Risk and Returns in Exchange Rate: Evidence from Commodity Currencies*

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Abstract:

Do risk-averse agents demand compensation for holding risky currencies? Is there a "volatility feedback" effect in exchange returns as observed in the aggregate equity market? We employ the asset market approach to develop a model of volatility feedback in exchange rate and show how both perceived and unexpected risks affect currency returns and volatility. Using the volatility of world commodity price to capture risks specific to "commodity currencies" such as Australia, New Zealand, Norway, Brazil, and Iceland, we provide empirical evidence that the market demands higher currency returns to compensate for perceived risk but it leads lower currency returns to price for unexpected risk. When the effects of volatility feedback are fully taken into account, there is statistically significant evidence of the negative volatility feedback effect, which implies a positive relationship between currency risk and returns in commodity exporting countries.

Key words: Exchange rate, commodity price, exchange returns, risk, volatility feedback

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I. Introduction

Understanding and explaining floating exchange rate behavior is one of the most important yet unresolved research agenda in international economics. The exchange rate links international assets and goods markets; it is the linchpin in all open economy macroeconomic models and analyses. Proper understanding of its determinants has obvious global policy implications as countries aim to control inflation, stabilize economies, and limit contagion from global market linkages. For this ultimate objective, we investigate exchange rate behavior in terms of risk and returns trade-off by answering the following two questions, i) Do risk-averse agents demand compensation for holding risky currencies?, and ii) Is there a "volatility feedback" effect in exchange returns as observed in the aggregate equity market? Based on the asset market approach to exchange rate, we develop a volatility feedback model in exchange rate, and provide empirical evidence how fundamental-induced risks, both perceived and unexpected, affect currency movements and currency returns.

In fact, risk and returns trade-off in financial literature has been well-documented theoretically and empirically. For example, French, Schwert, and Stambaugh (1987), Turner, Startz, and Nelson (1989), Campbell and Hentschel (1992), Scruggs (1998), and others find supportive evidence for a positive relationship between market volatility and equity premium.

The idea of volatility feedback has been well-documented in the finance literature. The idea is that if volatility is persistent and is priced, an increase in volatility raises the expected future volatility and thus the required return on stocks. The result is an immediate negative impact on the current price.

In contrast, empirical investigation in the context of exchange rate has been scarce due to two main difficulties. First, bringing standard structural models of exchange rate risk to the empirics has been confounded by the long-standing struggle to systematically connect exchange rate behavior to any theoretical fundamental determinants, such as cross-country differences in monetary policy, interest rates, output, and inflation rates. When tested against data from major industrialized economies, macro fundamental-based exchange rate models produce notoriously poor estimations. The lack of empirical support for these standard macroeconomic variables cast doubts on their relevance as measures of risk. Obsfeld and Rogoff (2000), Duarte and Stockman (2001) summarize the thin connection between exchange rate and macroeconomic variables as the exchange rate disconnect puzzle. Next, it is common in the asset pricing literature to measure risk by the conditional volatility of past returns. The higher conditional volatility captures perceived risk of the asset, and should induce investors to demand higher returns as compensation. This approach, however, does not work for the exchange market because exchange rate is the relative price of two currencies. It is thus impossible to know, based on high observed volatility alone, whether it reflects riskiness about the numeraire currency or the denominator currency, hence whether which currency should appreciate relative to the other.

We note that these empirical difficulties can be resolved by focusing on commodity currencies such as Australia, New Zealand, Norway, Iceland, and Brazil relative to a non-commodity-dependent benchmark currency (USD).[1] As Chen and Rogoff (2003, 2006) and Chen et al (2008) show, these countries are industrialized economies, which are highly integrated into global capital markets and are active participants in international trade. In terms of monetary policies, they have all been under a flexible exchange rate regime over time. In addition, Chen and Rogoff (2003) show that world commodity price movements induce a roughly one-to-one response in the currency values of major OECD commodity-exporters, and represent an essentially exogenous measures of terms-of-trade fluctuations for them, because primary commodities constitute a significant share of these small-open-economies' exports.[2] Thus, the conditional variance of global commodity price captures the riskiness of commodity currencies, and allows us to measure how exchange rate movements or exchange returns react to risk. For example, if the commodity market is expected to be very volatile compared to normal variation, perceived risk to holding these commodity currencies should demand higher expected exchange returns as compensation. On the other hand, when unexpectedly higher volatility than expectation in commodity market is observed, it initially generates additional exchange rate volatility as it adjusts in response to new information about future discounted expected returns. In particular, if the volatility of the commodity price is positively related to the exchange returns, then exchange rate, the relative price of the USD in terms of unit of the foreign country, should

[1] We define exchange rate as the price of USD per unit of the foreign currency. Thus, a positive growth rate of log exchange rate implies an appreciation of foreign currency.

[2] For example, commodities have maintained about 60% share of Australia and Norway's total exports. In New Zealand, primary commodities continue to account for more than half of its total exports. (See Chen and Rogoff (2003))

immediately move in the opposite to the level of commodity price volatility so that it suppress their exchange returns.[3]

Based on the asset market approach in Campbell and Hentschel (1992), we first develop a volatility feedback model in exchange rate by synthesizing the log-linear present value framework to derive an analytical expression for the volatility feedback parameter in terms of the other parameters associated with the underlying relationship between risk and returns. We then focus on commodity currencies to identify how exchange risk is priced in exchange market. By using the volatility of world commodity price as a main determinant of the expected exchange returns, we investigate exchange rate behavior of commodity exporting countries- Australia, New Zealand, Norway, Iceland, and Brazil, and compare with non-commodity-dependent countries- Japan, Mexico, Taiwan, and Thailand. The empirical estimation shows that for only commodity exporting countries, the market demands higher currency returns to compensate for perceived risk, while it leads lower currency returns to price for unexpected risk due to a negative volatility effect. This finding supports a positive relationship between exchange risk and returns as well as asset market.

The rest of the paper is organized as follows. Section II specifies a model of volatility feedback based upon the present value representation. Section III reports the empirical results of a volatility feedback effect in exchange rate. Section IV concludes.

II. Model Specification

[3] The idea of volatility feedback effect is well demonstrated in French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), and subsequent works and has been widely used to model the positive correlation between stock market returns and aggregate stock market volatility.

1. Present Value Representation

In this section, we develop a formal model to show risk and returns trade-off in exchange market. Following interest rate parity in international economics, we can express nominal exchange rate as the relation among past exchange rate, cross-country interest rate differentials, and risk premium.

$$S_{t+1} = \left(\frac{1+i_t}{1+i_t^*} \right) S_t + RP_{t+1} = S_{t+1}^* + RP_{t+1} \tag{1}$$

where nominal exchange rate, S_{t+1} is the price of U.S. dollar per unit of foreign currency, i_t is home interest rate (U.S.), i_t^* foreign interest rate, and risk premium, RP_{t+1} is the deviation from rational expectations of uncovered interest parity and can be interpreted as an expectation error for holding foreign currency during time $t+1$. The first part in right hand side is what uncovered interest parity implies, while the second part in right hand side captures other factors that affects exchange rate in next period. Then, the gross return of exchange rate can be defined as

$$\left(1+R_{t+1}\right) = \frac{S_{t+1}}{S_t} = \frac{S_{t+1}^* + RP_{t+1}}{S_t}. \tag{2}$$

Following Campbell and Shiller (1988), we obtain the one-period natural log exchange returns by using a first-order Taylor expansion.[4]

$$r_{t+1} \approx k + s_{t+1}^* - s_t + \left(1-a\right)\left(rp_{t+1} - s_{t+1}^*\right). \tag{3}$$

where lowercase letters denote log variables throughout, the parameter a is the steady-state ratio of $\tilde{s}^*$ to sum of $\tilde{s}^*$ and $\widetilde{rp}$, and k is a nonlinear function of a. By rearranging

[4] The detail derivation is in Appendix A.

equation (3), we obtain a function of s_t which consists of r_{t+1}, s^*_{t+1}, and rp_{t+1}.

$$s_t \approx k + (1-a)\, rp_{t+1} - r_{t+1} + a s^*_{t+1}. \tag{4}$$

In international economics, the risk-adjusted interest parity relationship can be expressed as a form of a first-order approximation from equation (1)

$$s_{t+1} \approx b s^*_{t+1} + (1-b) rp_{t+1}. \tag{5}$$

Substituting equation (5) into (4), we obtain the following relation as

$$s_t \approx k + (1-\rho)\, rp_{t+1} - r_{t+1} + \rho s_{t+1}. \tag{6}$$

where $\rho = {b}/{a}$ is considered as a discount rate. Solving forward with the terminal condition of $\lim_{j\to\infty} E\left[\rho^j s_{t+j} \mid I_t\right] = 0$ to rule out rational bubbles and taking expectation given information I_t, we obtain a log-linear present value relationship:

$$s_t = \frac{k}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty} \rho^j E\left[rp_{t+1+j} \mid I_t\right] - \sum_{j=0}^{\infty} \rho^j E\left[r_{t+1+j} \mid I_t\right]. \tag{7}$$

This asset price approach implies that log of nominal exchange rate, s_t reflects the discounted sum of all future expected risk premium and all future expected returns. As discussed in Engle and West (2005), it is intuitively identical to the present value approach in stock market proposed by Campbell and Shiller (1988), which shows that current price of aggregate stock market is discounted sum of all future expected returns and all future expected dividend growth of equity.

Using equation (7) to substituting s_t and s_{t+1} out of equation (6) gives a useful expression:

$$r_{t+1} = E\left[r_{t+1} \mid I_t\right] + f_{t+1} + \varepsilon_{t+1}.^{5} \tag{8}$$

where $f_{t+1} = -\sum_{j=1}^{\infty} \rho^j \left(E\left[r_{t+1+j} \mid I_{t+1}\right] - E\left[r_{t+1+j} \mid I_t\right]\right)$ reflects revisions in future expected returns, and $\varepsilon_{t+1} = (1-\rho)\sum_{j=0}^{\infty} \rho^j \left(E\left[rp_{t+1+j} \mid I_{t+1}\right] - E\left[rp_{t+1+j} \mid I_t\right]\right)$ reflects revisions in future expected risk premium by updating unexpected news on exchange market during time $t+1$. This equation implies that exchange returns should be explained by news on future expected returns and news on future expected risk premium.

2. Volatility Feedback Model of Commodity Currencies

A stylized fact in finance literature is that stock returns are negatively correlated with the volatility of subsequent returns. Thus, below-average returns are associated with increases in volatility, and vice versa. One explanation for the negative relationship is a volatility feedback effect. The idea of volatility feedback is as follows: If volatility is priced and an increase in volatility raises the expected future volatility, then risk-averse agents demand higher future returns to compensate future risk. As a result, it has an immediate negative impact on the current price so that current stock returns fall. In empirics, there have been a bunch of studies that show a volatility feedback effect. For example, French et al. (1987), Turner et al. (1989), Campbell and Hentschel (1992), Bekaert and Wu (2000), Wu (2001), and Mayfield (2004) find evidence in support of a negative volatility feedback effect in stock market.

In constrast, there is an obstacle in bringing the idea of a volatility feedback into

[5] The detail derivation is in Appendix B.

exchange market due to difficulty in identifying risk of exchange rate. Since exchange rate is the relative price of two currencies, there is no way to see which currency should be appreciated relative to the other by looking at higher volatiliry alone. The source of higher exchange return volatility could be shocks in home or foreign country. Using relative volatility of macroeconomic variables may be an alternative way to measure risk associated with two currencies. In fact, it has been theoretically accepted in international economics that exchange return is determined by macroeconomic fundamentals such as cross-country differences in monetary policy, interest rates, output, and inflation rates (see Engel and West (2004, 2005)). The empirical investigation, however, has not been successful to explain high volatility and persistence of exchange rate by using macroeconomic fundamentals. For example, Baxer and Stockman (1989) show that it is difficult to identify any stable systematic relationship between exchange rate and macroeconomic variables including policy variables such as interest rates and budget deficits. These empirical challenges led Frankel and Rose (1995, Handbook of International Economics) to conclude with doubts in the value of further time-series modeling of exchange rates using macroeconomic variables. Flood and Rose (1995), Obsfeld and Rogoff (2000), and others summarize that exchange risk seems to be disconnected from the real economy.

Recent studies including Chen and Rogoff (2003, 2006) and Chen et al (2008) show a systematic relation between world commodity price and commodity currencies. They find that the price of commodity exports has a strong and stable influence on currency value of commodity exporting countries such as Australia and New Zealand. For these countries, where are described as commodity economies due to the large portion of their production and export, the commodity price is a major determinant of their currency value (exchange rate). It means that the uncertainty associated with the commodity price can be a good measure of risk of exchange returns in commodity exporters. Thus, we now focus on the commodity currencies to develop a volatility feedback model.

Including the effect of volatility feedback into the model requires a number of assumptions. First, we use conditional variance of the commodity price to capture risk of commodity currencies by assuming log difference of the commodity price to follow ARCH (Autoregressive Conditional Heteroskedasticity) (p) process with constant mean.[6]

$$\Delta cp_{t+1} = \mu + u_{t+1},\ u_{t+1} \sim N\left(0, \sigma^2_{u,t+1}\right), \tag{9}$$

$$\sigma^2_{u,t+1} = \beta_0 + \beta_1 \sum_{i=1}^{p} k_i u^2_{t+1-i},\ k_i = \frac{p+1-i}{\sum_{i=1}^{p} i},\ \sum_{i=1}^{p} k_i = 1. \tag{10}$$

where Δcp_{t+1} is the log difference of the commodity price in time $t+1$ and k_i for $i = 1, 2, \cdots, p$ is weight parameter of ARCH process so that β_1 measures the persistence of volatility of commodity price. We employ ARCH model rather than GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is because of the possibility of spurious inference on conditional variance. Although GARCH model by Bollerslev (1986) has been widely used in financial and international literature, Ma et al (2007) point out that the GARCH estimate tends to have too small standard error to the true one when the ARCH parameter is small, even when sample size becomes very large. In combination with an upward bias in the GARCH estimate, the small standard error will often lead to the spurious inference that volatility is highly persistent when it is not. As a response to the danger of

[6] We assume that ARCH parameters are declining geometrically to reduce estimation burden.

spurious inference, they propose an empirical strategy based on a pure ARCH (p) approximation to GARCH model. (See Ma et al (2007))

Secondly, we assume that expected exchange returns are a linear function of the expected volatility of the commodity price as[7]

$$E\left[r_{t+1+j} \mid I_t\right] = \alpha_1 E\left[u_{t+1+j}^2 \mid I_t\right]. \tag{11}$$

These assumptions imply that the revision of expected exchange returns in time $t+1+j$ due to the unexpected volatility shock on the commodity price during time $t+1$ is a linear function of $\left(u_{t+1}^2 - \sigma_{u,t+1}^2\right)$, which has zero mean and is serially uncorrelated. Then, it is straight forward to show the news effects on discounted all future expected returns based on unexpected volatility that is occurred during trading period as[8]

$$f_{t+1} = -\sum_{j=1}^{\infty} \rho^j \left(E\left[r_{t+1+j} \mid I_{t+1}\right] - E\left[r_{t+1+j} \mid I_t\right]\right) = -\alpha_1 \beta_1 \frac{\sum_{i=1}^{p} \rho^i k_i}{1 - \sum_{i=1}^{p} \rho^i k_i}. \tag{12}$$

By substituting equation (11) and (12) into (8), we complete a volatility feedback model as

$$r_{t+1} = \alpha_1 \sigma_{u,t+1}^2 + \alpha_2 \left(u_{t+1}^2 - \sigma_{u,t+1}^2\right) + \varepsilon_{t+1}. \tag{13}$$

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[7] Campbell and Hentschel (1992) define expected stock returns as a linear function of expected market volatility including a constant term, which implies risk free rate. However, exchange rate, which is the relative price between two currencies does not have the concept of risk free rate. For example, if exchange rate is expected to be constant, its expected returns should be zero with constant commodity price. Thus, we do not include a constant term here.

[8] The detail derivation is in Appendix C.

where $\alpha_2 = -\alpha_1\beta_1 \frac{\sum_{i=1}^{p} \rho^i k_i}{1-\sum_{i=1}^{p} \rho^i k_i}$. Note that the parameter of linearization, ρ (in practice 0.999) should be less than 1. Thus, the coefficient α_2, which capture a price of risk based on the volatility of commodity price will be negative as long as the α_1 is positive. Conversely, any evidence of a negative volatility feedback effect implies a positive relationship between market volatility and excess currency return.

Third, we assume that the disturbance term ε_{t+1} in equation (13), which includes other factors excluding commodity price that effects exchange returns has the following ARCH (q) process.[9]

$$\varepsilon_{t+1} \sim N\left(0,\, \sigma^2_{\varepsilon,t+1}\right), \tag{14}$$

$$\sigma^2_{\varepsilon,t+1} = \gamma_0 + \gamma_1 \sum_{i=1}^{q} l_i \varepsilon^2_{t+1-i},\; l_i = \frac{q+1-i}{\sum_{i=1}^{q} i},\; \sum_{i=1}^{q} l_i = 1. \tag{15}$$

where news on future expected risk premium is assumed to be uncorrelated with the shocks on volatility of the commodity price.

Lastly, we address assumption about information available to risk-averse agents during each trading period. Risk-averse agents observe past exchange returns and volatility of the commodity price at the beginning of the trading period based on the conditional volatility of the commodity price. Given this information, they form expected exchange returns of the next period and then obtain news about the volatility of the commodity price through the process of trading. This additional information potentially produces volatility

[9] Similarly, we assume that ARCH parameters are declining geometrically to reduce estimation burden.

feedback by revising future expected exchange returns. For empirical investigation, we first estimate the dynamics of the commodity price and generate conditional volatility and unexpected volatility. We then estimate a volatility feedback model using generated regressors as independent variables.

III. Empirical Results

1. Data

Our sample consists of daily data from 2002 to 2007. For world commodity price, we use Moody's commodity price index from Global Financial Data.[10] The log of the commodity price and its first difference is plotted in Figure 1. As it is shown, the log of the commodity price seems to follow I(1) process. We find that the augmented Dickey-Fuller test fails to reject the presence of a unit root in the log of the commodity price.

Exchange rate is measured as the USD per unit of the foreign currency, and exchange returns are log difference of nominal exchange rate. Thus, positive exchange returns imply an appreciation of the foreign currency.[11] For empirical investigation, we consider exchange returns of 9 countries-Australia, New Zealand, Norway, Brazil, Iceland, Japan, Mexico, Taiwan, and Thailand. [12] Descriptive statistics for each country's exchange

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[10] When other commodity price index such as CRB (Commodity Research Bureau) index is used, the empirical results are qualitatively same as Moody's index.

[11] Some researchers in international economics have defined excess currency returns as the difference in the cross-country interest rate adjusting for the relative currency. Even when excess currency returns are used as a dependent variable, the results are very similar to what we obtain in this paper.

[12] The history of floating system is varying according to each country and they may have an inflation targeting framework. All of them, however, have adopted floating exchange rate at least since 2002.

rate and world commodity price are reported in Table 1. For convenience of inference, data are rescaled by multiplying 100. Figure 2 plots nominal exchange rate and exchange returns. The dynamics of exchange returns are slightly different among countries. The variation of exchange rate in Australia and New Zealand, considered as developed countries seems like less volatile than Brazil and Thailand as developing countries.

It is worth to noting how to determine which currencies belong to commodity currencies. Because there is no decision rule of defining commodity exporting countries, we use the weight of commodity exports out of total exports provided in Table 2. For making easy comparison of empirical results, we arbitrarily divide these countries into two groups according to whether the ratio of commodity exports to total exports is greater than 40%. Out of 9 countries, thus we consider Australia, New Zealand, Norway, Iceland, and Brazil as commodity dependent countries, while Japan, Mexico, Taiwan, and Thailand as non-commodity dependent countries.

2. Conditional Volatility and News on Volatility of Commodity Price

Table 3 presents ARCH process of the commodity price. When lag of 10 in ARCH process is imposed with the assumption of the geometrically declining ARCH parameters, the coefficient β_1 is statistically significant, which means that the conditional variance of the commodity price is mainly determined by the past volatility of the commodity price.[13] It is worth to addressing that persistency of conditional volatility in the commodity price is

[13] Even when different lag of ARCH process is imposed, the empirical results are qualitatively similar so that we report the results with lag of 10.

about 0.3, which is not high as expected. In order to capture change of future expected exchange returns due to unexpected shock, we generate news on volatility by subtracting actual volatility to expected volatility and use it as independent variable in exchange returns. Figure 3 plots actual and expected volatility, and news on volatility generated from the dynamics of the commodity price. Actual volatility is squared residual, while expected volatility is conditional variance of the commodity price. Compared to expected volatility, actual volatility is more volatile because ARCH process of the commodity price is not quite persistent. We thus expect this may lead small volatility feedback effect in magnitude.

3. Expected Exchange Returns and Volatility Feedback in Commodity Currencies

Exchange returns response to risk of the commodity price in two channels. If the commodity markets are expected to be highly volatile compared to normal level, this perceived risk should lead to higher expected exchange returns as commensation for higher risk. In contrast, when new information about the volatility of the commodity price is arrived during the trading period, it potentially produces volatility feedback by revising future expected exchange returns, not just the contemporaneous expected exchange returns but also all future expected exchange returns. Furthermore, it potentially provides a more powerful way to estimate the true sign of the relationship between exchange risk and returns because volatility feedback captures the effects of exchange risk on all future expected exchange returns.

3.1 No Restriction on Volatility Feedback Parameter

Table 4 reports maximum likelihood estimation results of the volatility feedback model without restriction on coefficient α_2 as a function of other parameters. First, we find that risk-averse agents holding commodity currencies demand higher compensation for perceived risk. For the null hypothesis of $H_0 : \alpha_1 = 0$ is rejected at least at the 10% level for the commodity exporting countries, but not for the non-commodity exporting countries. A positive value of α_1 presents that expected returns on commodity currencies are positively time varying according to riskiness of the commodity price, which is main determinant of the value of commodity currencies. Second, there is supportive evidence for a volatility feedback effect only in the commodity exporting countries. For the null hypothesis of no feedback, $H_0 : \alpha_2 = 0$, the *t*-statistics for the feedback term α_2 in Table 4 are significant at least at the 10% level in the commodity exporting countries, but not from the non-commodity exporting countries. This result confirms that risk-averse agents consider not only the partial effect on the contemporaneous expected return, but also a volatility feedback effect by summarizing the impact of an unanticipated change in the volatility of the commodity price on future expected exchange returns.

3.2 Restriction on Volatility Feedback Parameter

Table 5 reports maximum likelihood estimation results of the volatility feedback model with restriction on coefficient $\alpha_2 = -\alpha_1 \beta_1 \sum_{i=1}^{p} \rho^i k_i \Big/ \left(1 - \sum_{i=1}^{p} \rho^i k_i\right)$ where $\rho = 0.999$ and $k_i = (p+1-i) \Big/ \sum_{i=1}^{p} i$, $\sum_{i=1}^{p} k_i = 1$, $p = 10$. Similar findings are reported even when a restriction is imposed. The efficient of expected exchange returns, α_1 is still statistically significant at least at the 5% level for all commodity exporting countries, but not for non-commodity exporting countries. The coefficient of volatility feedback term, α_2 and its standard errors are ex-post calculated based on the estimation results reported in Table 3 and Table 5. It confirms a negative volatility feedback effect for commodity exporting countries at least at the 5% level. In addition, we find a positive relationship between volatility and expected change returns. For the null hypothesis of $\alpha_2 = -\alpha_1 \beta_1 \sum_{i=1}^{p} \rho^i k_i \Big/ \left(1 - \sum_{i=1}^{p} \rho^i k_i\right)$, the likelihood ratio tests fail to reject the null for all commodity exporting countries. The likelihood ratio statistics are 0.284 (*p*-value of 0.594) for Australia, 0.744 (*p*-value of 0.388) for New Zealand, 0.863 (*p*-value of 0.353) for Norway, 1.171 (*p*-value of 0.271) for Brazil, and 0.0014 (*p*-value of 0.970) for Iceland. As frequently mentioned in financial literature, a positive relationship between commodity currency risk and returns implies a negative relationship between exchange returns and volatility of the commodity price.

3.3 Conditional Variance of Exchange Returns

The fourth and fifth column in Table 4–5 report estimation results for ARCH process of error term with lag of 10. Note that conditional variance of error term captures news effects on future expected risk premium coming from shocks on other components excluding the commodity price. It seems that ARCH parameter of the error term is country specific, but it is statistically significant at the 1% level. This implies that capturing exchange risk via the volatility of the commodity price may not be enough to explain high and persistent exchange rate volatility. Thus, after controlling for the commodity price shocks, there still remains exchange rate disconnect puzzle in the residual. This is consistent with the findings of Baxer and Stockman (1989), and Flood and Rose (1995) that show exchange rate changes are very much volatile compared to other macroeconomic variables.

IV. Conclusion

The difficulties associated with measurement and identification of exchange rate risk between two currencies have been obstacles for empirical investigation in exchange market. We address that these issues can be resolved at least in the range of commodity currencies because the main determinant of value of commodity currencies is exogenous world commodity price. For empirical investigation, we develop a model of volatility feedback in commodity currencies and provide supportive evidence of a positive relationship between exchange risk and returns with a negative volatility feedback. When the effects of volatility feedback are fully taken into account, there is statistically significant evidence of the negative volatility feedback effect from commodity exporting countries-Australia, New Zealand, Norway, Brazil, and Iceland, which have floating system. These findings imply a positive relationship between exchange risk and returns.

Appendix

A. Derivation of log-linearization of return on exchange rate

The exchange returns can be defined as follows:

$$\left(1+R_{t+1}\right)=\frac{S_{t+1}}{S_t}=\frac{S_{t+1}^{*}+RP_{t+1}}{S_t}=\frac{S_{t+1}^{*}}{S_t}\left(1+\frac{RP_{t+1}}{S_{t+1}^{*}}\right) \tag{A1}$$

Taking natural log in both sides provides the following relation:

$$r_{t+1}=s_{t+1}^{*}-s_t+\log\left[1+\frac{RP_{t+1}}{S_{t+1}^{*}}\right]=s_{t+1}^{*}-s_t+\log\left[1+\exp\left(rp_{t+1}-s_{t+1}^{*}\right)\right] \tag{A2}$$

Let define $f\left(x_{t+1}\right)=\log\left[1+\exp\left(x_{t+1}\right)\right]$ where $x_{t+1}=rp_{t+1}-s_{t+1}^{*}$, then the first order linearization around the steady state can be expressed as follows:

$$\begin{aligned} f\left(x_{t+1}\right) &\approx f\left(\tilde{x}\right)+f\left(\tilde{x}\right)'\left(x_{t+1}-\tilde{x}\right) \\ &=\log\left[1+\exp\left(\tilde{x}\right)\right]+\frac{\exp\left(\tilde{x}\right)}{1+\exp\left(\tilde{x}\right)}\left(x_{t+1}-\tilde{x}\right) \\ &=\log\left[1+\exp\left(\tilde{x}\right)\right]-\frac{\exp\left(\tilde{x}\right)}{1+\exp\left(\tilde{x}\right)}\tilde{x}+\frac{\exp\left(\tilde{x}\right)}{1+\exp\left(\tilde{x}\right)}x_{t+1} \\ &=k+\left(1-a\right)x_{t+1} \end{aligned} \tag{A3}$$

where $a=\frac{1}{1+\exp\left(\tilde{x}\right)}$, and thus $\log\left[1+\exp\left(x_{t+1}\right)\right]=k+\left(1-a\right)\left(rp_{t+1}-s_{t+1}^{*}\right)$. By plugging equation (A3) into (A2), we obtain log- linearized exchange returns.

$$r_{t+1}\approx k+s_{t+1}^{*}-s_t+\left(1-a\right)\left(rp_{t+1}-s_{t+1}^{*}\right) \tag{A4}$$

B. Derivation of a Volatility Feedback Model

We can rewrite equation (7) as

$$s_t = \frac{k}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty}\rho^j E\left[rp_{t+1+j} \mid I_t\right] - \sum_{j=0}^{\infty}\rho^j E\left[r_{t+1+j} \mid I_t\right] \quad \text{(B1)}$$

$$s_{t+1} = \frac{k}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty}\rho^j E\left[rp_{t+2+j} \mid I_{t+!}\right] - \sum_{j=0}^{\infty}\rho^j E\left[r_{t+2+j} \mid I_{t+1}\right] \quad \text{(B2)}$$

Plugging equation (B1) and (B2) into (6) gives us the following relation.

$$\begin{aligned}
r_{t+1} &\approx k + (1-\rho) rp_{t+1} - s_t + \rho s_{t+1} \\
&= k + (1-\rho) rp_{t+1} - \left(\frac{k}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty}\rho^j E\left[rp_{t+1+j} \mid I_t\right] - \sum_{j=0}^{\infty}\rho^j E\left[r_{t+1+j} \mid I_t\right]\right) \quad \text{(B3)} \\
&\quad + \rho\left(\frac{k}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty}\rho^j E\left[rp_{t+2+j} \mid I_{t+!}\right] - \sum_{j=0}^{\infty}\rho^j E\left[r_{t+2+j} \mid I_{t+1}\right]\right) \\
&= \quad k - \frac{k}{1-\rho} + \rho\frac{k}{1-\rho} \quad .....i) \\
&\quad + \sum_{j=0}^{\infty}\rho^j E\left[r_{t+1+j} \mid I_t\right] - \rho\sum_{j=0}^{\infty}\rho^j E\left[r_{t+2+j} \mid I_{t+1}\right] \quad .....ii) \\
&\quad + (1-\rho) rp_{t+1} + (1-\rho)\sum_{j=0}^{\infty}\rho^j E\left[rp_{t+1+j} \mid I_t\right] + \rho(1-\rho)\sum_{j=0}^{\infty}\rho^j E\left[rp_{t+2+j} \mid I_{t+1}\right] \quad .....iii)
\end{aligned}$$

From the equation (B3), it can be easily shown that the constant term, part *i)* becomes zero.

$$k - \frac{k}{1-\rho} + \rho\frac{k}{1-\rho} = k - k\left(\frac{1}{1-\rho} - \frac{\rho}{1-\rho}\right) = 0 \quad \text{(B4)}$$

Secondly, the part *ii)* can be rewritten as the sum of news on expected exchange returns, f_{t+1} and the next period expected returns given information I_t.

$$\begin{aligned}
&-\rho\sum_{j=0}^{\infty}\rho^j E\left[r_{t+2+j} \mid I_{t+1}\right] + \sum_{j=0}^{\infty}\rho^j E\left[r_{t+1+j} \mid I_t\right] \\
&= -\sum_{j=1}^{\infty}\rho^j E\left[r_{t+1+j} \mid I_{t+1}\right] + \sum_{j=1}^{\infty}\rho^j E\left[r_{t+1+j} \mid I_t\right] + E\left[r_{t+1} \mid I_t\right]
\end{aligned} \quad \text{(B5)}$$

Lastly, news on risk premium, part *iii)* can be summarized as follows:

$$\begin{aligned}
&(1-\rho)rp_{t+1}-(1-\rho)\sum_{j=0}^{\infty}\rho^{j}E\left[rp_{t+1+j}\mid I_{t}\right]+\rho(1-\rho)\sum_{j=0}^{\infty}\rho^{j}E\left[rp_{t+2+j}\mid I_{t+1}\right]\\
&=(1-\rho)rp_{t+1}+\rho(1-\rho)\sum_{j=0}^{\infty}\rho^{j}E\left[rp_{t+2+j}\mid I_{t+1}\right]-(1-\rho)\sum_{j=0}^{\infty}\rho^{j}E\left[rp_{t+1+j}\mid I_{t}\right]\\
&=(1-\rho)\left\{E\left[rp_{t+1}\mid I_{t+1}\right]+\sum_{j=0}^{\infty}\rho^{j+1}E\left[rp_{t+2+j}\mid I_{t+1}\right]-\sum_{j=0}^{\infty}\rho^{j}E\left[rp_{t+1+j}\mid I_{t}\right]\right\}\\
&=(1-\rho)\sum_{j=0}^{\infty}\rho^{j}\left\{E\left[rp_{t+1+j}\mid I_{t+1}\right]-E\left[rp_{t+1+j}\mid I_{t}\right]\right\}
\end{aligned}\tag{B6}$$

By plugging equation (B4)-(B6) in the (B3), we obtain a volatility feedback model as follows:

$$\begin{aligned}
r_{t+1}=E\left[r_{t+1}\mid I_{t}\right]-\sum_{j=1}^{\infty}\rho^{j}\left(E\left[r_{t+1+j}\mid I_{t+1}\right]-E\left[r_{t+1+j}\mid I_{t}\right]\right)\\
+(1-\rho)\sum_{j=0}^{\infty}\rho^{j}\left(E\left[rp_{t+1+j}\mid I_{t+1}\right]-E\left[rp_{t+1+j}\mid I_{t}\right]\right)
\end{aligned}\tag{B7}$$

C. Derivation of Volatility Feedback Parameter

From the assumption of ARCH (p) process of the volatility of the commodity price, we can derive the volatility feedback parameter as a function of other parameters. An ARCH (p) process in equation (10) can be rewritten as AR(p) process.

$$u_{t+1}^{2}=\beta_{0}+\beta_{1}^{*}u_{t}^{2}+\beta_{2}^{*}u_{t-1}^{2}+\cdots+\beta_{p}^{*}u_{t+1-p}^{2}+\eta_{t+1}\tag{C1}$$

where $\beta_{1}^{*}=\beta_{1}k_{i}$ for $i=1,2,\cdots,p$ and $\eta_{t+1}=\left(u_{t+1}^{2}-\sigma_{u,t+1}^{2}\right)$ is mean zero innovations and serially uncorrelated. Then, we specify a state space representation for u_{t+1}^{2} as

$$\begin{bmatrix}u_{t+1}^{2}\\u_{t}^{2}\\u_{t-1}^{2}\\\vdots\\u_{t+2-p}^{2}\end{bmatrix}=\begin{bmatrix}\beta\\0\\0\\\vdots\\0\end{bmatrix}+\begin{bmatrix}\beta_{1}^{*}&\beta_{2}^{*}&\beta_{3}^{*}&\cdots&\beta_{p}^{*}\\1&0&\cdots&0&0\\0&1&\cdots&0&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&1&0\end{bmatrix}\begin{bmatrix}u_{t}^{2}\\u_{t-1}^{2}\\u_{t-2}^{2}\\\vdots\\u_{t+1-p}^{2}\end{bmatrix}+\begin{bmatrix}1\\0\\0\\\vdots\\0\end{bmatrix}\eta_{t+1}^{2}$$

$$\Updownarrow$$

$$Z_{t+1} \quad = \quad \Gamma \quad + \quad F\ Z_t \quad + \quad R\eta_{t+1} \tag{C2}$$

By solving backward the transition equation and taking expectation given information I_{t+1} and I_t, we obtain the followings:

$$E\left[Z_{t+1+j} \mid I_{t+1}\right] = \Gamma\sum_{i=0}^{j} F^i + F^j Z_t + \sum_{i=0}^{j} F^i R\eta_{t+1+j-i} \tag{C3}$$

$$E\left[Z_{t+1+j} \mid I_{t+1}\right] = \Gamma\sum_{i=0}^{j} F^i + F^j Z_t \tag{C4}$$

The innovations in the expectation of Z_{t+1+j} are given by

$$E\left[Z_{t+1+j} \mid I_{t+1}\right] - E\left[Z_{t+1+j} \mid I_t\right] = F^j R\eta_{t+1} \tag{C5}$$

which shows that the innovations in expectation for ARCH(p) model are in general a linear function of $\eta_{t+1} = \left(u_{t+1}^2 - \sigma_{u,t+1}^2\right)$. Thus, news on conditional volatility of the commodity price, $E\left[u_{t+1+j}^2 \mid I_{t+1}\right] - E\left[u_{t+1+j}^2 \mid I_t\right]$ is the (1,1) element of $F^j R\eta_{t+1}$. Finally, we solve the volatility feedback term, which implies the revisions of discounted future return of exchange rate as

$$\begin{aligned}
f_{t+1} &= -\sum_{j=1}^{\infty} \rho^j \left(E\left[r_{t+1+j} \mid I_{t+1}\right] - E\left[r_{t+1+j} \mid I_t\right]\right) \\
&= -\sum_{j=1}^{\infty} \rho^j \alpha_1 \left(E\left[u_{t+1+j}^2 \mid I_{t+1}\right] - E\left[u_{t+1+j}^2 \mid I_t\right]\right) \\
&= -\alpha_1 \sum_{j=1}^{\infty} \rho^j \left(1,1\right) element\ of\ F^j R\left(u_{t+1}^2 - \sigma_{u,t+1}^2\right) \\
&= -\alpha_1 \left(1,1\right)\ element\ of\ \rho F\left(I_p - \rho F\right)^{-1} R\left(u_{t+1}^2 - \sigma_{u,t+1}^2\right) \\
&= -\alpha_1 \frac{\sum_{i=1}^{p} \rho^i \beta_i^*}{1 - \sum_{i=1}^{p} \rho^i \beta_i^*}
\end{aligned} \tag{C6}$$

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Table 1. Basic Statistics of Exchange Returns and Commodity Price Growth Rate

	Mean	Max	Min	Std. Dev.	Skewness	Kurtosis
Australia	0.0340	2.2540	-4.0028	0.6604	-0.5650	5.0996
New Zealand	0.0382	2.7250	-2.8131	0.7335	-0.4084	3.8756
Norway	0.0314	2.2652	-2.5105	0.6582	-0.2130	3.5398
Brazil	0.0165	10.4069	-8.3009	1.0605	-0.2084	13.4771
Iceland	0.0312	3.8913	-4.4767	0.7311	-0.4738	6.0121
Japan	0.0203	2.0540	-1.8224	0.5007	-0.0870	3.5287
Mexico	-0.0115	2.1049	-2.8787	0.4772	-0.4443	5.0195
Taiwan	0.0048	1.1135	-1.4374	0.2487	-0.0200	5.6116
Thailand	0.0250	11.7376	-11.0720	0.6410	0.3963	141.2109
Commodity Price Growth Rate	0.0700	4.7241	-5.4298	0.6608	-0.4232	9.1139

Note. Exchange returns are defined as 100* log difference of nominal exchange rate, USD per unit of foreign currency. The commodity price growth rate is 100* log difference of commodity price index. Samples are daily data from 2002 to 2007 (observation = 1561).

Table 2. The Ratio of Commodity Exports out of Total Exports

	1995	2000	2005
Australia	57%	61%	62%
New Zealand	62%	61%	62%
Norway	58%	71%	74%
Brazil	41%	37%	43%
Iceland	78%	67%	61%
Japan	2%	1%	2%
Mexico	20%	16%	22%
Taiwan	14%	10%	6%
Thailand	26%	22%	21%

Note. This table is based upon the calculation of Chen and Rogoff(2003, 2006).

Table 3. ARCH Process of Commodity Price

	μ	β_0	β_1	*Log Likelihood*
Coefficient	0.0696 ***	0.2941***	0.3429***	-1538.2318
(Std Errors)	(0.0161)	(0.0247)	(0.0655)	

Note. The estimation results are based on the following model specification.

$$\Delta cp_{t+1} = \mu + u_{t+1},$$
$$u_{t+1} \sim N\left(0, \sigma_{t+1}^2\right), \ \sigma_{t+1}^2 = \beta_0 + \beta_1\left(l_1 u_{t-1}^2 + l_2 u_{t-2}^2 + \cdots + l_q u_{t-q}^2\right)$$
$$l_i = \frac{q+1-i}{\sum_{i=1}^{q} i}, \ \sum_{i=1}^{q} l_i = 1, \ q = 10$$

Standard errors are in parenthesis. *, **, and *** are significant at the 10%, 5%, and 1% level respectively.

Table 4. Volatility Feedback in Exchange Returns Without Restriction

	α_1	α_2	γ_0	γ_1	*Log Likelihood*
Australia	0.0771***	-0.0290***	0.2045***	0.5357***	-1492.4792
	(0.0339)	(0.0122)	(0.0297)	(0.0983)	
New Zealand	0.0985***	-0.0305***	0.2664***	0.5112***	-1666.5342
	(0.0375)	(0.0136)	(0.0192)	(0.0260)	
Norway	0.0612*	-0.0526***	0.3067***	0.2904***	-1531.4122
	(0.0341)	(0.0132)	(0.0284)	(0.0677)	
Brazil	0.1207***	-0.0374***	0.1921***	0.8803***	-1980.7687
	(0.0372)	(0.0139)	(0.0263)	(0.0637)	
Iceland	0.0787**	-0.0418***	0.3024***	0.4319***	-1650.9895
	(0.0381)	(0.0153)	(0.0248)	(0.0579)	
Japan	-0.0040	-0.0165	0.2116***	0.3474***	-1293.9307
	(0.0286)	(0.0120)	(0.0202)	(0.0696)	
Mexico	-0.0115	-0.0039	0.1237***	0.4610***	-1004.8132
	(0.0320)	(0.0089)	(0.0121)	(0.0651)	
Taiwan	0.0038	-0.0006	0.0145***	0.8616***	87.5364
	(0.0112)	(0.0040)	(0.0020)	(0.0681)	
Thailand	0.0253	-0.0086	0.0405***	0.6372***	-260.3474
	(0.0157)	(0.0049)	(0.0045)	(0.0730)	

Note. The estimation results are based on the following model specification.

$$r_{t+1} = \alpha_1 \sigma_{t+1}^2 + \alpha_2 \left(u_{t+1}^2 - \sigma_{t+1}^2 \right) + \varepsilon_{t+1},$$

$$\varepsilon_{t+1} \sim N\left(0, \sigma_{\varepsilon,t+1}^2\right), \ \sigma_{\varepsilon,t+1}^2 = \gamma_0 + \gamma_1 \left(k_1 \varepsilon_{t-1}^2 + k_2 \varepsilon_{t-2}^2 + \cdots + k_p \varepsilon_{t-p}^2 \right)$$

$$k_i = \frac{p+1-i}{\sum_{i=1}^{p} i}, \ \sum_{i=1}^{p} k_i = 1, \ p = 10$$

Standard errors are in parenthesis. *, **, and *** are significant at the 10%, 5%, and 1% level respectively.

Table 5. Volatility Feedback in Exchange Returns With Restriction

	α_1	α_2	γ_0	γ_1	*Log Likelihood*
Australia	0.0625***	-0.0324***	0.2049***	0.5345***	-1492.6214
	(0.0190)	(0.0136)	(0.0241)	(0.0739)	
New Zealand	0.0721***	0.0374***	0.2671***	0.5095***	-1666.9062
	(0.0208)	(0.0153)	(0.0371)	(0.0757)	
Norway	0.0872***	-0.0452***	0.3076***	0.2922***	-1531.8438
	(0.0203)	(0.0168)	(0.0280)	(0.0664)	
Brazil	0.0884***	-0.0459***	0.1921***	0.8804***	-1981.3540
	(0.0217)	(0.0174)	(0.0262)	(0.0630)	
Iceland	0.0798***	-0.0414***	0.3023***	0.4320***	-1650.9902
	(0.0227)	(0.0168)	(0.0240)	(0.0547)	
Japan	0.0223	-0.0128	0.2138***	0.3408***	-1295.0508
	(0.0167)	(0.0099)	(0.0203)	(0.0694)	
Mexico	0.0007	-0.0004	0.1238***	0.4611***	-1005.0023
	(0.0145)	(0.0075)	(0.0117)	(0.0619)	
Taiwan	0.0021	-0.0011	0.0145***	0.8612***	87.5185
	(0.0061)	(0.0032)	(0.0020)	(0.0680)	
Thailand	0.0108	-0.0092	0.0405***	0.6366***	-260.4283
	(0.0080)	(0.0059)	(0.0045)	(0.0728)	

Note. The estimation results are based on the following model specification.

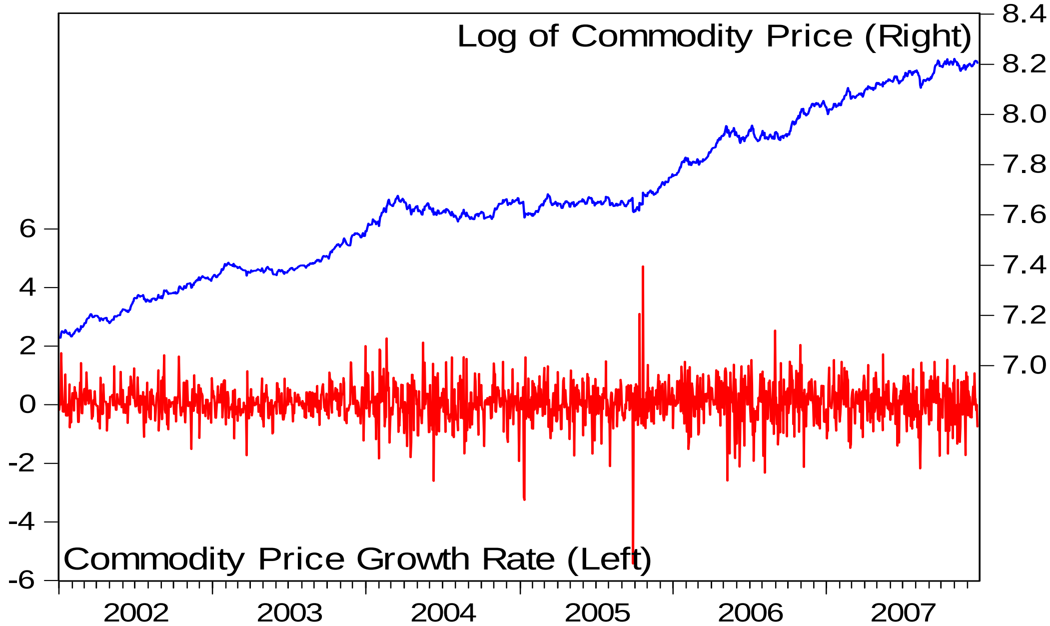
$$r_{t+1} = \alpha_1 \sigma_{t+1}^2 + \alpha_2 \left(u_{t+1}^2 - \sigma_{t+1}^2 \right) + \varepsilon_{t+1},$$

$$\varepsilon_{t+1} \sim N\left(0, \sigma_{\varepsilon,t+1}^2\right), \ \sigma_{\varepsilon,t+1}^2 = \gamma_0 + \gamma_1 \left(k_1 \varepsilon_{t-1}^2 + k_2 \varepsilon_{t-2}^2 + \cdots + k_p \varepsilon_{t-p}^2 \right)$$

$$\alpha_2 = -\frac{\alpha_1 \beta_1 \sum_{i=1}^{p} \rho^i k_i}{1 - \beta_1 \sum_{i=1}^{p} \rho^i k_i}, k_i = \frac{p+1-i}{\sum_{i=1}^{p} i}, \ \sum_{i=1}^{p} k_i = 1, \ p = 10$$

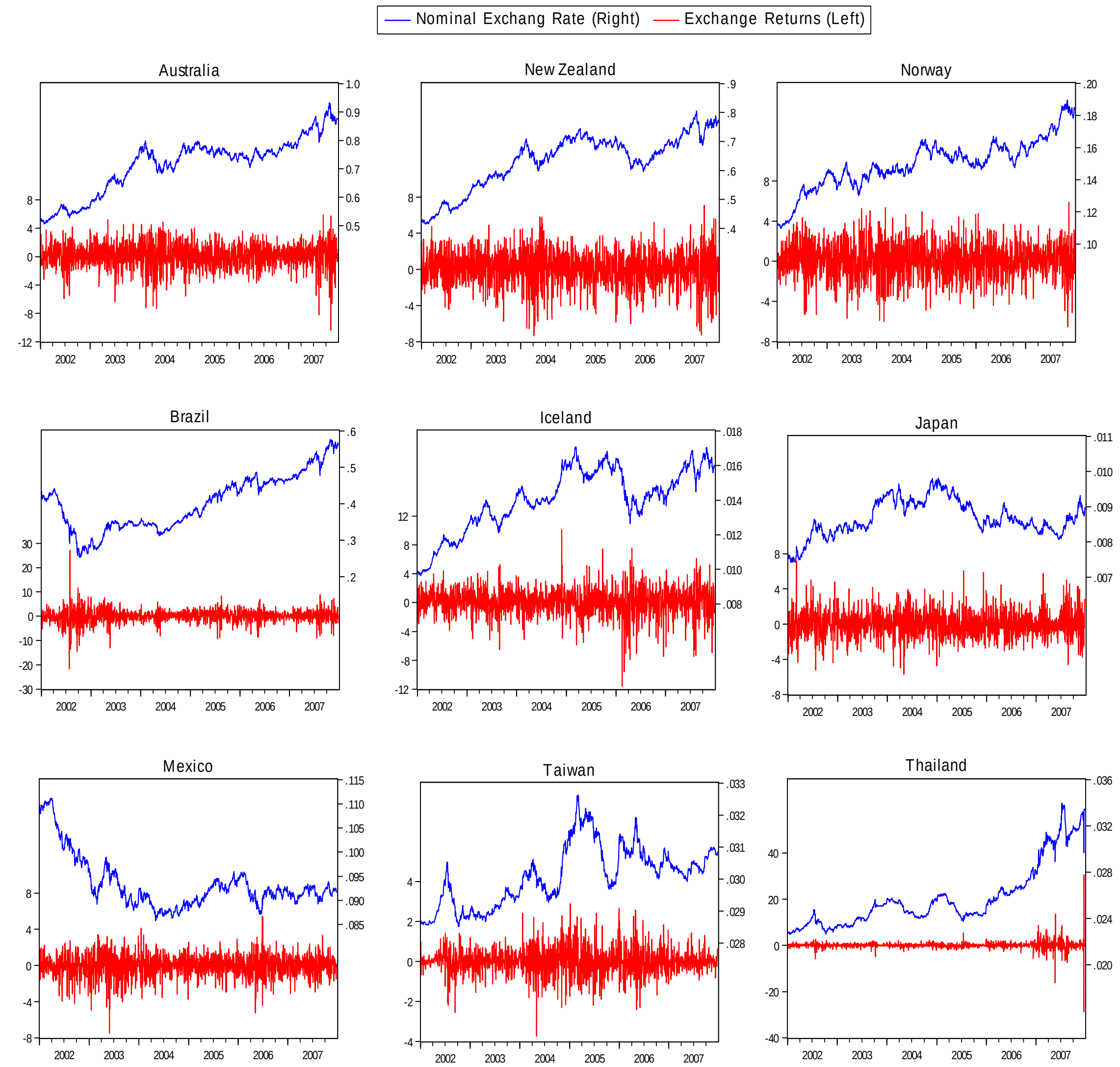
Standard errors are in parenthesis. *, **, and *** are significant at the 10%, 5%, and 1% level respectively.

Figure 1. The Commodity Price Index



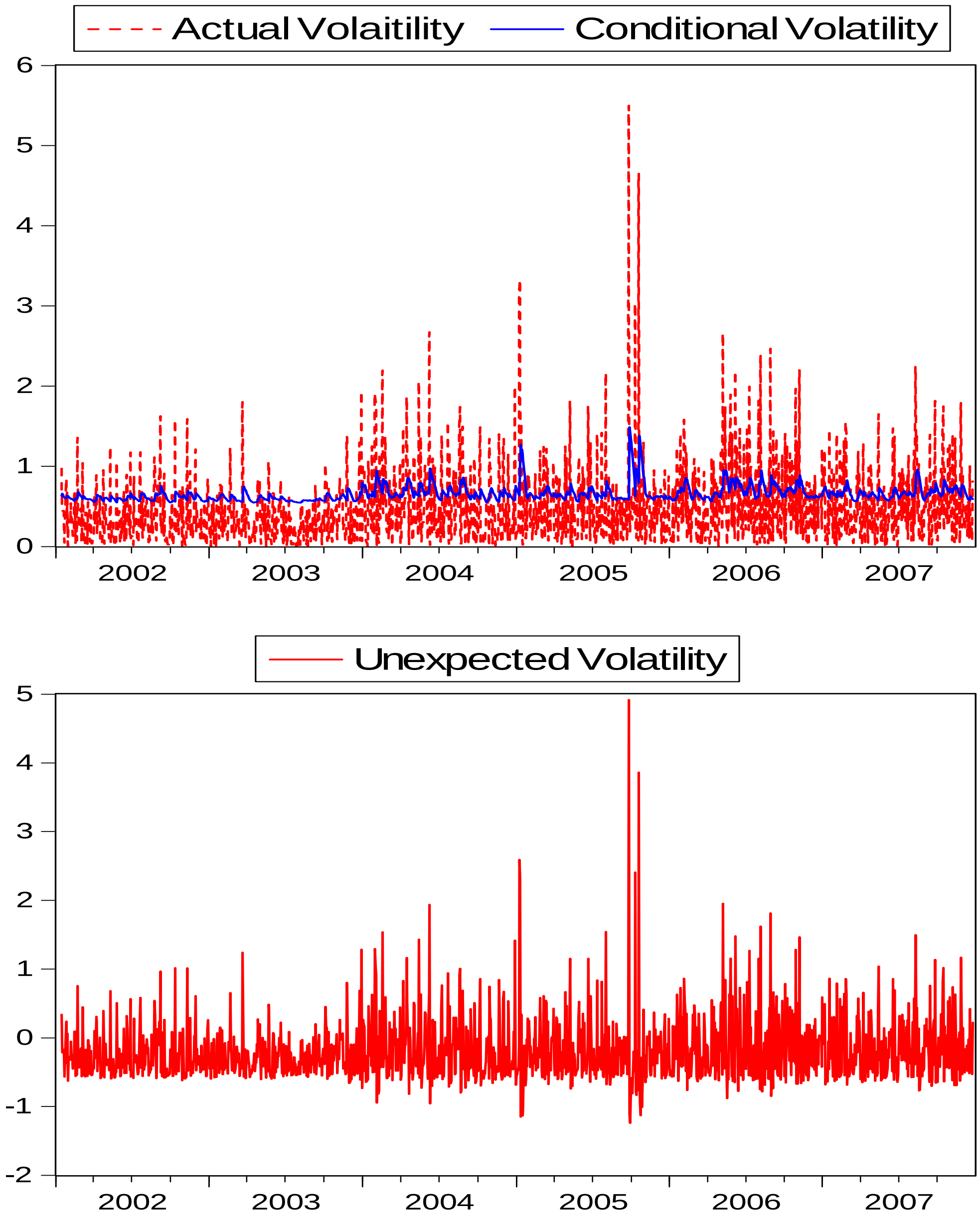
Note. The Moody's commodity price index is used from Global Financial Data. The commodity price growth rate is defined as log difference of commodity price * 100.

Figure 2. Nominal Exchange Rate and Exchange Returns



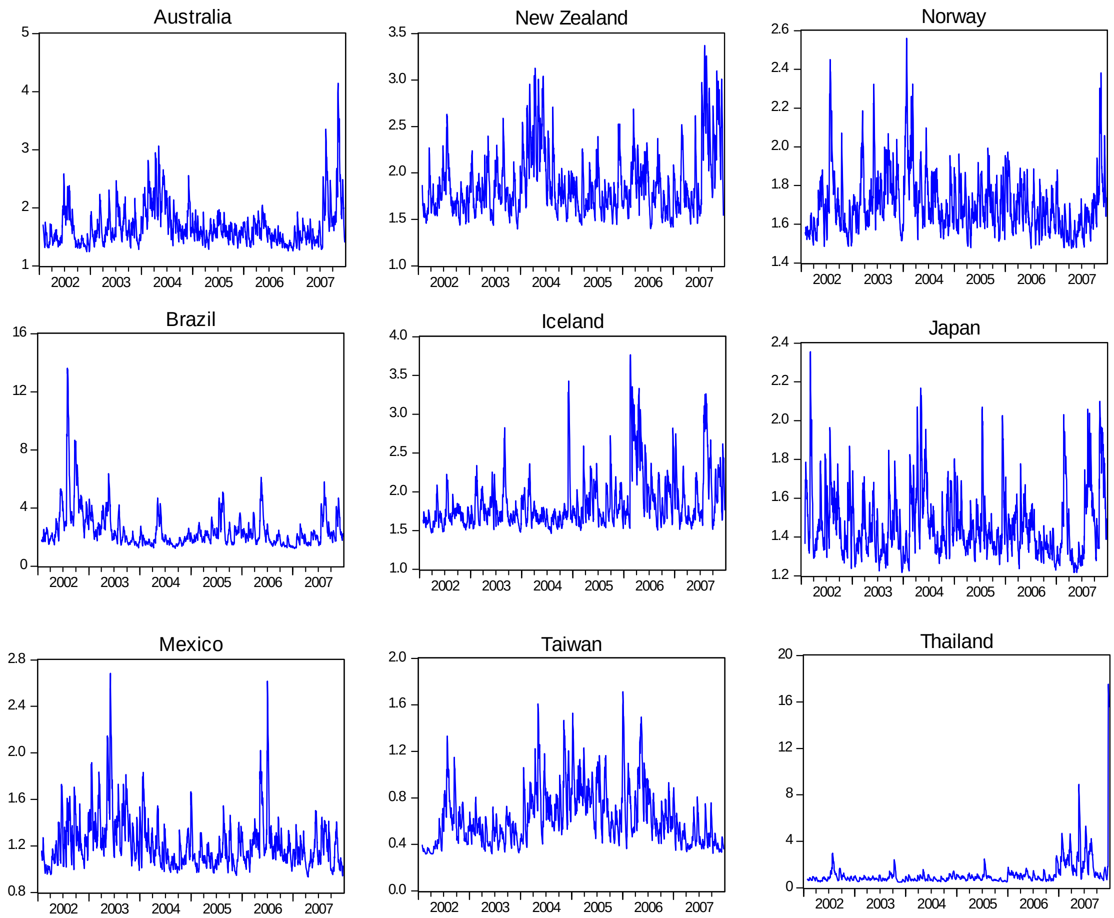
Note. Daily data are used from 2002 to 2007. The right axis is for nominal exchange rate and the left axis is for exchange returns.

Figure 3. Actual and Conditional Volatility, and Unexpected Volatility



Note. This figure is based on the estimation results reported in Table 3. Actual volatility is squared residual, while conditional volatility is conditional variance of the commodity price in each time. Unexpected volatility is obtained by subtractive actual volatility to conditional volatility.

Figure 4. Conditional Volatility of Exchange Returns



Note. This figure is based on the estimation results of the disturbance term, ε_{t+1}. where $\varepsilon_{t+1} \sim N\left(0,\ \sigma^2_{\varepsilon,t+1}\right),\ \sigma^2_{\varepsilon,t+1} = \gamma_0 + \gamma_1 \sum_{i=1}^{q} l_i \varepsilon^2_{t+1-i},\ l_i = \frac{q+1-i}{\sum_{i=1}^{q} i},\ \sum_{i=1}^{q} l_i = 1,\ q = 10.$