises the return to capital; See García-Peñalosa and Turnovsky (2006, p.38).

effect on income inequality, yet tightens the welfare distribution.

5.2 Positive adjustment costs

The three examples above demonstrate that the growth-inequality relation – whether positive, negative, or unrelated – depends crucially on the sources of the shocks and channels of transmission. Fig. 2 analyzes the same structural changes and illustrates the relevant equilibria under positive adjustment costs. The two main differences from Fig. 1 are that the RR curve now describes the quadratic locus (31b), and that the WY curve now commences from a value $(y_i - 1) < 0$, in accordance with (25a). In order to satisfy the transversality condition, $r > \psi$, the intersection of the PP and RR curves in the NW quadrant must always be to the right of the dashed line. Thus the initial equilibrium growth rate and rate of return on capital is at A. Given that value of r_K, B yields the corresponding equilibrium leisure, with C and D yielding the resulting welfare and income inequality. The three structural changes can be represented by shifts in the various curves as in Fig. 1, and generally the same qualitative conclusions prevail.

6. Numerical Results

To obtain further insights into the growth-income inequality relationship we provide some numerical examples. In doing so, we employ the aggregate equilibrium CES production function, $Y = A\left[\alpha L^{-\rho} + (1-\alpha)\right]^{-1/\rho} K$, where $\varepsilon = 1/(1+\rho)$ denotes the elasticity of substitution. We use the following, mostly standard, parameter values:

Parameter Values

Production	$A = 0.65;\ \alpha = 0.65;\ \varepsilon = 0.8, 1.0, 1.2;\ h = 0, 10$
Preferences	$\beta = 0.04;\ \gamma = -1.5;\ \theta = 1.5$
Foreign interest rate	$r = 0.075$

The choice of distributive parameter $\alpha = 0.65$ is conventional. It implies that for the Cobb-Douglas case 65% of output accrues to labor, consistent with empirical evidence. The rate of time preference of 4% is commonly used in calibrations for the US economy, while the choice of the elasticity on leisure, $\eta = 1.5$, is standard in the real business cycle literature. For $\varepsilon = 0.8$, a value close to recent consensus estimates, it implies that around 72% of time is devoted to leisure, generally consistent with empirical evidence. Estimates of the intertemporal elasticity of substitution (IES) are more variable throughout the literature. With few exceptions they lie in the range (0,1) and our choice of 0.4 is in line with early estimates based on consumption.[16] The choice of the scale parameter A = 0.65, is set to yield a plausible value for the equilibrium capital-output ratio.

Tables 1 and 2 report the equilibrium growth rates, and measures of wealth and income inequality in the case of (i) zero adjustment costs and (ii) $h = 10$, respectively. In both cases we consider three values of the elasticity of substitution, $\varepsilon = 0.8,\ 1,\ 1.2$. Taking the Cobb-Douglas case, $\varepsilon = 1$ as a benchmark, the comparison with $\varepsilon = 0.8$, ε=1.2 suggests that even small deviations from this benchmark, well within typical sampling errors, can yield different implications for the growth-inequality tradeoff.

In each panel, the first line reports the benchmark equilibrium for our base parameters. In addition to reporting the equilibrium growth rate and time allocation to leisure, it all indicates the sensitivity of wealth inequality and income inequalilty to the underlying heterogeneity in the endowments of capital and bonds. The following three lines report sequentially (i) an increase in the level of technology, A, from 0.65 to 0.75; (ii) a decrease in the rate of time preference from 0.04 to 0.02; (iii) an increase in the world interest rate from 0.075 to 0.085.

Examination of the tables suggests the following pattern of numerical effects, which clearly illustrate our message that in an open economy, the relation between growth and inequality depends on the source of structural shocks, the size of the adjustment costs, and most importantly, on the relative distributions of endowments of bonds and capital. There is no consistent correlation as observed in the closed economy.

6.1 Increase in *A*

(1a) An increase in A will decrease (increase) wealth inequality if the endowment of

[16]More recent empirical evidence, based on stock market data, suggests that it is substantially higher, perhaps 2/3 or even higher; see, for example, the discussion in Guvenen, (2006). Our qualitative conclusions are insensitive to the chosen value of γ.

capital is more (less) unequal than that of bonds. The sensitivity declines with higher elasticity of substitution. This applies independently of adjustment costs.

(1b) An increase in A reduces the impact of capital stock inequality on income inequality (Ω_k), but increases the impact of bond inequality on income inequality (Ω_b). Overall, the latter effect is dominant, so that with uniform asset inequality ($b_{i,0} = k_{i,0}$), an increase in A raises income inequality. In all cases, the sensitivity declines with higher elasticity of substitution and is independent of adjustment costs.

(1c) Since the growth rate is independent of A, there is no growth-inequality tradeoff.

6.2 Decrease in β

(2a) In the absence of adjustment costs, a decrease in β will decrease (increase) wealth inequality if the endowment of capital is more (less) unequal than that of bonds. With adjustment costs, a decrease in β will increase (decrease) wealth inequality if the endowment of capital is more (less) unequal than that of bonds. In this case the adjustment costs give a reversal. The sensitivity declines with the elasticity of substitution.

(2b) In the absence of adjustment costs, a decrease in β always increases the impact of bond inequality and generally that of capital inequality on income inequality. In all cases, the sensitivity declines with the elasticity of substitution, and indeed for $\varepsilon \geq 1.2$ a decrease in β causes the effect of capital inequality to decline. In all cases, the effect of bond inequality dominates so that with uniform inequality, a decrease in β raises income inequality.

With adjustment costs, a decrease in β decreases the impact of bond inequality on income inequality for low values of ε, and increases it if ε is sufficiently large (certainly greater than unity). In all cases it increases the effect of capital inequality on income inequality. In all cases, the bond effect dominates, so that with uniform inequality, a decrease in β reduces income inequality for low values of ε and increases it as ε increases.

(2c) In most cases the increase in the growth rate resulting from a decrease in β is associated with an increase in income inequality, implying a positive growth-inequality tradeoff. But there are exceptions; for example if there are adjustment costs and the elasticity of substitution is low.

6.3 Increase in r

(3a) An increase in r will increase (decrease) wealth inequality if the endowment of capital is more (less) unequal than that of bonds. The sensitivity declines with the elasticity of substitution. For low values of ε, in the presence of adjustment costs, it causes the country to become an international debtor.

(3b) An increase in r increases the impact of capital inequality and decreases that of bond inequality on income inequality. In all cases, the effect of bond inequality dominates so that with uniform inequality, an increase in r reduces income inequality. The sensitivity declines with the elasticity of substitution.

(3c) To the extent that income inequality is due to heterogeneity of capital (bond) endowment, an increase in r generates a positive (negative) growth-income inequality In the presence of adjustment costs and tradeoff. In all cases the bond effect dominates, so that with uniform inequality, an increase in r generates a negative growth-inequality tradeoff.

7. Concluding comments

Despite intensive research effort, the relationship between growth and inequality remains largely unresolved. Empirical evidence is inconclusive, some studies finding these two variables to be negatively related, while others obtain a positive relationship. Upon reflection, the ambiguity of the empirical findings is not surprising when one considers that both variables are endogenous, so that their co-movements are likely to depend upon the underlying structural changes impinging on the economy.

The existing literature analyzes the growth-inequality relationship within a closed economy. In this paper we have considered a small open economy which has access to a perfectly competitive world financial market. Output is produced by an AK technology and inequality is driven by agent heterogeneity, the particular form of which we consider pertains to the agents' initial endowments of capital and foreign bond holdings. The key mechanism whereby this initial distribution of capital endowments influences the distribution of income is through their differential wealth effects, and their impact on labor supply.

The extension to the small open economy enriches the potential growth-inequality tradeoffs from those of the corresponding closed economy. First, to the extent that the relative endowments of capital differs from that of bonds, structural shocks will affect wealth inequality, as well as income inequality, by impacting on the relative price of capital. Overall, whether a structural change is associated with a positive or negative growth-inequality tradeoff depends critically upon whether the underlying heterogeneity originates primarily with the initial endowment of bonds or with capital.

In this respect the ambiguity of our results contrasts with that of the analogous closed economy model, which tended to support a positive relationship between inequality and growth for most structural changes; García-Peñalosa and Turnovsky (2006). But at the same time it is generally more consistent with the ambiguous relationships that characterize the existing empirical evidence.

Finally, we conclude with a caveat. While the simple AK model has the advantage of providing a tractable framework for highlighting the growth-inequality relationship, it also has the limitation that the economy is always on its balanced growth path. It therefore cannot address issues pertaining to the dynamics of wealth and income distribution. This is particularly important if one wishes to study the distributional aspects of foreign aid which clearly evolve gradually over time. Building on the dynamic model of Turnovsky and García-Peñalosa (2008), we believe that the approach developed in this paper can be adapted to provide a tractable framework for investigating this important issue.

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Appendix

A.1 Derivation of macroeconomic equilibrium

We begin by dividing (7b) by (7a) to obtain

$$\theta C_i = \omega(l) l_i K \tag{A.1}$$

and taking the time derivative of this equation yields

$$\frac{\dot{C}_i}{C_i} - \frac{\dot{l}_i}{l_i} = \frac{\omega'(l) l}{\omega(l)} \frac{\dot{l}}{l} + \frac{\dot{K}}{K} \tag{A.2}$$

Next, taking the time derivative of equation (7a) and combining it with (8a) we see that all agents will choose the same growth rate for their marginal utility regardless of their capital endowment:

$$(\gamma - 1)\frac{\dot{C}_i}{C_i} + \theta\gamma \frac{\dot{l}_i}{l_i} = \frac{\dot{\lambda}_i}{\lambda_i} = \beta - r \tag{A.3}$$

Since equations (A.2) and (A.3) hold for all agents *i*, it then follows that all agents will choose the same growth rates for their individual consumption and leisure. Moreover, aggregating over the individuals, it follows that aggregate consumption, *C*, and leisure, *l*, will grow at the same rate, i.e.

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C}; \quad \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l} \text{ for } i \tag{A.4}$$

Aggregating (A.1) over the *i* agents, the following aggregate consumption-capital ratio is obtained:

$$\theta \frac{C}{K} = \omega(l) l \tag{A.1'}$$

We next consider the growth rate of the aggregate capital stock. Combining equation (7c), with (5), we can express individual *i*'s growth rate of private capital as

$$\psi_i(t) \equiv \frac{\dot{K}_i}{K_i} = \frac{q_i - 1}{h} \tag{A.5}$$

which implies that at time *t*, agent *i*'s capital stock has reached the level

$$K_i(t) = K_{i,0} e^{\int_0^t \psi_i(s)ds} \tag{A.6}$$

Combining equations (8a) and (A.3), we can express the transversality condition (9) as

$$\lim_{t\to\infty} q_i(t)\lambda_0 e^{(\beta-r)t} K_{i,0} e^{\int_0^t \psi_i(s)ds} e^{-\beta t} = \lim_{t\to\infty} q_i(t)\lambda_0 K_{i,0} e^{\int_0^t \psi_i(s)ds - rt} = 0 \tag{A.6'}$$

which means that $\int_0^t \psi_i(s)ds < rt$.

Now consider any two individuals i and j and subtract their respective capital returns equation (8b). This yields the following equation

$$(\dot{q}_i - \dot{q}_j) + \frac{(q_i - 1)^2 - (q_j - 1)^2}{2h} = r(q_i - q_j) \tag{A.7}$$

Letting $x \equiv q_i - q_j$, we can rewrite (A.7) in the form

$$\dot{x} + \frac{x(q_i + q_j - 2)}{2h} = rx\ ,$$

and using (A.5) we can obtain the following solution for $x(t)$:

$$x(t) = x_0 e^{rt - \frac{1}{2}\int_0^t (\psi_i(s) + \psi_j(s))ds}\ . \tag{A.8}$$

Since we know from above that $\int_0^t \psi_i(s)ds < rt$ for all i, the only stable solution to (A.8) is $x(t) \equiv 0$, implying that $q_i = q_j = q$ for all individuals i and j, and therefore the aggregate. Hence, the growth rate of aggregate capital equals the common growth rates of all individual capital stocks, namely

$$\psi(t) \equiv \frac{\dot{K}}{K} = \frac{\dot{K}_i}{K_i} = \frac{q-1}{h} \tag{A.5'}$$

Taking the time derivative of equation (1b), and substituting from eq. (A.5'), we obtain

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} - \frac{f'}{f}\dot{l} = \frac{q-1}{h} - \frac{(1-l)f'}{f}\frac{\dot{l}}{(1-l)} \tag{A.9}$$

Next, differentiating the equilibrium consumption-wealth ratio, yields

$$\frac{\dot{C}}{C}-\frac{\dot{Y}}{Y}=\frac{\dot{l}}{l}+\frac{\dot{l}}{1-l} \tag{A.10}$$

We can then combine equations (A.10), (A.9), (A.4) and (A.3), to express the evolution of l in terms of the current level of q and l:

$$(\gamma-1)\left[\frac{q-1}{h}-\frac{(1-l)f'}{f}\frac{\dot{l}}{(1-l)}+\frac{\dot{l}}{l}+\frac{\dot{l}}{1-l}\right]+\theta\gamma\frac{\dot{l}}{l}=\beta-r$$

Rearranging terms in this equation and substituting $q_i = q$ into (8b), the macroeconomic equilibrium is summarized by the pair of equations

$$\dot{l}=\frac{l}{\Gamma(l)}\left[r-\beta-(1-\gamma)\frac{(q-1)}{h}\right] \tag{A.11a}$$

$$\frac{\dot{q}}{q}=r-\frac{r_k(l)}{q}-\frac{(q-1)^2}{2hq} \tag{A.11b}$$

where $\Gamma(l)=\frac{1-\gamma(1+\theta)}{l}+(1-\gamma)\frac{(1-l)f'}{(1-l)f}+\frac{1-\gamma}{1-l}>0$

A.2 Distribution of wealth

With q identical and constant across agents, we define the wealth of agent i, V_i by

$$V_i \equiv B_i + qK_i \tag{A.12}$$

the time derivative of which is

$$\dot{V}_i \equiv \dot{B}_i + q\dot{K}_i \tag{A.13}$$

Using (7c), and (11), and the symmetry condition for q, we can write the individual budget constraint (6) as:

$$\dot{B}_i = \omega(l)[1-l_i-\frac{l_i}{\theta}]K+[r_k-(\frac{q^2-1}{2h})]K_i+rB_i \tag{A.14}$$

Substituting (A.14), (5) and (7c) above, along with equation (8b) into (A.13) yields

$$\dot{V}_i = \omega(l)\left(1-l_i-\frac{l_i}{\theta}\right)K+\left(rq-\frac{(q-1)^2}{2h}-\frac{q^2-1}{2h}+\frac{q(q-1)}{h}\right)K_i+rB_i$$

Enabling us to express the growth rate of agent *i*'s wealth in the form

$$\frac{\dot{V}_i}{V_i} = r+\omega(l)\left(1-l_i-\frac{l_i}{\theta}\right)\frac{K}{V_i} \qquad \text{(A.15)}$$

Aggregating (A.15) over all individuals, the aggregate wealth V grows at:

$$\frac{\dot{V}}{V} = r+\omega(l)\left(1-l-\frac{l}{\theta}\right)\frac{K}{V} \qquad \text{(A.16)}$$

Denoting individual *i*'s share of aggregate wealth as: $v_i \equiv \frac{V_i}{V}$, equations (A.15) and (A.16) imply that its dynamics

$$\frac{\dot{v}_i}{v_i} \equiv \frac{\dot{V}_i}{V_i}-\frac{\dot{V}}{V}$$

can be described by

$$\dot{v}_i(t) = \omega(l)\frac{K}{V}\left[\left(1-l_i-\frac{l_i}{\theta}\right)-\left(1-l-\frac{l}{\theta}\right)v_i(t)\right] \qquad \text{(A.17)}$$

With l_i, l both constants, this is a simple linear differential equation, the properties of which depend upon the coefficient of $v_i(t)$, and which we can determine from the transversality condition. Adding the transversality conditions (9a) and (9b), we see:

$$\lim_{t\to\infty}\lambda(qK_i+B_i)e^{-\beta t} = \lim_{t\to\infty}\lambda V_i e^{-\beta t} = 0$$

Summing over the individuals leads to the aggregate condition:

$$\lim_{t\to\infty}\lambda V e^{-\beta t} = \lim_{t\to\infty}\lambda_0 V e^{-rt} = 0$$

which implies that $\dot{V}/V < r$. It then follows from (A.16), that:

$$l > \frac{\theta}{1+\theta} \qquad \text{(A.18)}$$

Now returning to (A.17) we see from (A.18) that the coefficient of $v_i(t)$ is positive implying that the only solution consistent with long-run stability and the transversality condition is that the right hand side of (A.17) be zero, so that $\dot{v}_i(t) = 0$ for all time. Setting the right hand side of (A.17) to zero, implies that agents select their respective leisure, l_i, in accordance with the "relative labor supply" function

$$l_i - l = \left(l - \frac{\theta}{1+\theta} \right)(v_i - 1) \tag{A.19}$$

From $\dot{v}_i = 0$, we know $\frac{B_i + qK_i}{B + qK} = \frac{B_{i,0} + qK_{i,0}}{B_0 + qK_0}$, so that after an initial jump following a structural change there is no further evolution of the distribution of wealth.

A.3 Intertemporal viability

Solving (A.5) for $K(t) = K_0 e^{\psi t}$ and substituting into equation (A.15), we may rewrite the latter as

$$\dot{V}_i = rV_i + \omega(l)\left(1 - l_i - \frac{l_i}{\theta}\right) K_0 e^{\psi t} \tag{A.20}$$

Starting from the initial condition, $V_i(0)$, the solution to (A.20) is

$$V_i(t) = \left[V_i(0) - \frac{\omega(l)[1 - l_i - \frac{l_i}{\theta}]K_0}{\psi - r} \right] e^{rt} + \frac{\omega(l)\left[1 - l_i - \frac{l_i}{\theta}\right]}{\psi - r} K_0 e^{\psi t} \tag{A.21}$$

In order for the transversality condition, $\lim_{t\to\infty} V_i(t) e^{-rt} = 0$, to hold, we require

$$V_i(0) = \frac{\omega(l)\left[1 - l_i - \frac{l_i}{\theta}\right] K_0}{\psi - r} \tag{A.22}$$

and substituting into (A.21) and aggregating we see

$$\frac{\dot{V}_i(t)}{V_i(t)} = \frac{\dot{V}(t)}{V(t)} = \psi \tag{A.23}$$

so that wealth grows at the same rate as capital. It then follows from (12) and (13) that

$$\frac{\dot{B}_i(t)}{B_i(t)} = \frac{\dot{B}(t)}{B(t)} = \psi \tag{A.24}$$

bonds grow at the same constant rate.

Now consider (A.19) and rewrite it in the form:

$$\left(1 - l_i - \frac{l_i}{\theta}\right)V_0 = \left(1 - l - \frac{l}{\theta}\right)V_i(0) \tag{A.25}$$

Substituting this expression into (A.22) we may re-express the feasibility condition in the form

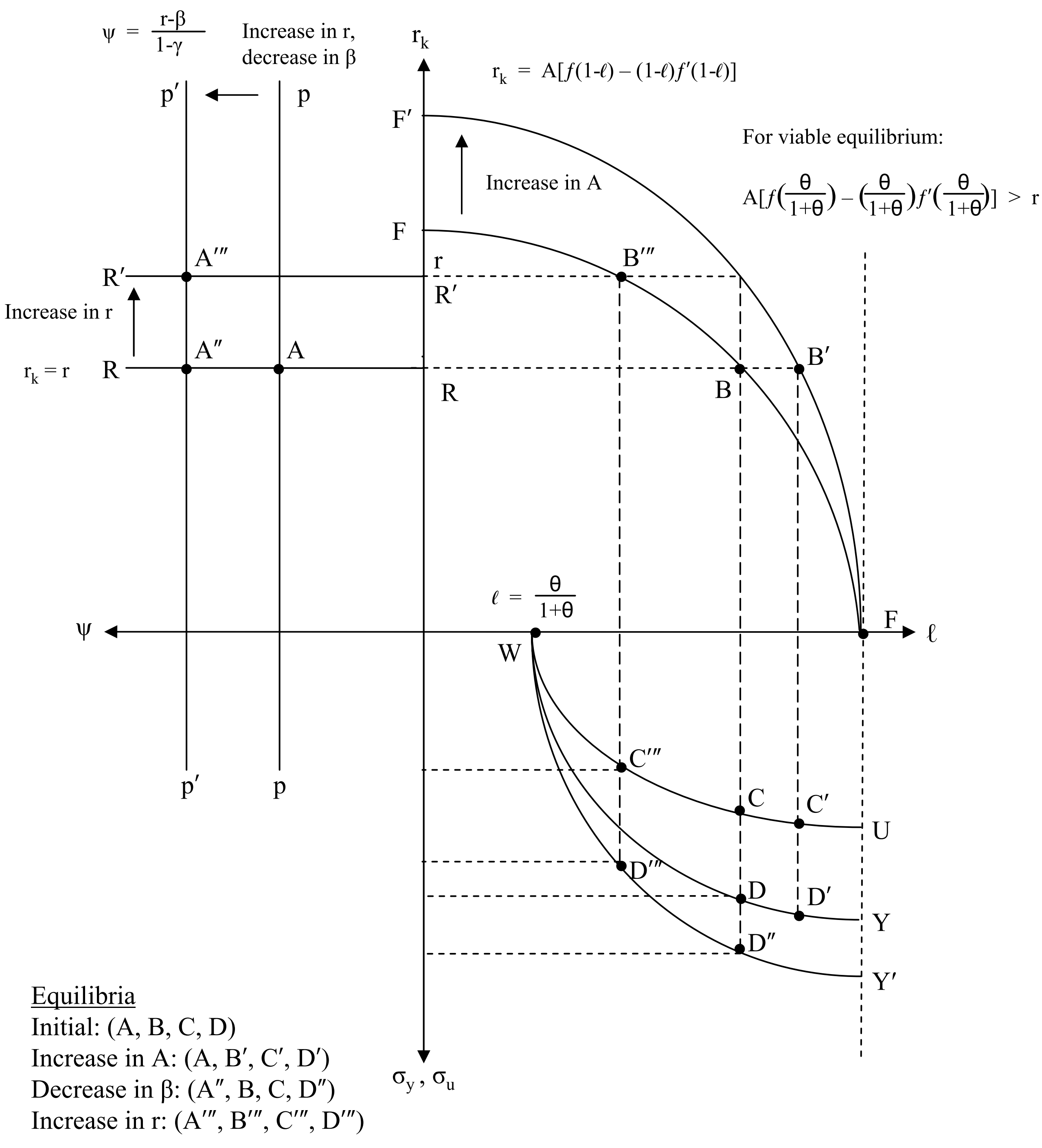
$$V(0) + \frac{\omega(l)\left(1 - l - \frac{l}{\theta}\right)}{r - \psi}K_0 = 0 \tag{A.26}$$

Rearranging, we see that in order for the economy to be intertemporally viable, the initial ratio of bonds to capital must satisfy the relationship

$$\frac{B_0}{K_0} = \frac{\omega(\tilde{l})\left(\tilde{l}\left(1 + \frac{1}{\theta}\right) - 1\right)}{r - \tilde{\psi}} - \tilde{q} \tag{A.27}$$

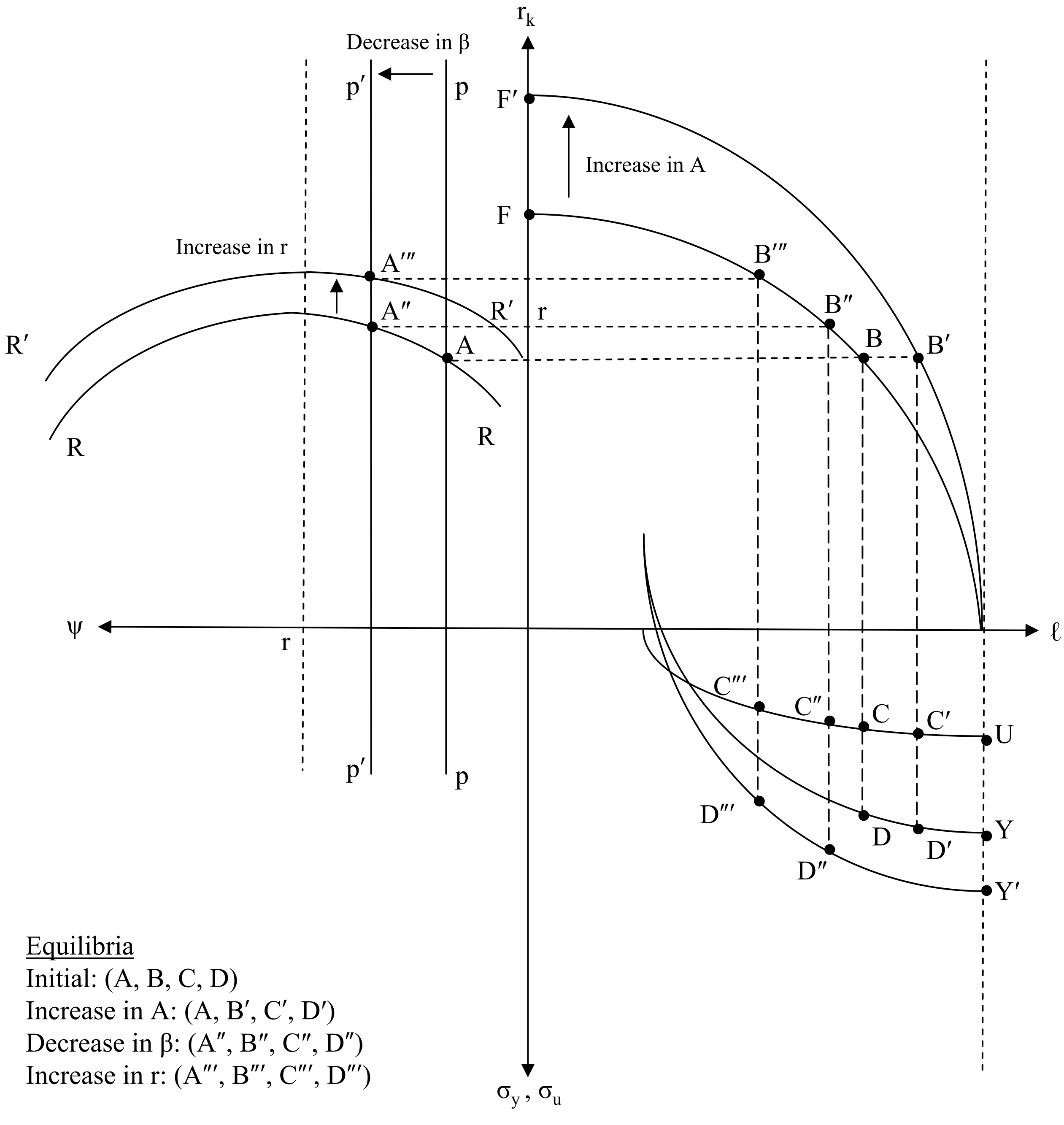
Equation (A.25) implies that the economy is viable in the aggregate if and only if each individual is intertemporally viable.

Figure 1. Equilibria under No Adjustment Costs



ψ and *l* are the equilibrium growth rate and time allocation to leisure, respectively; r_k is the rate of return on capital; σ_y and σ_u measure income and welfare inequality, respectively. See text for details.

Figure 2: Equilibria under Positive Adjustment Costs



ψ and *l* are the equilibrium growth rate and time allocation to leisure, respectively; r_k is the rate of return on capital; σ_y and σ_u measure income and welfare inequality, respectively. See text for details.

Table 1: $h = 0$

(i) Elasticity of substitution $\varepsilon = 0.8$

	ψ	l	Wealth inequality			Income inequality		
			Γ_k	Γ_b	$\Gamma_k + \Gamma_b$	Ω_k	Ω_b	$\Omega_k + \Omega_b$
Benchmark $A = 0.65, \beta = 0.04, r = 0.075$	0.014	0.726	0.413	0.587	1	0.103	0.146	0.249
Increase A to 0.75	0.014	0.766	0.257	0.743	1	0.078	0.226	0.304
Decrease β to 0.02	0.022	0.726	0.359	0.641	1	0.108	0.192	0.300
Increase r to 0.085	0.018	0.685	0.707	0.293	1	0.136	0.056	0.192

(ii) Elasticity of substitution $\varepsilon = 1$

	ψ	l	Wealth inequality			Income inequality		
			Γ_k	Γ_b	$\Gamma_k + \Gamma_b$	Ω_k	Ω_b	$\Omega_k + \Omega_b$
Benchmark $A = 0.65, \beta = 0.04, r = 0.075$	0.014	0.819	0.218	0.782	1	0.079	0.285	0.364
Increase A to 0.75	0.014	0.854	0.150	0.850	1	0.060	0.340	0.400
Decrease β to 0.02	0.022	0.819	0.189	0.811	1	0.081	0.345	0.426
Increase r to 0.085	0.018	0.780	0.311	0.689	1	0.104	0.230	0.334

(iii) Elasticity of substitution $\varepsilon = 1.2$

	ψ	l	Wealth inequality			Income inequality		
			Γ_k	Γ_b	$\Gamma_k + \Gamma_b$	Ω_k	Ω_b	$\Omega_k + \Omega_b$
Benchmark $A = 0.65, \beta = 0.04, r = 0.075$	0.014	0.888	0.146	0.854	1	0.063	0.368	0.431
Increase A to 0.75	0.014	0.893	0.135	0.865	1	0.061	0.388	0.449
Decrease β to 0.02	0.022	0.888	0.127	0.873	1	0.063	0.432	0.495
Increase r to 0.085	0.018	0.855	0.200	0.800	1	0.083	0.332	0.415

Table 2: $h = 10$

(i) Elasticity of substitution $\varepsilon = 0.8$

	ψ	l	Wealth inequality			Income inequality		
			Γ_k	Γ_b	$\Gamma_k + \Gamma_b$	Ω_k	Ω_b	$\Omega_k + \Omega_b$
Benchmark $A = 0.65, \beta = 0.04, r = 0.075$	0.014	0.687	0.716	0.284	1	0.130	0.053	0.183
Increase A to 0.75	0.014	0.733	0.383	0.617	1	0.097	0.160	0.257
Decrease β to 0.02	0.022	0.668	0.870	0.130	1	0.157	0.024	0.181
Increase r to 0.085	0.018	0.627	2.778	-1.778	1	0.190	-0.126	0.064

(ii) Elasticity of substitution $\varepsilon = 1$

	ψ	l	Wealth inequality			Income inequality		
			Γ_k	Γ_b	$\Gamma_k + \Gamma_b$	Ω_k	Ω_b	$\Omega_k + \Omega_b$
Benchmark $A = 0.65, \beta = 0.04, r = 0.075$	0.014	0.782	0.318	0.682	1	0.101	0.221	0.322
Increase A to 0.75	0.014	0.825	0.207	0.793	1	0.075	0.295	0.370
Decrease β to 0.02	0.022	0.764	0.339	0.661	1	0.116	0.238	0.354
Increase r to 0.085	0.018	0.723	0.581	0.419	1	0.145	0.108	0.253

(iii) Elasticity of substitution $\varepsilon = 1.2$

	ψ	l	Wealth inequality			Income inequality		
			Γ_k	Γ_b	$\Gamma_k + \Gamma_b$	Ω_k	Ω_b	$\Omega_k + \Omega_b$
Benchmark $A = 0.65, \beta = 0.04, r = 0.075$	0.014	0.856	0.206	0.794	1	0.081	0.320	0.401
Increase A to 0.75	0.014	0.894	0.139	0.861	1	0.059	0.375	0.434
Decrease β to 0.02	0.022	0.840	0.213	0.787	1	0.091	0.355	0.446
Increase r to 0.085	0.018	0.802	0.332	0.668	1	0.116	0.242	0.358