New Product Design and Pricing in a Duopoly Market with Consumer Rational Expectations

by

Aysun Ozler
Ted Klastorin
Yong-Pin Zhou

ISOM Department
Michael G Foster School of Business
University of Washington
Seattle, WA 98195-3500

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Abstract

In this paper, we consider the development and introduction of a new product into a durable goods duopoly market. We assume that there is an innovator firm who initially begins the development of the product; after a random time period, information about this product “leaks” to the market and an imitator firm begins to develop a competing product. Depending on the quality/complexity of each firm’s respective product, either the innovator or the imitator firm could be first to introduce their product into the market and enjoy a monopoly until the other firm enters the market. We assume that consumers purchase at most one unit of the product when they have maximum positive utility surplus that is determined by the characteristics of the product, the consumer’s marginal utility, and the consumer’s discounted utility for future expected products and prices. Using our model, we derive implications for both profit-maximizing firms with respect to the design and pricing of their respective products.
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1. Introduction

The introduction of many durable goods follows a common pattern: an innovator firm initially develops a new product and is then followed by one or more imitator firms who subsequently produce competing products after learning of the innovator firm’s efforts. Many well-known technology products have followed this pattern; for example, consider the ubiquitous digital audio player (DAP). In 1997, Saehan Information Systems began development of the first mass-produced MP3 digital audio player (the “MPMan”) that it introduced to the US market in the summer of 1998. Following Saehan’s initial development, Diamond Multimedia began developing a competing product, the Rio PMP300, that was introduced a few months later (in September, 1998). Both players had similar technical characteristics (e.g., they both used 32MB flash memory) and similar prices (the MPMan sold for $250 while the Rio PMP300 sold for $299). The Rio PMP300 was smaller and more stylish; however, both products were eclipsed by the Apple iPod when it was introduced in 2001. The research described in this paper suggests that Diamond Multimedia erred by producing a product that was technologically similar to its competitor; the Apple iPod’s success, on the other hand, partly relies on differentiating itself from its competitors (the Apple iPod used a hard disk instead of flash memory).

The history of digital audio players (DAPs) illustrates numerous factors that are common in the development and introduction of many durable goods. Saehan (the innovator firm) introduced their DAP into the US market first but enjoyed only a brief monopoly period before the Rio PMP300 was available. While it is unclear when Diamond Multimedia began to develop their DAP, it is likely that they started development after learning of the efforts by Saehan but prior to the introduction of the “MPMan”. Furthermore, although Saehan had a (brief) monopoly on DAPs during the summer of 1998, potential consumers during this time were undoubtedly aware of the impending introduction of the Rio PMP300 later that year—knowledge that may have affected their purchase decisions.
In this paper, we analyze this process of developing and introducing a durable good such as the DAP in a market that is characterized by two homogeneous firms. We assume that the product has a finite lifespan and that there is a set of potential consumers who enter the market uniformly throughout this period and purchase at most one unit. Furthermore, we assume that the product represents an incremental improvement of an existing product so that the information diffusion process can be considered exogenously and the potential demand is constant at any time during the product life cycle (Klastorin and Tsai 2004, Bayus et al. 1997, Cohen et al. 1996).

In our model, the innovator firm begins development of a new product; after some (random) time, information about the innovator firm’s efforts “leaks” to the market. As a result, the second firm (the imitator firm) begins developing a competing product. Following Moorthy (1988) and others, we assume that the development time and variable cost of each product are functions of the characteristics of each product that can be measured by a single scalar. Depending on the development time function, the complexity of the product, as well as the time that the information becomes available, either the innovator or imitator firm could be the first firm to introduce their product to the market.

In this paper, we model this product introduction process as a Stackelberg game where consumers’ utility surplus and discounted expectations impact their purchase decisions. We show that duopoly price never exceeds the initial monopoly price for the same product and show how firm characteristics and consumer expectations impact overall profitability and market share. To the best of our knowledge, our work is the first to investigate product entry times and design and pricing decisions in a duopoly market where consumers are forward-looking and make their purchasing decisions based their discounted utility surplus.

1.1 Literature Review

Our work is related to numerous previous papers in both the marketing and operations management literatures. Dockner and Jorgensen (1988) generalized dynamic pricing strategies in a differential game model in dynamic oligopolies. Kouvelis and Mukhopadhyay (1999) studied pricing and product design quality in a competitive diffusion model setting. Klastorin and Tsai (2004) extended their work by including pricing and timing decisions in a duopoly market when
the product lifespan is finite. Our work is also related to previous research on time-based competition. Cohen et al. (1996) analyzed the time-to-market decision for product replacement over a given entry period when demand was a function of product design/quality level (they did not consider alternative pricing however). Bayus et al. (1997) studied the trade-off between time-to-market and product quality in a duopoly market over an infinite time horizon. They analyzed how one firm’s decision affects the other firm’s design and product entry-timing decisions. Bayus (1997) analyzed the optimum product quality and entry timing decisions for different cost, market and demand conditions. Morgan et al. (2001) extended these previous papers by studying a multi-generation product with fixed cost of product development.

With respect to product and price competition, our work is related to Hotelling (1929), Moorthy (1988), and Schmidt and Porteus (2000). Moorthy (1988) studied a duopoly competition where two firms simultaneously determine their respective product designs and then subsequently set product prices. In his paper, Moorthy assumed that consumers were myopic and purchased the product that maximized their utility surplus at the time that they entered the market (a similar assumption was made by Klastorin and Tsai, 2004). Schmidt et al. (2000) considered the case when a firm developed a new product that competes with an existing product in a market with linear reservation price curves.

Moorthy and Png (1992) studied product positioning in a dynamic context (i.e., when should a supplier introduce a product sequentially?). Dhebar (1994) analyzed a two period durable-good monopolist who introduces a new product in the first period and an upgraded version in the second period. In Dhebar’s model, the firm determines the product prices in both periods as well as the upgraded product scope/design to maximize its expected net present profit over both periods. Consumers in Dhebar’s model are forward-looking in the sense that they consider their expectation of the upgraded product when making a purchase decision in the first period. Kornish (2001) extended Dhebar’s model to the case when the upgrade design/scope was exogenously determined. Ramachandran and Krishnan (2008) studied entry timing decisions in a monopoly market with a rapidly improving product design. Padmanabhan et al. (1997) analyzed a sequential product introduction problem and investigated the implications of consumer uncertainty regarding network externalities. In this work, the authors showed that it may be beneficial to the firm to
provide “private” information to consumers. Fudenberg and Tirole (1998) studied the monopoly pricing of overlapping generations of a durable good under different market conditions. Bala and Carr (2009) analyzed the upgrade pricing policies by characterizing the relationship between the product upgrade magnitudes and upgrade pricing structures along with the user upgrade costs. Yin et al. (2009) studied the impact of used goods on the product upgrade and pricing decisions in a monopoly market. Bhaskaran and Gilbert (2005) studied the selling and leasing strategies of a durable goods manufacturer in the presence of a complementary product.

1.2 Overview and Contributions

Our work extends these previous papers in several important ways. First, we model the product introduction process as a Stackelberg game where an innovator firm is the market leader and a second firm (the imitator firm) is the follower. In our model, the imitator firm only begins developing a competing product when they learn of the innovator firm’s efforts; we assume that the time between the start of the innovator firm’s development effort and the time when this information “leaks” to the market is a random variable (truncated by the time when the innovator firm introduces their product to the market). Finally, our model allows the consumers to be forward-looking; that is, a consumer may postpone the purchase of a product with current positive utility surplus in order to wait for an expected future product that offers a greater (discounted) utility surplus.

This paper offers several important contributions. First, our model provides a comprehensive framework for analyzing product development decisions in a duopoly market with consumers who have “rational expectations”. Second, we derive conditions that describe how firms should act in such a market to maximize their respective profits; our results provide likely explanations for the success or failure of many well-known durable consumer goods. Finally, we show how the values of various parameters (e.g., new product development time, discount rate) impact the decisions of both firms with respect to product design and pricing decisions.

The paper is organized as follows. In the second section, we present our assumptions and describe the model. In the third section, we define conditions for defining when equilibrium solutions exist and derive results for profit-maximizing firms. In the fourth section, we present
the results of our numerical experiments and discuss the managerial implications. Finally, we summarize our results and discuss several implications that our work offers for other fields (e.g., supply chain management).

2. Model Description

There are two competing firms; we will denote firm X as the innovator firm and firm Y as the imitator firm (throughout the remainder of this paper, we will use subscripts X and Y to indicate firm-specific variables). The firms are homogeneous with respect to cost and development time characteristics. Each firm develops a single product that is characterized by a scalar $s_k, k = X, Y$; once a firm has determined the product design level, we assume that it cannot be changed. The design level of the product reflects the number of features, the performance level and the overall quality of the product. The time it takes to develop the product, $g(S)$, is a linear function of the product design level $S$: $g(S) = \beta S$ where $\beta > 0$ is the time to develop an additional unit of design level. This assumption is based on the empirical studies that demonstrated that the speed of product performance improvement can be represented by a Cobb-Douglas model (Cohen et al. 1996). Following Moorthy (1988) and Klastorin and Tsai (2004), we assume that the variable production cost with design level $S$ is given by $c(S) = \alpha S^2$ where $\alpha > 0$ is the product cost coefficient.

Firm X initially sets the design level $S_X$ of its product. After learning of firm X’s product design level, $S_X$, firm Y begins to develop a competing product. The leadtime between firm X’s initial development and firm Y learning of this effort, which we call the information release time, is given by $L$, which is defined as a truncated exponential distribution with rate $\lambda$. We will denote its cumulative distribution function (CDF) by $F(\cdot)$.

We model both firms’ product development and pricing decisions as a Stackelberg game. Let $[0,T]$ define the product lifespan. At time 0, stage 1 starts and the innovator firm (firm X) sets the design level of its product ($S_X$). When firm Y learns the value of $S_X$ at the beginning of stage 2 (time $L$), it sets its own product design level $S_Y$. Depending on the values of $S_X$, $S_Y$, and $L$, either product X or product Y could be the first to launch its product:
• If \( L + g(S_Y) > g(S_X) \), product X is released first at the beginning of stage 3 (see Figure 1a) and sets the monopoly price \( P_{XM} \) at that time. When product Y is released at the beginning of stage 4, both firms simultaneously set their respective duopoly prices \( (P_{XD} \text{ and } P_{YD}) \).

• If \( L + g(S_Y) < g(S_X) \), product Y enters the market before product X (see Figure 1b). Firm Y sets its monopoly price \( P_{YM} \) at the beginning of stage 3 and both firms set their respective duopoly price \( (P_{XD} \text{ and } P_{YD}) \) at the beginning of stage 4.

Insert Figure 1 Here

We assume there are \( M \) potential consumers in total and they enter the market uniformly over the product lifespan \([0, T]\) (Klastorin and Tsai 2004, Bayus et al. 1997, Cohen et al. 1996). Consumers’ willingness to purchase a new product is determined by the price and design/quality of the current product(s), as well as their expectations of future products’ design and price. Some consumers who arrive during stage 2 will remain in the market based on their expectation of future products and prices even though no products are available in this stage. During stage 3, some consumers will purchase the monopoly firm’s product while others will choose to wait (expecting lower prices and/or better products).

Each consumer derives a utility \( u_k = vS_k \), \((k=X,Y)\) for each firm \( k \)'s product where \( v \) is the marginal consumer valuation of each unit of product design. Without loss of generality, we assume \( v \) to be uniformly distributed over \([0, 1]\). If \( P_k \) denotes the price set by firm \( k \) \((k = X,Y)\) for its respective product, the consumer’s utility surplus is defined by \( U_k = vS_k - P_k \). If consumers are myopic, upon entering the market they would purchase the product that provides the greater (positive) utility surplus. In our model, however, consumers are forward-looking and may choose to defer a current purchasing decision if her discounted utility for an expected future product is greater than her current utility surplus. Following Dhebar (1994) and Kornish (2001), we assume that future utilities and prices are discounted by a rate \( \delta \).
To simplify our model, we assume that consumers who arrive during any stage indicated in Figure 1 arrive at the beginning of that stage. Since no new information is available to consumers who arrive during any stage, this assumption is equivalent to changing the discounting of cash flows resulting from consumers’ purchases during a stage. Our numerical analyses indicate that this assumption has a negligible impact on our results.

While firm X begins developing its product at time $t = 0$, the market only becomes aware of this effort at time $t = L$. While no products are available in stage 2, some consumers who arrive during this stage will remain in the market based on their expectation of future products and prices. Since there are no products on the market until stage 3 (the monopoly period), we can group these consumers together with those who arrive at the beginning of the monopoly period without loss of generality, and refer to them as Group M. Finally, more consumers will arrive in stage 4 (the duopoly period). We will refer to them as Group D.

Both firms seek to maximize their total discounted profit by optimally setting their own product design level and duopoly and/or monopoly prices in this Stackelberg game. The respective profit functions of each firm, however, depend in large part on who launches the product first. To simplify exposition in the remainder of the paper, we will denote the first firm to launch its product by firm $i$, and the second the market firm $j$. Also let $l_m$ $(m=1, 2, 3, 4)$ be the length of the four stages in Figure 1.

- In case A where $L + g(S_1) \geq g(S_X)$ and firm X launches first, we have $i=X, j=Y, l_1=L, l_2=g(S_X)-L, l_3=L + g(S_1) - g(S_X)$, and $l_4= T - L - g(S_1)$.
- In case B where $L + g(S_1) < g(S_X)$ and firm Y launches first, we have $i=Y, j=X, l_1=L, l_2=g(S_1), l_3= g(S_X) - L - g(S_1)$, and $l_4= T - g(S_X)$.

The unified timeline using this notation can be found in Figure 1c.

Below is a summary of all the variables and parameters that we have discussed.

$S_i$: design level of product $k$ where $k=X$ or $Y$

$\beta$: time to develop an additional unit of design level
\( \alpha: \) cost to develop a unit of design level

\( L: \) information release time

\( \lambda: \) rate of information release time

\( T: \) lifespan of the product

\( P_{im}: \) price of product \( k \) in period \( m \) where \( k=X \) or \( Y \) and \( m=M \) (monopoly) or \( D \) (duopoly)

\( v: \) marginal consumer valuation of each unit of product design

\( U_k: \) consumer’s utility surplus for product \( k \) where \( k=X \) or \( Y \)

\( \delta: \) discount rate

\( M: \) total number of potential consumers arriving to the market during the lifespan of the product

\( D_{kmn}: \) the size of the market segment that will purchase product \( k, \ k=X \) or \( Y; \)

\( m \) indicates the consumer arrival period; \( m=D \) (arrival during duopoly period) or \( M \) (arrival before duopoly period)

\( n \) indicates the consumer purchase period; \( m=D \) (duopoly period) or \( M \) (monopoly period)

We will now develop the decision models and derive the optimization solutions using backward induction. At the beginning of the monopoly period, firm \( i \) sets monopoly price \( P_{iM} \) and then, at the beginning of the duopoly period, both firms set duopoly prices \( P_{iD} \) and \( P_{iD} \). Since consumers arrive to the market uniformly, the size of Group M is \( \frac{M}{T-L}(l_2+l_3) \). These consumers will make one of four purchasing decisions: 1) buy product \( i \) during monopoly period, 2) buy product \( i \) during duopoly period, 3) buy product \( j \) during duopoly period, and 4) buy nothing and leave the market. We will denote the sizes of the first three segments by \( D_{iMM}, D_{iMD}, \) and \( D_{jMD}. \) Here the first subscript index indicates the product that is purchased, the second index indicates which group (M or D) the consumers belong to, and third index indicates the purchase period (M or D) of these consumers. Similarly, the size of Group D is \( \frac{M}{T-L}l_4 \), and these consumers will
make one of three decisions: 1) buy product $i$ during duopoly period, 2) buy product $j$ during duopoly period, and 3) buy nothing and leave the market. The sizes of the first two consumer segments will be similarly denoted by $D_{iDD}$ and $D_{jDD}$. All the $D$ variables here are determined by consumers’ utility surplus and the prices $P_{XM}$, $P_{XD}$, and $P_{YD}$. Their explicit expressions will be given later in Section 3.

At the beginning of the duopoly period, design levels $S_i$ and $S_j$, time $L$, and monopoly price $P_{iM}$ are already known. Both firms solve the following optimization problems simultaneously:

$$
\begin{align*}
\pi_{jd} (S_i, S_j, L, P_{iM}) &= \max_{p_{id}} \left( P_{id} - \alpha S_i^2 \right) \left( D_{iMD} + D_{iDD} \right) \\
\pi_{jd} (S_i, S_j, L, P_{iM}) &= \max_{p_{jd}} \left( P_{jd} - \alpha S_j^2 \right) \left( D_{jMD} + D_{jDD} \right)
\end{align*}
$$

Let the optimal (equilibrium) duopoly prices be $P_{iD}^* (S_i, S_j, L, P_{iM})$ and $P_{jD}^* (S_i, S_j, L, P_{iM})$.

At the beginning of the monopoly period, $S_i$, $S_j$, and time $L$ are known, and firm $i$ aims to optimize total discounted future profit by solving

$$
\pi_{iM} (S_i, S_j, L) = \max_{p_{iM}} \left( P_{iM} - \alpha S_i^2 \right) D_{iMM} + \delta S \pi_{jd} (S_i, S_j, L, P_{iM}).
$$

Let the optimal solution be $P_{iM}^* (S_i, S_j, L)$. Firm $j$ does not make any decision here, $\pi_{jd} (S_i, S_j, L)$ is simply calculated as $\pi_{jd} (S_i, S_j, L) = \delta S \pi_{jd} (S_i, S_j, L)$.

Further going back in time, since firm $Y$ is the imitator, when it sets its design level at the beginning of stage 2, both $S_X$ and time $L$ are known, so firm $Y$ solves

$$
\begin{align*}
\pi_Y (S_X, L) &= \max_{S_Y} \left\{ \pi_{YM} (S_X, S_Y, P_{XM}^* (S_X, S_Y, L)) \mid L + g (S_Y) \geq g (S_X) \right\} \\
&\cup \left\{ \pi_{YM} (S_Y, S_X, P_{YM}^* (S_Y, S_X, L)) \mid L + g (S_Y) < g (S_X) \right\}
\end{align*}
$$

Denote the optimal solution by $S_Y^* (S_X, L)$. 

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All of the above optimization problems are solved after time $L$ has already occurred. When firm $X$ sets its design level $S_X$ at the beginning, however, $L$ is still a random variable. Thus, its optimization problem has to consider the possibility of both cases above

$$\pi_X = \max_{S_X} \left[ \int_{L + g(S^*_Y) > g(S_X)} \pi_{XM} \left( S_X, S_Y^*(S_X, L), L \right) dF(L) + \int_{L + g(S^*_Y) \leq g(S_X)} \pi_{XM} \left( S_Y^*(S_X, L), S_X, L \right) dF(L) \right]$$

We will denote the optimal product design by $S^*_X$.

In the next section, we will derive the optimal solutions. For clarity of exposition, the state-dependency of all the solutions will be suppressed when it is clear from the context.

### 3. Model Solutions

Both firms have to make product design level and pricing decisions over time. The following proposition simplifies our analysis by showing that firm $Y$ will always differentiate its product from firm $X$’s product.

**Proposition 1:** Firm $Y$ would never set $S_Y$ such that $S_Y = S_X$.

**Proof:** For all the proofs in this paper, please see Appendix A.

The intuition behind Proposition 1 is straightforward. If firm $Y$ were to choose $S_Y = S_X$, it will always be the second to the market. During the duopoly period, both firms would have identical products; the duopoly would be a purely competitive market with downward sloping demand curves where firm $Y$ would not earn a positive profit. Thus, the only rational strategy for firm $Y$ is to set $S_Y \neq S_X$. This conclusion is further illuminated by the competition between Saehan Information Systems and Diamond Multimedia in the DAP industry that we described in the introduction.

To maintain analytic tractability, we will only consider two levels of product design: high ($S_{H}$) and low ($S_{L}$) in the remainder of the paper. Following Proposition 1, there are only two possible cases: when firm $X$ chooses $S_X = S_L$, firm $Y$ must choose $S_Y = S_H$, and vice versa.
Recall that firm $i$’s product is the first to be released to the market, so during the monopoly period only firm $i$’s product is available to consumers in Group M. With rational expectation of future products and prices in the duopoly period, Group M consumers can either purchase $i$’s product during the monopoly period or wait. They will make the purchase if and only if they have a positive utility surplus in the monopoly period, and it is greater than the discounted utility surplus that can be expected for products in the duopoly period. Hence, we have:

**Lemma 1** A consumer in Group M with valuation $v$ will buy product $i$ in the monopoly period if and only if

$$
\nu S_i - P_{IM} \geq \max \left\{ 0, \delta^i \left( \nu S_i - P_{ID} \right), \delta^i \left( \nu S_j - P_{JD} \right) \right\}
$$

(1)

Furthermore, let $v_1 = \frac{P_{IM}}{S_i}$, $v_2 = \frac{P_{IM} - \delta^i P_{ID}}{S_i (1 - \delta^i)}$ and $v_3 = \frac{P_{IM} - \delta^i P_{JD}}{S_i - S_j \delta^i}$. Depending on the value of $S_i - S_j \delta^i$, (1) may become one of the following three conditions:

1. $S_i - S_j \delta^i > 0$ and $v \geq \max \{v_1, v_2, v_3\}$;
   
   \[
   \text{in this case, firm } i \text{'s monopoly sales is } D_{IMM} = \frac{M}{T-L} l_3 \left[ 1 - \max \{v_1, v_2, v_3\} \right]^+ \text{ where } x^+ = \max\{x,0\}.
   \]

2. $S_i - S_j \delta^i < 0$ and $\max \{v_1, v_2\} \leq v \leq v_3$;

   \[
   \text{in this case, firm } i \text{'s monopoly market share } D_{IMM} = \frac{M}{T-L} l_3 \left[ v_3 - \max \{v_1, v_2\} \right]^+ .
   \]

3. $S_i - S_j \delta^i = 0$, $P_{IM} - \delta^i P_{JD} \leq 0$, and $v \geq \max \{v_1, v_2\}$;

   \[
   \text{in this case, firm } i \text{'s monopoly market share } D_{IMM} = \frac{M}{T-L} l_3 \left[ 1 - \max \{v_1, v_2\} \right]^+ .
   \]

Recall that $l_3 = L + g(S_T) - g(S_X)$ in case A and $l_3 = g(S_X) - L - g(S_T)$ in case B. It is thus a continuous random variable for which any particular value happens with probability zero. Consequently, the case shown in equation (4) happens with probability zero. We will therefore ignore the case represented by (4) and focus on the other two cases instead.
For those in Group M who do not purchase during the monopoly period, some may still purchase during the duopoly period. We will analyze them together with the consumers in Group D. During the duopoly period, consumers have three choices: 1) buy product \( i \), 2) buy product \( j \), and buy nothing. The utility surplus of these choices are \( vS_i - P_{id} \), \( vS_j - P_{jd} \), and 0 respectively. Therefore, consumers in Group D with valuation \( v \) will buy firm \( i \)'s product in the duopoly period if

\[
\begin{cases}
\frac{P_{id}}{S_i} \leq v \leq \frac{P_{id} - P_{id}}{S_j - S_i} & \text{if } S_i = S_L < S_H = S_j \\
v \geq \max \left\{ \frac{P_{id}}{S_i}, \frac{P_{id} - P_{id}}{S_j - S_i} \right\} & \text{if } S_i = S_H > S_L = S_j
\end{cases}
\]

(5)

Similarly, consumers in Group D with valuation \( v \) will buy firm \( j \)'s product if

\[
\begin{cases}
v \geq \max \left\{ \frac{P_{jd}}{S_j}, \frac{P_{jd} - P_{jd}}{S_j - S_i} \right\} & \text{if } S_i = S_L < S_H = S_j \\
\frac{P_{jd}}{S_j} \leq v \leq \frac{P_{jd} - P_{jd}}{S_j - S_i} & \text{if } S_i = S_H > S_L = S_j.
\end{cases}
\]

(6)

The form of (5) and (6) make sense. Once the duopoly period starts, what matters most is what design level each firm has, not which product comes to the market first. We note that the firm with a lower design level must also have a low-enough price during the duopoly period to attract consumers. The lemma below shows that the price ratio threshold is exactly \( S_L/S_H \).

**Lemma 2:** When \( S_i = S_L \) and \( S_j = S_H \), \( P_{id} \) must satisfy \( P_{id} < (S_L/S_H)P_{jd} \); when \( S_i = S_H \) and \( S_j = S_L \), \( P_{jd} \) must satisfy \( P_{jd} < (S_L/S_H)P_{id} \).

Following Lemma 2, we can simplify and combine (5) and (6) to obtain the market segmentation during the duopoly period:

**Corollary 1:** When \( S_i = S_L \) and \( S_j = S_H \), the consumers during the duopoly period will buy products from firm \( i \) and \( j \) if their valuation \( v \) satisfy the following conditions:
• buy product $i$ if $\frac{P^D_i - P^D_j}{S_j - S_i} \leq v \leq \frac{P^D_i - P^D_j}{S_j - S_i}$ and buy product $j$ if $v > \frac{P^D_i - P^D_j}{S_j - S_i}$. \hfill (7)

When $S_i = S_H$ and $S_j = S_L$, the consumers during the duopoly period will buy products from firm $i$ and $j$ if their valuation $v$ satisfy the following conditions:

• buy product $i$ if $v > \frac{P^D_i - P^D_j}{S_j - S_i}$ and buy product $j$ if $\frac{P^D_i - P^D_j}{S_j - S_i} \leq v \leq \frac{P^D_i - P^D_j}{S_j - S_i}$. \hfill (8)

Furthermore, in order to have a non-zero market segment, the firm with higher product design level will set its price so that $\frac{P^D_i - P^D_j}{S_j - S_i} < 1$.

So far, we have derived customer segmentations during monopoly and duopoly periods. The duopoly market condition can be very simply specified by (7) and (8). The monopoly market segmentation condition (1), however, is more complex, and must be simplified before it can be further used. We have done that preliminarily by simplifying (1) to (2), (3), or (4), depending on the value of $\delta_i$. Below we describe a more complete analysis that further simplifies (2), (3), and (4) into 7 mutually exclusive scenarios. For example, when $S_i - S_j \delta_i > 0$ and $1 \geq v_3 \geq v_2 \geq v_1$, (2) becomes $v_2 \leq v \leq v_3$, but when $S_i - S_j \delta_i > 0$ and $v_3 > 1 \geq v_2 \geq v_1$, (2) becomes $v_2 \leq v \leq 1 \leq v_3$. In our list they are called scenarios 1 and 2 respectively. For the complete list please refer to Table 1.

Insert Table 1 Here

Now that we have completely segmented the consumers in the monopoly and duopoly periods separately, we proceed to derive both firms’ total market share (throughout the time horizon) and their profit function. This requires us to combine the market segmentations in monopoly and duopoly periods. Specifically, each distinct scenario in Table 1 can be combined with the duopoly conditions (7) and (8) to generate a Market Configuration (MC). We give a complete list of all the possible combinations in Table 2. We show, however, many of some combinations are infeasible (i.e. leading to an empty set) and some combinations are clearly dominated by the others.
Eliminating all the infeasible or dominated combinations, only 10 MCs remain. We have derived both firms’ profit function under all the MCs, but will give details here only for MC1. The analysis of the other MCs are similar. A complete list of all the profit functions are given in Table 3; all the optimal design levels and prices for both firms can be derived from these functions.

**Analysis of Market Configuration (MC) 1:**

Market configuration 1 (MC1) is the combination of monopoly period scenario 1 in Table 1 and the duopoly conditions (7) and (8). Under MC1 firm X develops a low design level product and enters the market first. Therefore, we have \( i=X, S_i=S_L, j=Y, \) and \( S_j=S_H. \) All the monopoly period and duopoly period conditions lead to the following constraints:

\[
P_{XM} \leq \frac{P_{YD} S_L (1-\delta^i)} {S_H - S_L} - P_{XD} S_L + P_{XD} S_H \delta^i
\]  
(9)

\[
P_{XM} \geq P_{XD}
\]  
(10)

\[
P_{XM} - \delta^i P_{YD} \leq S_L - S_H \delta^i
\]  
(11)

\[
\frac{P_{XD}} {S_L} < \frac{P_{YD}} {S_H}
\]  
(12)

\[
\frac{P_{YD} - P_{XD}} {S_H - S_L} < 1.
\]  
(13)

We also need the following constraints to represent the fact that both firms will price their products higher than its (non-zero) unit production cost:

\[
P_{XD} \geq \alpha S_L^2
\]  
(14)

\[
P_{XM} \geq \alpha S_L^2
\]  
(15)

\[
P_{YD} \geq \alpha S_H^2
\]  
(16)

Hence, MC1 is the set of all feasible parameter defined by (9)-(16). Consumer segmentations for both Group M and Group D can be easily deducted from Table 1 scenario 1 and conditions (7) and (8), and they are presented in Figure 2.

*Insert Figure 2 Here*
Remark 1 Under MC1, we must have $P_{AD} \leq P_{AM}$. This means firm X takes advantage of its monopoly period and price its product higher than when it’s competing with firm Y in the duopoly period. However, under MC1, since monopoly period (i.e. $l_3$) is relatively small ($S_L - S_H S^h < 0$), firm Y’s entry into the market is relatively soon. All the consumers are aware of this so firm X cannot set price in the monopoly period too high. In fact, \[ \frac{P_{XM}}{S_L} < \frac{P_{YD}}{S_H} \] Overall, \[ \frac{P_{XD}}{S_L} < \frac{P_{XM}}{S_L} < \frac{P_{YD}}{S_H} \] implies $P_{AD} < P_{XM} < P_{YD}$.

From Figure 2, we obtain the following market share values:

\[
D_{XMM} = \frac{M}{T-L} (l_2 + l_3) (v_3 - v_2), \quad D_{XMD} = \frac{M}{T-L} (l_2 + l_3) \left( v_2 - \frac{P_{XD}}{S_L} \right),
\]

\[
D_{XDD} = \frac{M}{T-L} l_4 \left( \frac{P_{YD} - P_{XD}}{S_H - S_L} - \frac{P_{XD}}{S_L} \right), \quad D_{YMD} = \frac{M}{T-L} (l_2 + l_3) (1 - v_3),
\]

\[
D_{YDD} = \frac{M}{T-L} l_4 \left( 1 - \frac{P_{YD} - P_{XD}}{S_H - S_L} \right).
\] (17)

Abusing the notation slightly, we express both firms’ profit functions (before prices are optimized) as follows:

\[
\pi_{XD} = (D_{XMD} + D_{XDB}) \left( P_{XD} - \alpha S_L^2 \right)
\]

\[
= \frac{M}{T-L} \left( l_2 + l_3 \right) \left( v_2 - \frac{P_{XD}}{S_L} \right) + l_4 \left( \frac{P_{YD} - P_{XD}}{S_H - S_L} - \frac{P_{XD}}{S_L} \right) \left( P_{XD} - \alpha S_L^2 \right), \] (18)

\[
\pi_{XM} = D_{XMM} \left( P_{XD} - \alpha S_L^2 \right) + \delta^l \pi_{XD}
\]

\[
= \frac{M}{T-L} \left\{ l_3 (v_3 - v_2) + \left[ l_3 \left( v_2 - \frac{P_{XD}}{S_L} \right) + l_4 \left( \frac{P_{YD} - P_{XD}}{S_H - S_L} - \frac{P_{XD}}{S_L} \right) \right] \delta^l \right\} + \left( P_{XD} - \alpha S_L^2 \right), \] (19)

\[
\pi_{YD} = (D_{YMD} + D_{YDD}) \left( P_{YD} - \alpha S_H^2 \right) = \frac{M}{T-L} \left\{ l_3 (1 - v_3) + l_4 \left( 1 - \frac{P_{YD} - P_{XD}}{S_H - S_L} \right) \right\} \left( P_{YD} - \alpha S_H^2 \right), \] (20)

\[
\pi_{YM} = \delta^l \pi_{YD}.
\] (21)
Thus far, we have characterized market shares and profits for both firms under MC1 without optimizing the prices. Next, we go backwards in time to find the equilibrium solution for both firms. To find $P_{XD}$ and $P_{YD}$, we solve the following problems for firms $i$ and $j$ simultaneously:

$$\begin{align*}
\max_{P_{XD}} \pi_{XD} & \quad \text{s.t. Constraints (9)-(16)} \\
\max_{P_{YD}} \pi_{YD} & \quad \text{s.t. Constraints (9)-(16)}
\end{align*}$$

**Lemma 3** The profit functions in (18) and (20) are concave in $P_{XD}$ and $P_{YD}$ respectively.

Lemma 3 allows us to solve the first order conditions of (18) and (20) to derive the constrained equilibrium duopoly prices, $P^*_X$ and $P^*_Y$.

**Proposition 2** Under Market Configuration 1, the unconstrained equilibrium duopoly prices are:

$$\begin{align*}
P^*_X &= \frac{A_1 E_i + B_1}{1 - A_1 D_1} P_{XM} + \frac{A_1 F_i + C_1}{1 - A_1 D_1} \\
P^*_Y &= \left( \frac{A_1 E_i + B_1}{1 - A_1 D_1} D_1 + E_i \right) P_{YM} + \left( \frac{A_1 F_i + C_1}{1 - A_1 D_1} D_1 + F_i \right)
\end{align*}$$

where

$$\begin{align*}
A_1 &= \frac{S_L l_4 (1-\delta^i)}{2 S_H \left[ (T - L)(1-\delta^i) - \beta (S_L - S_H \delta^i) \right]} > 0 \\
B_1 &= \frac{\beta (S_H - S_L)}{\left[ (T - L)(1-\delta^i) - \beta (S_L - S_H \delta^i) \right]} > 0 \\
C_1 &= \frac{\alpha S_H S_L^2 l_4 (1-\delta^i)}{2 S_H \left[ (T - L)(1-\delta^i) - \beta (S_L - S_H \delta^i) \right]} > 0 \\
D_1 &= \frac{l_4 (S_L - S_H \delta^i)}{2 \left[ (T - L)(S_L - S_H \delta^i) - \beta S_L S_H (1-\delta^i) \right]} > 0 \\
E_1 &= \frac{-\beta S_H (S_H - S_L)}{2 \left[ (T - L)(S_L - S_H \delta^i) - \beta S_L S_H (1-\delta^i) \right]} > 0
\end{align*}$$
\[ F_i = \frac{\left( S_h - S_L \right) (T - L) \left( S_L - S_h \delta^{t_h} \right) + \alpha S_{h}^{2} \left( T - L \right) \left( S_L - S_h \delta^{t_h} \right) - S_{h}^{2} S_L \alpha \beta \left( 1 - \delta^{t_h} \right)}{2 \left[ \left( T - L \right) \left( S_L - S_h \delta^{t_h} \right) - \beta S_L S_{h} \left( 1 - \delta^{t_h} \right) \right]} > 0. \]

Furthermore, \( A_1D_1 < 1 \).

**Remark 2** \( A_1D_1 < 1 \) implies the equilibrium duopoly prices \( P_{XD}^{*} \) and \( P_{YD}^{*} \) are both increasing in firm \( X \)'s monopoly price \( P_{XM} \). This reflects the impact of consumer rational expectation: As the monopoly price goes up, purchasing in the duopoly period becomes more attractive and more consumers in Group M will postpone their purchases. This allows both firms to charge higher prices in the duopoly period.

The next step is to find firm \( X \)'s optimal monopoly price, \( P_{XM}^{*} \), by solving the following problem:

\[
\max_{P_{XM}} \pi_{XM} \quad \text{s.t. Constraints (9)-(16)}. \]

Again, we solve the problem above and find \( P_{XM}^{*} \) using FOCs. We have not been able to show the concavity of \( \pi_{XM} \) with respect to \( P_{XM} \), but it is confirmed in all the numerical cases we have tested.

Henceforth, we have completed the analysis of MC1. Analysis of the other MCs can be performed similarly, and we have summarized the results in Table 3.

4. **Numerical Analyses and Managerial Implications**

As described in the previous sections, firm X (the innovator firm) sets the design/quality of its product knowing only that the information about its product will become known to the market after some time, \( L \). Firm Y, on the other hand, sets the design level \( S_Y \) of its product after learning the exact values of \( S_X \) and \( L \). Thus, the information release rate \( \lambda \) should impact firm X’s profitability and design decisions, while firm Y’s decisions and profitability would be dependent on the realized value of \( L \). The decisions of both firms and their resultant profits also depend on the values of \( \alpha \) (variable production cost per unit of design), \( \beta \) (development time coefficient), and the discount rate \( \delta \). In this section, we analyze the impact of these parameters on the firms’
profitability, market share, design decisions and product prices. In these analyses, we focus on the case when firm X is the first firm to enter the market. When firm Y was the first firm to enter the market, we found the results were similar to those reported here.

To estimate realistic parameter values, we followed the approach used by Klastorin and Tsai (2004) who defined product indices for a number of well known consumer durable goods. The ratio B can also be viewed as the product of the ratio of the maximum price to cost for a good with design level $S$, $\frac{bS}{\alpha S^2}$, times the ratio representing the relative difficulty of developing a product with a lifespan $T$, $\frac{BS}{T}$. In our analyses, we used the value of the product index reported by Klastorin and Tsai (2004) for the Hewlett-Packard DeskJet Printer ($B = 4.02$). Without loss of generality, we normalized the product lifespan $T$ and consumer’s utility upper bound $b$ to 1. Accordingly, we initially set $\beta = 0.41$, $\alpha = 0.1$, $S_L = 1$ and $S_H = 1.2$. Following Savin and Terweisch (2005), we set the interest rate $\delta = 0.8$.

The total profit for firms X and Y are indicated in Figure 3 for various values of $L$ (information release time). When firm X produces a high quality product (i.e., $S_X = S_H$), the profits of firm Y decrease as its market entry is delayed. Firm X’s profits and market share, on the other hand, increase as $L$ increases and firm Y enters the market later. Our analysis shows that firm Y’s market share decreases as the information release time $L$ increases when firm X produces a high quality product. When firm X produces a low quality product, the market shares of both firms remain relatively constant for most values of $L$.

Insert Figure 3 Here

Somewhat unexpected, however, is the observation that firm X’s profits decrease as the information release time increases when firm X designs a lower quality product (i.e., $S_X = S_L$). On the other hand, both the monopoly and duopoly prices for firm X decrease when $S_X=S_L$ and $S_Y=S_H$ as indicated in Figure 4. While firm X’s market share decreases with decreasing values of $L$, product X’s prices increase to compensate for a smaller market share. As a result, firm X prefers small values of $L$ when they design a lower quality product; in this case, firm X may have an incentive to release information early about their product.
When firm X designs a high quality product and enters the market first, they increase their monopoly price as the information release time, L, increases. On the other hand, firm X will decrease their duopoly price as L increases to make forward-looking consumers find their product more attractive.

Furthermore, since firm X makes its product design decision before knowing when information about its product will be released, we found that the expected profit for firm X is very sensitive to the rate of information release, λ. When firm X produces a high quality product, increases in λ significantly reduces firm X’s profits as indicated in Figure 5. Conversely, when firm X produces a low quality product, increases in the information release rate increases firm X’s profits.

The value of the product variable cost, α, will impact both firms’ pricing decisions, market shares, and resulting profitability. In this case, we set β=0.41, S_L=1, S_H=1.5, δ=0.8, L=0.2, and λ=5. As indicated in Figure 6, the value of α has a significant impact on both firms’ profits. When firm X produces the high quality product, its profits decrease rapidly as α increases, while firm Y’s profits increase somewhat at firm X’s expense. Clearly, the firm that produces the high quality product has a greater incentive to reduce product costs; the difference between the price that the firm can charge and its costs defines the amount of consumer surplus that the high quality firm earns. Market share for each firm is similar to their profitability (for example, firm X’s market share decreases as it increases it price to compensate for the increase in product variable cost).

To investigate the impact of the product development coefficient β on market characteristics, we set α=0.1, S_L=1, S_H=1.5, δ=0.8, L=0.1, and λ=5 and varied values of β from 0.2 to 0.6. When firm X produces a high quality product, its expected profit decreases approximately linearly with increasing values of β as indicated in Figure 8; firm Y’s profits
similarly decrease. In this case, profits decrease as a result of the increased discounting of earned profits over the longer development cycle. Firm X’s expected profit, however, increases significantly with $\beta$ when it produces a low quality product; since firm Y is producing the high quality product in this case, firm X has a relatively longer monopoly period that it can exploit for greater profitability. In addition, increasing values of $\beta$ increase the risk that firm Y (given the randomness of the information release time, $L$) may not be able to enter the market at all. Hence, when $\beta$ is large, firm X may prefer the low quality product in order to enter the market as quickly as possible.

*Insert Figure 8 Here*

The discount rate $\delta$ impacts both firms’ profits and consumers’ expectations. Following Dhebar (1994) and Kornish (2001), we varied $\delta$ between .6 and 1.0 (no discounting) and used previous parameter values (e.g., $\alpha=0.1, \beta=0.41, S_L=1, S_H=1.5, L=0.2, \lambda=5$). Since increases in $\delta$ reflect less discounting, the value of a product offered in the duopoly period is more attractive to consumers who enter the market in a prior stage as $\delta$ increases. As a result, the prices set by both firms in the duopoly are likely to increase as $\delta$ increases in order to gain additional consumer surplus during the duopoly. This is illustrated in Figure 9 for the case when firm X produces the low quality product (and enters the market first). It is interesting to note that firm X also increases its monopoly price as $\delta$ increases, knowing that firm Y will likely raise its duopoly price. This pricing behavior illustrates the impact of consumer expectations on markets; as consumers become more “forward-looking”, firms (even monopolies) face increasing competition from expected future markets.

*Insert Figure 9 Here*

Figure 10 indicates the impact of the discount rate on both firms’ profitability when firm X develops the low quality product. As $\delta$ increases, profitability increases as profits are discounted at a lower rate and future competition is reduced. At higher values of $\delta$, consumers are more likely to purchase the higher quality product from firm Y in the duopoly period, indicating that firm Y’s profits increase at a rate greater than firm X as the discount rate approaches one.

*Insert Figure 10 Here*
5. Conclusions and Extensions

In this paper, we studied a durable goods market with two competing homogeneous firms when one firm is an innovator and the second firm is an imitator. The imitator firm begins developing a competing product after learning of the innovator firm’s design decision. We assume that the time needed to develop a product is a function of the design/quality of the product that is measured by a single scalar. Since the firms are homogeneous, the innovator firm will be the first firm to enter the market if it designs a lower quality product. If the innovator firm designs a higher quality product, it may or may not be the first firm to enter the market depending on the imitator firm’s design decision and the time when the innovator firm’s design decision is made known to the marketplace.

Our work combines several important concepts in a single model. In addition to modeling a durable good market with a finite lifespan as a Stackelberg game, we assume that consumers are forward-looking and make decisions based on both the value of current goods as well as their expectation of future products and prices. Using this model, we were able to derive several important results. First, we showed that the imitator firm is always better off differentiating their product from the innovator; if they duplicate the innovator’s product, they will make no profit. Second, we showed that the duopoly price will always be less than that in the monopoly period for the same product. Furthermore, for given product design levels, we derived the possible market configurations and found closed form solutions for the duopoly pricing decisions for both firms. We showed that the information release rate, \( \lambda \), and subsequent realization of \( L \) is critically important to the innovator firm’s profitability, explaining why many firms who develop high quality products make serious efforts to prevent information about their new product from “leaking” to the market. On the other hand, we showed cases when early information release benefits the innovator firm (e.g., when they produce a lower quality product) that may explain why some firms voluntarily release information about their product well before the product is available.

Our model also shows the impact of variable costs, development times, and discount rates on product decisions, prices, and profitability. Since we showed that the imitator firm should always differentiate its product from that of the innovator firm, the innovator firm makes the critical
design decision. However, prices set during the monopoly and duopoly periods will impact both firms’ long-term profitability.

The model presented in this paper has implications for many areas, including marketing and operations management. Clearly, a firm’s product design decision impacts the resultant supply chain; a higher quality product is likely to require a more complex supply chain that may, in turn, increase the likelihood that information about the innovator firm’s product “leaks” to the market. Another important extension is suggested by relaxing our assumption that the product development time is an exogenous function (equal to $\beta S$). In reality, firms can usually reduce the development time by increasing the level of resources allocated to product development (a modification of the time-cost trade-off problem discussed in the project management literature). This extension would allow a firm to study the impact of its development costs on resultant market share and profitability.
REFERENCES


FIGURES

Figure 1  New Product Introduction Timelines

a) \( L + g(S_Y) > g(S_X) \)

\[ \begin{array}{cccc}
0 & L & g(S_X) & L + g(S_Y) \\
\hline
\text{Stage 1:} & \text{Stage 2:} & \text{Stage 3:} & \text{Stage 4:} \\
\text{Firm X sets } S_X & S_X \text{ revealed} & \text{Product X released} & \text{Product Y released} \\
& \text{Firm Y sets } S_Y & \text{Firm X sets } P_{XM} & S_Y \text{ revealed} \\
& & \text{Firm X sets } P_{XO} \text{ and Firm Y sets } P_{YO} & \\
\end{array} \]

b) \( L + g(S_Y) < g(S_X) \)

\[ \begin{array}{cccc}
0 & L & L + g(S_Y) & g(S_X) \\
\hline
\text{Stage 1:} & \text{Stage 2:} & \text{Stage 3:} & \text{Stage 4:} \\
\text{Firm X sets } S_X & S_X \text{ revealed} & \text{Product Y released} & \text{Product X released} \\
& \text{Firm Y sets } S_Y & S_Y \text{ revealed} & S_X \text{ revealed} \\
& & \text{Firm Y sets } P_{YM} & \text{Firm X sets } P_{XO} \text{ and Firm Y sets } P_{YO} \\
\end{array} \]

c) Unified New Product Introduction Timelines \((i = X, j = Y \text{ or } i = Y, j = X)\)

\[ \begin{array}{cccc}
l_1 \text{=} L & l_2 & l_3 & l_4 \\
\hline
\text{Stage 1:} & \text{Stage 2:} & \text{Stage 3:} & \text{Stage 4:} \\
\text{Firm X sets } S_X & S_X \text{ revealed} & \text{Product } i \text{ released} & \text{Product } j \text{ released} \\
& \text{Firm Y sets } S_Y & \text{Firm } i \text{ sets } P_{iM} & \text{Firm } i \text{ sets } P_{iD} \text{ and Firm } j \text{ sets } P_{jD} \\
\end{array} \]
Figure 2    Consumer segmentations under Market Configuration 1 (MC1)

a: Consumer Group M

Buy nothing   Buy i in duopoly   Buy i in monopoly   Buy j in duopoly

\[ \frac{P_i \omega}{s_i} \quad v_1 \quad v_2 \quad \frac{P_j \rho - P_i \omega}{s_j - s_i} \quad v_3 \quad 1 \]

Consumer valuation, \( v \)

b: Consumer Group D

Buy nothing   Buy i   Buy j

\[ \frac{P_j \rho - P_i \omega}{s_j - s_i} \quad 1 \]

Consumer valuation, \( v \)

Figure 3     Firms’ Profits as Function of the Information Release Time, \( L \)

a) When \( S_X = S_H \) and \( S_Y = S_L \)

b) When \( S_X = S_L \) and \( S_Y = S_H \)
Figure 4  Firms’ Pricing Decisions for Values of L

a) When $S_X = S_H$ and $S_Y = S_L$

b) When $S_X = S_L$ and $S_Y = S_H$

Figure 5  Expected Firm X Profit as a function of the Information Release Rate, $\lambda$

Case 5a: $S_X = S_H$ and $S_Y = S_L$

Case 5b: $S_X = S_L$ and $S_Y = S_H$
Figure 6  Firm Profitability versus Variable Cost Coefficient, $\alpha$

a) When $S_X=S_H$ and $S_Y=S_L$

b) When $S_X=S_L$ and $S_Y=S_H$

Figure 7  Firms’ Market Shares versus Variable Cost Coefficient, $\alpha$

a) When $S_X=S_H$ and $S_Y=S_L$

b) When $S_X=S_L$ and $S_Y=S_H$
Figure 8  Expected Profit for firm X for Product Development Time Coefficients

Figure 9  Firms’ Pricing Decisions when $S_X = S_L$ and $S_Y = S_H$

a) Firm X’s Monopoly Pricing Decisions

b) Duopoly Pricing Decisions
Figure 10  Firm Profitability for Different Discount Rates when $S_X=S_L$ and $S_Y=S_H$
# TABLES

## Monopoly period market segmentation scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Conditions under which a consumer will buy product $i$ in the monopoly period</th>
<th>The segment of consumers who will buy product $i$ in the monopoly period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$S_i - S_j \delta^i &lt; 0$, $1 \geq v_j \geq v_2 \geq v_1$</td>
<td>$v_2 \leq v \leq v_3$</td>
</tr>
<tr>
<td>2</td>
<td>$S_i - S_j \delta^i &lt; 0$, $v_j &gt; 1 \geq v_2 \geq v_1$</td>
<td>$v_2 \leq v \leq 1$</td>
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<tr>
<td>3</td>
<td>$S_i - S_j \delta^i &lt; 0$, $1 \geq v_j \geq v_1 &gt; v_2$</td>
<td>$v_1 \leq v \leq v_3$</td>
</tr>
<tr>
<td>4</td>
<td>$S_i - S_j \delta^i &lt; 0$, $v_j &gt; 1 \geq v_1 &gt; v_2$</td>
<td>$v_1 \leq v \leq 1$</td>
</tr>
<tr>
<td>5</td>
<td>$S_i - S_j \delta^i &gt; 0$, $S_i = S_L$, $S_j = S_H$, $v_j &gt; v_2$, $v_1 \geq v_2$, $v_2 \leq 1$</td>
<td>$v_2 \leq v \leq 1$</td>
</tr>
<tr>
<td>6</td>
<td>$S_i - S_j \delta^i &gt; 0$, $S_i = S_L$, $S_j = S_H$, $v_j &gt; v_2$, $v_1 \geq v_2$, $v_2 \leq 1$</td>
<td>$v_2 \leq v \leq 1$</td>
</tr>
<tr>
<td>7</td>
<td>$S_i - S_j \delta^i &gt; 0$, $S_i = S_L$, $S_j = S_H$, $v_3 \geq v_1$, $v_2 \geq v_1$, $v_2 \leq 1$</td>
<td>$v_2 \leq v \leq 1$</td>
</tr>
<tr>
<td>8</td>
<td>$S_i - S_j \delta^i &gt; 0$, $S_i = S_L$, $S_j = S_H$, $v_j &gt; v_1$, $v_3 \geq v_2$, $v_3 \leq 1$</td>
<td>$v_3 \leq v \leq 1$</td>
</tr>
<tr>
<td>9</td>
<td>$S_i - S_j \delta^i &gt; 0$, $S_i = S_H$, $S_j = S_L$, $v_3 \geq v_1$, $v_3 \geq v_2$, $v_3 \leq 1$</td>
<td>$v_3 \leq v \leq 1$</td>
</tr>
</tbody>
</table>

### Conditions on $v$ under which a consumer makes no purchase in the monopoly period

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Conditions on $v$ under which a consumer makes no purchase in the monopoly period</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$S_i - S_j \delta^i &lt; 0$ and $(v_j &lt; \max{v_1, v_2}$ or $v_j \geq \max{v_1, v_2} &gt; 1)$</td>
</tr>
<tr>
<td>9</td>
<td>$S_i - S_j \delta^i &gt; 0$, $S_i = S_L$, $S_j = S_H$ and $\max{v_1, v_2, v_3} &gt; 1$</td>
</tr>
<tr>
<td>13</td>
<td>$S_i - S_j \delta^i &gt; 0$, $S_i = S_H$, $S_j = S_L$ and $\max{v_1, v_2, v_3} &gt; 1$</td>
</tr>
</tbody>
</table>
Table 2  All possible market segmentation combinations

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Monopoly conditions</th>
<th>Duopoly conditions</th>
<th>Result of Combining Monopoly and Duopoly Conditions</th>
</tr>
</thead>
</table>
| 1        | $S_i - S_j \delta^i < 0$,  
1 $v_1 > v_2, v_1 > v_2$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | When $S_i = S_L$ and $S_j = S_H$, MC 1  
When $S_i = S_H$ and $S_j = S_L$, MC 7 |
| 2        | $S_i - S_j \delta^i < 0$,  
$v_3 > 1 \geq v_2, v_1$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | When $S_i = S_L$ and $S_j = S_H$, MC 2  
When $S_i = S_H$ and $S_j = S_L$, MC 8 |
| 3        | $S_i - S_j \delta^i < 0$,  
1 $v_3 \geq v_2, v_1$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | Does not exist (See Appendix B) |
| 4        | $S_i - S_j \delta^i < 0$,  
$v_1 > 1 \geq v_2, v_2$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | Does not exist (See Appendix B) |
| 5        | $S_i - S_j \delta^i < 0$,  
$v_3 < \max\{v_1, v_2\}$ or  
$v_3 \geq \max\{v_1, v_2\}$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | Does not exist (See Appendix B) |
| 6        | $S_i - S_j \delta^i > 0$, $S_i = S_L$, $S_j = S_H$,  
$v_1 > v_2, v_3 \geq v_2, v_1 \leq 1$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | Does not exist (See Appendix B) |
| 7        | $S_i - S_j \delta^i > 0$, $S_i = S_L$, $S_j = S_H$,  
$v_2 \geq v_1, v_2 \geq v_3, v_2 \leq 1$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | When $S_X = S_L$ and $S_Y = S_H$, MC 3  
When $S_X = S_H$ and $S_Y = S_L$, MC 9 |
| 8        | $S_i - S_j \delta^i > 0$, $S_i = S_L$, $S_j = S_H$,  
$v_3 > v_1, v_3 \geq v_2, v_3 \leq 1$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | When $S_X = S_L$ and $S_Y = S_H$, MC 4  
When $S_X = S_H$ and $S_Y = S_L$, MC 10 |
| 9        | $S_i - S_j \delta^i > 0$, $S_i = S_L$, $S_j = S_H$,  
$\max\{v_1, v_2, v_3\} > 1$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | Does not exist (See Appendix B) |
| 10       | $S_i - S_j \delta^i > 0$, $S_i = S_H$, $S_j = S_L$,  
$v_1 > v_2, v_1 \geq v_3, v_1 \leq 1$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | Does not exist (See Appendix B) |
| 11       | $S_i - S_j \delta^i > 0$, $S_i = S_H$, $S_j = S_L$,  
$v_2 \geq v_1, v_2 \geq v_3, v_2 \leq 1$ | $P_{id} < P_{jd}$, $P_{jd} - P_{id} < 1$ | MC 5 |
<table>
<thead>
<tr>
<th>12</th>
<th>$S_i - S_j \delta_{ij} &gt; 0$, $S_i = S_{H_i}$, $S_j = S_{L_j}$, $v_3 &gt; v_1$, $v_3 \geq v_2$, $v_3 \leq 1$</th>
<th>$P_{jD} &lt; \frac{P_{jD}}{S_j}$, $\frac{P_{jD} - P_{jD}}{S_i - S_j} &lt; 1$</th>
<th>MC 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>$S_i - S_j \delta_{ij} &gt; 0$, $S_i = S_{H_i}$, $S_j = S_{L_j}$, max ${v_1, v_2, v_3} &gt; 1$</td>
<td>$P_{jD} &lt; \frac{P_{jD}}{S_j}$, $\frac{P_{jD} - P_{jD}}{S_i - S_j} &lt; 1$</td>
<td>Does not exist (See Appendix B)</td>
</tr>
</tbody>
</table>

### Table 3: Profit Functions and Constraints for the Market Configurations

<table>
<thead>
<tr>
<th>MC</th>
<th>Profit Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi_{XM} = \frac{M}{T-L} \beta S_H (v_1 - v_2) (P_{XM} - \alpha S^2_i) \delta_{HS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{Y XD} = \frac{M}{T-L} \beta S_H (v_2 - \frac{P_{YD}}{S_L}) (P_{YD} - \alpha S^2_L) \delta_{YS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{Y MD} = \frac{M}{T-L} \beta S_H (1 - v_1) (P_{YD} - \alpha S^2_H) \delta_{YS}^\beta$</td>
</tr>
<tr>
<td>2</td>
<td>$\pi_{XM} = \frac{M}{T-L} \beta S_H (1 - v_2) (P_{XM} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{X MD} = \frac{M}{T-L} \beta S_H (v_2 - \frac{P_{YD}}{S_L}) (P_{YD} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{YMD} = 0$</td>
</tr>
<tr>
<td>3</td>
<td>$\pi_{XM} = \frac{M}{T-L} \beta S_H (1 - v_2) (P_{XM} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{X MD} = \frac{M}{T-L} \beta S_H (v_2 - \frac{P_{YD}}{S_L}) (P_{YD} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{YMD} = 0$</td>
</tr>
<tr>
<td>4</td>
<td>$\pi_{XM} = \frac{M}{T-L} \beta S_H (1 - v_1) (P_{XM} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{X MD} = \frac{M}{T-L} \beta S_H (P_{YD} - P_{YD}) (P_{YD} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{YMD} = \frac{M}{T-L} \beta S_H (v_3 - \frac{P_{YD}}{S_L}) (P_{YD} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td>5</td>
<td>$\pi_{XM} = \frac{M}{T-L} \beta S_L (1 - v_2) (P_{XM} - \alpha S^2_H) \delta_{HS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{X MD} = \frac{M}{T-L} \beta S_L (v_2 - \frac{P_{YD}}{S_L}) (P_{YD} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>$\pi_{YMD} = \frac{M}{T-L} \beta S_L (P_{YD} - P_{YD}) (P_{YD} - \alpha S^2_L) \delta_{LS}^\beta$</td>
</tr>
<tr>
<td></td>
<td>[ \pi_{\text{YMM}} = \frac{M}{T - L} \beta S_L \left( 1 - v_3 \right) \left( P_{\text{YM}} - \alpha S_H^2 \right) \delta \beta S_u ]</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>[ \pi_{\text{YMD}} = 0 ]</td>
</tr>
<tr>
<td></td>
<td>[ \pi_{\text{XMM}} = \frac{M}{T - L} \beta S_L \left( v_3 - \frac{P_{\text{YD}}}{S_L} \right) \left( P_{\text{YM}} - \alpha S_H^2 \right) \delta L + \beta S_L ]</td>
</tr>
<tr>
<td></td>
<td>[ \pi_{\text{XMD}} = 0 ]</td>
</tr>
</tbody>
</table>

|   | \[ \pi_{\text{YMM}} = \frac{M}{T - L} \left( \beta S_H^2 - L \right) \left( v_3 - v_2 \right) \left( P_{\text{YM}} - \alpha S_H^2 \right) \delta L + \beta S_L \] |
|   | \[ \pi_{\text{YMD}} = \frac{M}{T - L} \left( \beta S_H - L \right) \left( v_2 - \frac{P_{\text{YD}}}{S_L} \right) \left( P_{\text{YM}} - \alpha S_H^2 \right) \delta \beta S_u \] |
|   | \[ \pi_{\text{XMM}} = \frac{M}{T - L} \left( \beta S_H^2 - L \right) \left( 1 - v_3 \right) \left( P_{\text{XD}} - \alpha S_H^2 \right) \delta \beta S_u \] |
|   | \[ \pi_{\text{XMD}} = \frac{M}{T - L} \left( \beta S_H - L \right) \left( 1 - v_3 \right) \left( P_{\text{YD}} - \alpha S_H^2 \right) \delta \beta S_u \] |

|   | \[ \pi_{\text{YMM}} = \frac{M}{T - L} \left( \beta S_H^2 - L \right) \left( 1 - v_2 \right) \left( P_{\text{YM}} - \alpha S_H^2 \right) \delta L + \beta S_L \] |
|   | \[ \pi_{\text{YMD}} = \frac{M}{T - L} \left( \beta S_H - L \right) \left( v_2 - \frac{P_{\text{YD}}}{S_L} \right) \left( P_{\text{YM}} - \alpha S_H^2 \right) \delta \beta S_u \] |
|   | \[ \pi_{\text{XMM}} = \frac{M}{T - L} \left( \beta S_H^2 - L \right) \left( 1 - v_2 \right) \left( P_{\text{XD}} - \alpha S_H^2 \right) \delta \beta S_u \] |
|   | \[ \pi_{\text{XMD}} = \frac{M}{T - L} \left( \beta S_H - L \right) \left( 1 - v_2 \right) \left( P_{\text{YD}} - \alpha S_H^2 \right) \delta \beta S_u \] |

|   | \[ \pi_{\text{YMM}} = \frac{M}{T - L} \left( \beta S_H^2 - L \right) \left( 1 - v_3 \right) \left( P_{\text{YM}} - \alpha S_H^2 \right) \delta L + \beta S_L \] |
|   | \[ \pi_{\text{YMD}} = \frac{M}{T - L} \left( \beta S_H - L \right) \left( v_3 - \frac{P_{\text{YD}}}{S_L} \right) \left( P_{\text{YM}} - \alpha S_H^2 \right) \delta \beta S_u \] |
|   | \[ \pi_{\text{XMM}} = \frac{M}{T - L} \left( \beta S_H^2 - L \right) \left( 1 - v_3 \right) \left( P_{\text{XD}} - \alpha S_H^2 \right) \delta \beta S_u \] |
|   | \[ \pi_{\text{XMD}} = \frac{M}{T - L} \left( \beta S_H - L \right) \left( v_3 - \frac{P_{\text{YD}}}{S_L} \right) \left( P_{\text{YD}} - \alpha S_H^2 \right) \delta \beta S_u \] |
Appendix A

**Proof of Proposition 1:** Firm X is the innovator. Assume that the imitator firm, firm Y, chooses $S_Y$ that is equal to $S_X$. A consumer in Group D buys product X in the duopoly period if:

$$v_k > \frac{P_{XD}}{S_X}$$

(22)

$$v_k S_X - P_{XD} > v_k S_X - P_{YD}$$

(23)

After a quick simplification, Condition (23) becomes $P_{XD} \leq P_{YD}$.

Similarly, a consumer buys product Y in the duopoly period if:

$$v_k > \frac{P_{YD}}{S_Y}$$

(24)

$$P_{XD} > P_{YD}$$

(25)

Apparently, (25) and (23) cannot be satisfied at the same time. This means that each firm tends to decrease its price to steal the market share from the other firm. Since this is a non-cooperative game between two firms, the price would go down to production cost and the profit goes down to zero if the imitator firm chooses the same design level with the innovator firm. Thus, Firm Y would always try to differentiate itself by choosing a different design level. □

**Proof of Lemma 1:** Equation (1) was written as the combination of the following inequalities:

$$vS_i - P_{IM} \geq 0$$

(26)

$$vS_i - P_{IM} \geq \delta^h (vS_i - P_{ID})$$

(27)

$$vS_i - P_{IM} \geq \delta^h (vS_j - P_{jD})$$

(28)

Under the condition of $S_i - S_j \delta^h > 0$, inequalities (26)-(28) can be further simplified to the following three inequalities respectively:

$$v \geq \frac{P_{IM}}{S_i}$$

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\[ v \geq \frac{P_{iM} - \delta_i^h P_{iD}}{S_i (1 - \delta_i^h)} \]

\[ v \geq \frac{P_{iM} - \delta_j^h P_{jD}}{S_j (1 - \delta_j^h)} \]

So, condition (2) is proven. Under the condition of \( S_i - S_j \delta_j^h < 0 \), inequalities (26)-(28) can be further simplified to the following inequalities respectively:

\[ v \geq \frac{P_{iM}}{S_i} \]

\[ v \geq \frac{P_{iM} - \delta_i^h P_{iD}}{S_i (1 - \delta_i^h)} \]

\[ v \leq \frac{P_{iM} - \delta_j^h P_{jD}}{S_j (1 - \delta_j^h)} \]

Therefore, condition (3) is proven. Under the condition of \( S_i - S_j \delta_j^h = 0 \), inequalities (26)-(28) can be further simplified to the following inequalities respectively:

\[ v \geq \frac{P_{iM}}{S_i} \]

\[ v \geq \frac{P_{iM} - \delta_i^h P_{iD}}{S_i (1 - \delta_i^h)} \]

\[ P_{iM} - \delta_i^h P_{jD} \leq 0 \]

**Proof of Lemma 2:**

When \( S_i=S_L \) and \( S_j=S_H \), \( P_{id} < (S_L/S_H) P_{jd} \) has to be satisfied in order for firm i to have a positive duopoly market share depicted in (5). Otherwise, firm i will have zero duopoly market share whereas firm j has a duopoly market share of \( 1 - \frac{P_{jd}}{S_j} \) since the consumers in group D with value
$v \geq \frac{P_{jd}}{S_j}$ buy product j in the duopoly period. Firm i will surely avoid that. When $S_i = S_H$ and $S_j = S_L$, it is vice versa.

**Proof Lemma 3:** It is clear that both second derivatives are negative:

$$\frac{\partial^2 \pi_X}{\partial P_{XD}^2} = M \delta^{l_i+l_j+l_h} \left[ 2 S_H (T-L)(1-\delta^h) + 2 \beta S_H (-S_L + S_H \delta^h) \right] \left( S_H - S_L \right) \left( -1 + \delta^h \right) < 0$$

$$\frac{\partial^2 \pi_Y}{\partial P_{YD}^2} = M \delta^{l_i+l_j+l_h} \left( - \frac{2l_i}{S_H - S_L} + \frac{2 \beta S_H \delta^h}{S_L - S_H \delta^h} \right) \frac{S_H - S_L}{T-L} < 0$$

**Proof of Proposition 2:**

$$P_{XD}^* = A_i P_{YD}^* + B_i P_{XM}^* + C_i$$ (29)

$$P_{YD}^* = D_i P_{XD}^* + E_i P_{XM}^* + F_i$$ (30)

where

$$A_i = \frac{S_i l_4 (1-\delta^h)}{2 S_j \left[ (T-L)(1-\delta^h) - \beta \left( S_i - S_j \delta^h \right) \right]} > 0$$

$$B_i = \frac{\beta (S_j - S_i)}{\left[ (T-L)(1-\delta^h) - \beta \left( S_i - S_j \delta^h \right) \right]} > 0$$

$$C_i = \frac{\alpha S_j S_l^2 l_4 (1-\delta^h)}{2 S_j \left[ (T-L)(1-\delta^h) - \beta \left( S_i - S_j \delta^h \right) \right]} > 0$$

$$D_i = \frac{l_4 \left( S_i - S_j \delta^h \right)}{2 \left[ (T-L)(S_i - S_j \delta^h) - \beta S_i S_j (1-\delta^h) \right]} > 0$$

$$E_i = \frac{-\beta S_j (S_j - S_i)}{2 \left[ (T-L)(S_i - S_j \delta^h) - \beta S_i S_j (1-\delta^h) \right]} > 0$$
\[
F_i = \frac{(S_j - S_i)(T - L)(S_i - S_j\delta^h) + \alpha S_i^2 (T - L)(S_i - S_j\delta^h) - S_i^3 S_i \alpha \beta (1-\delta^h)}{2\left[(T - L)(S_i - S_j\delta^h) - \beta S_i S_j(1-\delta^h)\right]} > 0.
\]

Since all the coefficients in (29) and (30) are positive, and \(P_{yd}^*\) are strategic complements. If one firm increases its price, the other firm will also increase its price. For a given firm \(X\) monopoly price \(P_{xm}\), we solve to get the corresponding duopoly prices:

\[
P_{xd}^* = \frac{A_i E_1 + B_1}{1 - A_i D_1} P_{xm} + \frac{A_i F_1 + C_1}{1 - A_i D_1}
\]

\[
P_{yd}^* = \left(A_i E_1 + \frac{B_i}{1 - A_i D_1} D_1 + E_1\right) P_{xm} + \left(A_i F_1 + \frac{C_1}{1 - A_i D_1} D_1 + F_1\right).
\]

Furthermore, we need to know whether \(A_1 D_1\) is less than zero in order to find the relationship among the equilibrium duopoly prices and firm \(X\)'s monopoly price. When we multiply with \(D_1\), we get the following:

\[
A_i D_1 = \frac{-S_L l_i^2 \left(S_L - S_H\delta^h\right)(1-\delta^h)}{4 S_H \left[-(T - L)^2 \left(S_L - S_H \delta^h\right)(1-\delta^h) + G\right]}
\]

where

\[
G = \beta S_L S_H \left(1-\delta^h\right)^2 (T - L) - \beta (L - T) \left(S_L - S_H \delta^h\right)^2 - \beta^2 S_L S_H \left(1-\delta^h\right) \left(S_L - S_H \delta^h\right) > 0
\]

\(A_i D_1 < 1\) since the following relation holds:

\[
0 < -S_L l_i^2 \left(S_L - S_H \delta^h\right)(1-\delta^h) < 4 S_H \left[-(T - L)^2 \left(S_L - S_H \delta^h\right)(1-\delta^h)\right]
\]
Appendix B: Proof of Results in Table 3

Proof of scenario 3 not generating a market configuration: Combining scenario 3 conditions with the duopoly market conditions will give us the following constraint set along with the profit functions given.

\[ v_3 \geq v_1 \left( \frac{P_{iM}}{S_i} \leq \frac{P_{jD}}{S_j} \right) \]

\[ v_1 > v_2 \left( P_{iM} < P_{iD} \right) \]

\[ v_3 \leq 1 \]

\[ \frac{P_{iD}}{S_i} < \frac{P_{jD}}{S_j} \]

\[ \frac{P_{jD} - P_{iD}}{S_j - S_i} < 1 \]

\[ P_{iD} \geq \alpha S_i^2 \]

\[ P_{iM} \geq \alpha S_i^2 \]

\[ P_{jD} \geq \alpha S_j^2 \]

*Insert Figure 11 Here*

Now we’ll define the following profit functions to inspect the total profit function for firms in three separate levels: customers in Group M buying product i in monopoly period, customers in Group M buying one of the products in duopoly period, customers in Group D buying one of the products in duopoly period. Note that we are going to use this notation throughout Appendix B.

\[ \pi_{kMr}: \text{firm } k's (k=i \text{ or } j) \text{ profit function generated from the customers in group M and buy firm k’s product in period } r (r=M (monopoly) or D(duopoly)) \]

\[ \pi_{kDD}: \text{firm } k's (k=i \text{ or } j) \text{ profit function generated from the customers in group M and buy firm k’s product in period } r (r=M (monopoly) or D(duopoly)) \]

Based on Figure 11 above, customers in group M make firms earn the following amounts:
\[ \pi_{iMM} = \frac{M}{T} \beta S_j (v_3 - v_i) \left( P_{IM} - \alpha S_i^2 \right) \delta^{l_2+l_3} \]

\[ \pi_{iMD} = 0 \]

\[ \pi_{jMD} = \frac{M}{T} \beta S_j (1-v_3) \left( P_{jD} - \alpha S_j^2 \right) \delta^{l_2+l_3+l_j} \]

Duopoly market shares of firms X and Y are as shown in Figure 2b. Thus, the profit functions, \( \pi_{iDD} \) and \( \pi_{jDD} \):

\[ \pi_{iDD} = \frac{M}{T-L} l_4 \left( \frac{P_{iD} - P_{iD}}{S_j - S_i} - \frac{P_{iD}}{S_i} \right) \left( P_{iD} - \alpha S_i^2 \right) \delta^{l_2+l_3+l_j} \quad (31) \]

\[ \pi_{jDD} = \frac{M}{T-L} l_4 \left( 1 - \frac{P_{jD} - P_{jD}}{S_j - S_i} \right) \left( P_{jD} - \alpha S_j^2 \right) \delta^{l_2+l_3+l_j} \quad (32) \]

Now, let’s define a firm’s myopic monopoly policy as a policy such that a firm maximizes its profit in each period only. Denote \( P_{iMM} \) as firm i’s myopic optimal monopoly price and \( P_{iMD} \) as firm i’s myopic duopoly price.

Firm i’s myopic monopoly profit is \( \frac{M}{T-L} \left( l_2 + l_3 \right) (v_3 - v_i) \left( P_{iMM} - \alpha S_i^2 \right) \delta^{l_2+l_3} \) and firm X’s myopic duopoly profit is \( \frac{M}{T-L} l_4 \left( P_{jD} - P_{iD} \right) \left( P_{iD} - \alpha S_i^2 \right) \delta^{l_2+l_3+l_j} \). From the FOCs, \( P_{iMD} = P_{iMM} = \frac{\alpha S_i^2 S_j + P_{jD} S_i}{2 S_j} \).

Next, we prove \( P_{iM}^* \geq P_{iMM} \). We do this by contradiction. Assume \( P_{iM}^* \) is the optimal monopoly price and \( P_{iM}^* < P_{iMM} \). Now, let’s define \( \pi_k \left( P_{iM}, P_{iD}, P_{jD} \right) \) as the firm k’s (k=i or j) total profit function for given prices \( P_{iM}, P_{iD}, \) and \( P_{jD} \). We are going to use that notation throughout Appendix B. Firm i’s total profits by using \( P_{iM}^* \) and \( P_{iMM} \) for all fixed \( P_{iD} \) and \( P_{jD} \) are respectively as follows:

42
\[
\pi_i(P_{iM}^*, P_{iD}, P_{iD}^*) = \frac{M}{T-L} \left( l_2 + l_3 \right) (v_i - v_i) \left( P_{iM}^* - \alpha S_i^2 \right) \theta^{t+1} + \frac{M}{T-L} \left( l_2 + l_3 \right) \left( v_i - \frac{P_{iD}}{S_i} \right) \left( P_{iD}^* - \alpha S_i^2 \right) \theta^{t+1} + \frac{M}{T-L} \left( l_4 \right) \left( P_{iD} - \frac{P_{iM}^*}{S_i} \right) \left( P_{iD}^* - \alpha S_i^2 \right) \theta^{t+1} + \frac{M}{T-L} \left( l_4 \right) \left( P_{iD} - \frac{P_{iD}^*}{S_i} \right) \left( P_{iD}^* - \alpha S_i^2 \right) \theta^{t+1} + \frac{M}{T-L} \left( l_4 \right) \left( P_{iD} - \frac{P_{iD}^*}{S_i} \right) \left( P_{iD}^* - \alpha S_i^2 \right) \theta^{t+1}.
\]

The first terms in the profit functions are monopoly period profits using prices \( P_{iM}^* \) and \( P_{iMM} \). Since \( P_{iMM} \) is found from the myopic policy that maximizes that first term (monopoly period profit), the first term in \( \pi_i \left( P_{iMM}, P_{iD}, P_{iD}^* \right) \) is larger than the first term in \( \pi_i \left( P_{iM}^*, P_{iD}, P_{iD}^* \right) \) (Remember \( v_i = P_{iM}^*/S_i \)). The second term in \( \pi_i \left( P_{iMM}, P_{iD}, P_{iD}^* \right) \) is no less than the second term in \( \pi_i \left( P_{iM}^*, P_{iD}, P_{iD}^* \right) \) (Remember \( v_i = P_{iM}^*/S_i \)). The third terms are equal. Therefore, \( \pi_i \left( P_{iM}^*, P_{iD}, P_{iD}^* \right) < \pi_i \left( P_{iMM}, P_{iD}, P_{iD}^* \right) \) for \( P_{iM}^* < P_{iMM} \) for all fixed \( P_{iD} \) and \( P_{iD}^* \). Let \( \{ P_{iD}^1, P_{iD}^2 \} \) and \( \{ P_{iD}^1, P_{iD}^2 \} \) be the optimal prices for the cases where firm \( i \) sets its optimal price to \( P_{iM}^* \) and \( P_{iMM} \) respectively. According to what we have just shown, \( \pi_i \left( P_{iM}^*, P_{iD}^1, P_{iD}^1 \right) < \pi_i \left( P_{iMM}, P_{iD}^1, P_{iD}^1 \right) \). Since \( P_{iD}^2 \) and \( P_{iD}^2 \) are the optimal prices for \( P_{iMM}, P_{iMM}, P_{iD}^2, P_{iD}^2 \) are the optimal prices for \( \pi_i \left( P_{iMM}, P_{iD}^2, P_{iD}^2 \right) \). That means \( \pi_i \left( P_{iM}^*, P_{iD}^1, P_{iD}^1 \right) < \pi_i \left( P_{iMM}, P_{iD}^2, P_{iD}^2 \right) \). Thus, \( P_{iM}^* \) cannot be firm \( i \)’s optimal monopoly price. So, \( P_{iMM} \leq P_{iM}^* \). Up to now, we have shown that \( P_{iMD} = P_{iMM} \leq P_{iM}^* \).

Now, assume \( P_{iD}^* \) is the optimal duopoly price and \( P_{iD}^* > P_{iM}^* \). Therefore, \( P_{iMD} < P_{iD}^* \). We will show that \( P_{iMD} \) cannot be less than \( P_{iD}^* \), thus \( P_{iD}^* \leq P_{iM}^* \) by contradiction. Firm \( i \)’s total profits by using \( P_{iD}^* \) and \( P_{iMD} \) are respectively as follows:
\[
\pi_i(P_{IM}^*, P_{ID}^*, P_{JD}) = \frac{M}{T - L} (l_2 + l_3)(v_i - v_l)(P_{IM}^* - \alpha S_l^2) \delta^{l_1 + l_2} + \frac{M}{T - L} (l_2 + l_3) \left( v_i - \frac{P_{ID}^*}{S_l} \right) \left( P_{ID}^* - \alpha S_l^2 \right) \delta^{l_1 + l_2 + l_3} + \frac{M}{T - L} l_4 \left( P_{JD} - P_{ID}^* \right) \left( P_{ID}^* - \alpha S_l^2 \right) \delta^{l_1 + l_2 + l_3}
\]

The first terms in the profit functions above are equal. There are no customers left from monopoly period to buy product \(i\) in the duopoly period since \(P_{iMM}^* < P_{iDD}^*\). That means \(\left( v_i - P_{ID}^* \right)\) (second term in \(\pi_i(P_{IM}^*, P_{ID}^*, P_{JD})\)) equals zero. Moreover, the third term in \(\pi_i(P_{IM}^*, P_{ID}^*, P_{JD})\) is less than the third term in \(\pi_i(P_{IM}^*, P_{IMD}^*, P_{JD})\) since \(P_{IMD}^*\) maximizes the myopic duopoly period profit (third terms in the profit functions). Thus, \(\pi_i(P_{IM}^*, P_{ID}^*, P_{JD}) < \pi_i(P_{IM}^*, P_{IMD}^*, P_{JD})\). Let \(P_{JD}^1\) be the optimal price for \(\{P_{IM}^*, P_{ID}^*\}\) and \(P_{JD}^2\) be the optimal price for \(\{P_{IM}^*, P_{IMD}^*\}\). With the same logic we followed above, \(\pi_i(P_{IM}^*, P_{ID}^*, P_{JD}^1) < \pi_i(P_{IM}^*, P_{IMD}^*, P_{JD}^2)\). Thus, \(P_{ID}^*\) cannot be firm \(i\)'s optimal duopoly price. So, \(P_{ID}^* \leq P_{IMM}^*\). Combining the previous results we’ve shown with this result gives us \(P_{iD}^* \leq P_{iMM} = P_{iMM} \leq P_{iM}^*\). This result conflicts with the constraint \(P_{iM} < P_{iD}\). Thus, the market configuration we could get by combining scenario 4 and the duopoly conditions creates an infeasible set.
Proof of scenario 4 not generating a market configuration: Combining scenario 4 conditions with the duopoly market conditions will give us the following constraint set along with the profit functions given:

\[ v_j \geq v_i \left( \frac{P_{id}}{S_j} \leq \frac{P_{jd}}{S_j} \right) \]

\[ v_i > v_2 \left( P_{im} < P_{id} \right) \]

\[ v_3 > 1 \]

\[ v_i \leq 1 \]

\[ \frac{P_{id}}{S_j} < \frac{P_{jd}}{S_j} \]

\[ \frac{P_{jd} - P_{id}}{S_j - S_i} < 1 \]

\[ P_{id} \geq \alpha S_i^2 \]

\[ P_{im} \geq \alpha S_i^2 \]

\[ P_{jd} \geq \alpha S_j^2 \]

Insert Figure 12 Here

Based on Figure 12 above, customers coming during the monopoly period make firms earn the following amounts:

\[ \pi_{iMM} = \frac{M}{T-L} \left( l_2 + l_1 \right) \left( 1-v_1 \right) \left( P_{im} - \alpha S_i^2 \right) \delta^{l_1+l_2} \]

\[ \pi_{iMD} = \pi_{jMD} = 0 \]

Duopoly market shares of firms X and Y are as shown in Figure 2b. Thus, the profit functions, \( \pi_{iDD} \) and \( \pi_{jDD} \) are (31) and (32). Essentially, the proof is similar to the previous proof where scenario 3 is proven not to generate a market configuration. The only difference is in the first part of the proof where the relationship between \( P_{iMM} \) and \( P_{iMD} \) is found. \( P_{iMM} \) is found from FOC of
the firm i’s myopic monopoly profit which is \( \pi_{iMM} = \frac{M}{T-L}(l_2 + l_3)(1-v_i)(P_{iM} - \alpha S_i^2)\delta^{i,t} \).

\( P_{iMM} \) is equal to \( \frac{\alpha S_i^2 + S_j}{2} \). \( P_{iMD} = \frac{\alpha S_i^2 S_j + P_{jD} S_i}{2 S_j} < P_{iMM} \). The rest of the proof is similar to the previous one. Finally, the relationship \( P_{iD}^* \leq P_{iMD} < P_{iMM} \leq P_{iM}^* \) is found. That conflicts with the constraint \( P_{iM} < P_{iD} \). Thus, the market configuration we could get by combining scenario 4 and the duopoly conditions creates an infeasible set.

**Proof of scenario 5 not generating a market configuration:**

We will show here that combining scenario 5 conditions and duopoly conditions will either be dominated by the market configuration formed by combining scenario 1 conditions and the duopoly conditions or lead us either to an infeasible set.

Let’s first focus on \( v_3 \geq \max\{v_1, v_2\} > 1 \) part of the scenario. If \( v_2 > v_1 \), the condition here becomes \( v_3 \geq v_2 > 1 \). Combining \( v_3 \geq v_2 > 1 \) with the duopoly market conditions \( \frac{P_{jD}}{S_i} < \frac{P_{jD}}{S_j} \) and

\[ \frac{P_{jD} - P_{iD}}{S_j - S_i} < 1 \]

will give us an infeasible set. When we work on the monopoly conditions, we get the relationship \( 1 < v_2 < \frac{P_{jD} - P_{iD}}{S_j - S_i} < v_3 \) that conflicts with the second condition in the duopoly market condition set that is \( \frac{P_{jD} - P_{iD}}{S_j - S_i} < 1 \). Thus, this does not lead us to a feasible duopoly market configuration. If \( v_1 > v_2 \), the condition here becomes \( v_3 \geq v_1 > 1 \). Combining \( v_3 \geq v_1 > 1 \) with duopoly market conditions \( \frac{P_{jD}}{S_i} < \frac{P_{jD}}{S_j} \) and \( \frac{P_{jD} - P_{iD}}{S_j - S_i} < 1 \) will give us an infeasible set. When we work on the conditions, we get the relationship \( 1 \leq v_1 < \frac{P_{jD}}{S_i} < \frac{P_{jD} - P_{iD}}{S_j - S_i} < v_3 \) that conflicts with the condition...
\[
\frac{P_{jD} - P_{iD}}{S_j - S_i} < 1 \text{ in the duopoly market condition set. Thus, this does not lead us to a feasible duopoly market configuration either.}
\]

Now, let's focus on \( v_3 < \max\{v_1, v_2\} \) part of the scenario. Combining \( v_3 < \max\{v_1, v_2\} \) with the duopoly conditions will give us the following constraint set:

\[
v_3 < \max\{v_1, v_2\} \quad (33)
\]

\[
\frac{P_{iD}}{S_i} < \frac{P_{jD}}{S_j}, \quad (34)
\]

\[
\frac{P_{jD} - P_{iD}}{S_j - S_i} < 1 \quad (35)
\]

\[
P_{iD} \geq \alpha S_i^2 \quad (36)
\]

\[
P_{iM} \geq \alpha S_i^2 \quad (37)
\]

\[
P_{jD} \geq \alpha S_j^2 \quad (38)
\]

In this market configuration, nobody would like to buy product \( i \) in the monopoly period. If this market configuration exists, customers purchase a product only in the duopoly period. (33) can be written in two possible ways:

**Possibility 1:** \( v_1 > v_2 \) and \( v_3 < v_1 \)

**Possibility 2:** \( v_1 < v_2 \) and \( v_3 < v_2 \)

We will show each one of those two conditions above is either dominated by a market configuration or infeasible. Possibility 1 creates an infeasible set. \( v_3 < v_1 \) is equivalent to \( P_{iM} < P_{jD} \) while \( v_3 < v_2 \) is equivalent to \( \frac{P_{iM}}{S_i} > \frac{P_{jD}}{S_j} \). Those two conditions together conflicts with one of the duopoly market conditions that is \( \frac{P_{iD}}{S_i} < \frac{P_{jD}}{S_j} \). That means the possibility 1 creates an infeasible set.
Possibility 2 is dominated by market configuration 1. For this possibility, we add \( v_3 < 1 \) as a constraint on top of (33)-(38) since \( v > 1 \) is not going to happen. The reason behind this is the following: When we work on the constraints (33)-(38), we come up with the relationship \( v_3 < \frac{P_{jD} - P_{iD}}{S_j - S_i} < v_2 \). Since \( \frac{P_{jD} - P_{iD}}{S_j - S_i} \) cannot be greater than 1 in order to have a duopoly period, \( v_3 \) cannot be greater 1. After that, we can compare this with the market configuration 1 in order to prove that the market configuration created under the possibility 2 (let us denote this configuration as MC-P2) is dominated by the market configuration created as a result of the combination of scenario 1 and the duopoly conditions (let us denote this configuration as MC-S1).

The only difference between MC-P2 and MC-S1 is that MC-P2 has the constraint

\[
P_{iM} > \frac{P_{jD} S_i \left(1 - \delta^h\right) - P_{iD} S_i + P_{iD} S_j \delta^h}{S_j - S_i}
\]

and MC-S1 has the constraint

\[
P_{iM} < \frac{P_{jD} S_i \left(1 - \delta^h\right) - P_{iD} S_i + P_{iD} S_j \delta^h}{S_j - S_i}.
\]

Let us define \( P_{iM}^{MC-S1} \) and \( P_{iM}^{MC-P2} \) as the optimal monopoly prices in MC-S1 and MC-P2 respectively. For fixed \( P_{iD} \) and \( P_{jD} \), profit functions for MC-S1 and MC-P2 are as follows:

\[
\pi_i^{MC-S1} \left(P_{iM}^{MC-S1}, P_{iD}, P_{jD}\right) = \frac{M}{T - L} \left(l_2 + l_3\right) \left(v_3 - v_2\right) \left(P_{iM}^{MC-S1} - \alpha S_i^2\right) \delta^{h_i + l_i} + \frac{M}{T - L} \left(l_2 + l_3\right) \left(P_{jD} - P_{iD}\right) \left(P_{iM} - \alpha S_i^2\right) \delta^{h_i + l_i + l_i} + \frac{M}{T} \left(l_2 + l_3\right) \left(P_{jD} - P_{iD}\right) - v_2 \left(P_{iD} - \alpha S_i^2\right) \delta^{h_i + l_i + l_i} + \frac{M}{T} l_4 \left(P_{jD} - P_{iD}\right) \left(P_{iD} - \alpha S_i^2\right) \delta^{h_i + l_i + l_i}
\]

\[
\pi_i^{MC-P2} \left(P_{iM}^{MC-P2}, P_{iD}, P_{jD}\right) = \frac{M}{T - L} \left(l_2 + l_3\right) \left(P_{jD} - P_{iD}\right) \left(P_{iM} - \alpha S_i^2\right) \delta^{h_i + l_i + l_i} + \frac{M}{T - L} l_4 \left(P_{jD} - P_{iD}\right) \left(P_{iD} - \alpha S_i^2\right) \delta^{h_i + l_i + l_i}
\]
In order to see the relationship between those two profit functions, we need to work on the difference between them. We want to show whether or not the difference shown below is greater than zero:

\[
\begin{align*}
\pi_i^{MC-S1} & \left( P_{IM}^{MC-S1}, P_{ID}, P_{JD} \right) \\
& - \pi_i^{MC-P2} \left( P_{IM}^{MC-P2}, P_{ID}, P_{JD} \right) \\
& = \frac{M}{T-L} \left( l_2 + l_3 \right) \left( v_3 - v_2 \right) \left( P_{IM}^{MC} - \alpha S_i^2 \right) \delta_i^{L} \delta_i^{L} \\
& - \frac{M}{T-L} \left( l_2 + l_3 \right) \left( \frac{P_{JD} - P_{ID}}{S_j - S_i} - v_2 \right) \left( P_{ID} - \alpha S_i^2 \right) \delta_i^{L} \delta_i^{L} \\
& > 0
\end{align*}
\]

After some manipulations, we get the following:

\[
\begin{align*}
\left( P_{IM}^{MC-S1} - \alpha S_i^2 \right) & \left( P_{IM}^{MC-S1} \left( S_j - S_i \right) - P_{ID} S_i \left( 1 - \delta_i^L \right) + P_{ID} \left( S_i - S_j \delta_i^L \right) \right) \\
& \left( S_i - S_j \delta_i^L \right) > \\
\left( P_{ID} - \alpha S_i^2 \right) & \left( P_{IM}^{MC-S1} \left( S_j - S_i \right) + P_{JD} S_j \left( 1 - \delta_i^L \right) - P_{ID} \left( S_i - S_j \delta_i^L \right) \right) \\
& \left( S_i - S_j \delta_i^L \right)
\end{align*}
\]

Since \( P_{IM} > P_{ID} \) for MC-S1, \( P_{IM}^{MC-S1} - \alpha S_i^2 \) is greater than \( P_{ID} - \alpha S_i^2 \). It is straightforward to show that \[
\frac{P_{IM}^{MC-S1} \left( S_j - S_i \right) - P_{ID} S_i \left( 1 - \delta_i^L \right) + P_{ID} \left( S_i - S_j \delta_i^L \right)}{\left( S_i - S_j \delta_i^L \right)}
\]
is greater than \[
\frac{-P_{IM}^{MC-S1} \left( S_j - S_i \right) + P_{JD} S_j \left( 1 - \delta_i^L \right) - P_{ID} \left( S_i - S_j \delta_i^L \right)}{\left( S_i - S_j \delta_i^L \right)}. \]
So, the result is that \( \pi_i^{MC-S1} \left( P_{IM}^{MC-S1}, P_{ID}, P_{JD} \right) \)
is greater than \( \pi_i^{MC-P2} \left( P_{IM}^{MC-P2}, P_{ID}, P_{JD} \right) \) for fixed \( P_{ID} \) and \( P_{JD} \). Let \( \left\{ P_{ID}^1, P_{JD}^1 \right\} \) and \( \left\{ P_{ID}^2, P_{JD}^2 \right\} \) be the optimal prices for \( P_{IM}^{MC-S1} \) and \( P_{IM}^{MC-P2} \). Thus, we got this following relationship:

\[
\pi_i^{MC-S1} \left( P_{IM}^{MC-S1}, P_{ID}^1, P_{JD}^1 \right) > \pi_i^{MC-S1} \left( P_{IM}^{MC-S1}, P_{ID}^2, P_{JD}^2 \right) > \pi_i^{MC-P2} \left( P_{IM}^{MC-P2}, P_{ID}^2, P_{JD}^2 \right)
\]
Therefore, the market configuration created as a result of the possibility 2 is dominated by Market Configuration obtained by combining scenario 1 conditions with the duopoly conditions. Therefore, scenario 5 does not lead us to a feasible market configuration. □

**Proof of Scenario 6 not generating a market configuration:**

Combining the scenario 6 and the duopoly period constraints will give us the following constraints:

\[ v_i > v_2 \left( P_{iM} < P_{iD} \right) \]

\[ v_i \geq v_3 \left( \frac{P_{iM}}{S_i} \leq \frac{P_{iD}}{S_j} \right) \]

\[ v_i \leq 1 \]

\[ \frac{P_{iD}}{S_i} < \frac{P_{jD}}{S_j} \]

\[ \frac{P_{jD} - P_{iD}}{S_j - S_i} < 1 \]

\[ P_{jD} \geq \alpha S_j^2 \]

\[ P_{iM} \geq \alpha S_i^2 \]

\[ P_{jD} \geq \alpha S_j^2 \]

*Insert Figure 13 Here*

Based on Figure 13 above, customers coming during the monopoly period make firms earn the following amounts:

\[ \pi_{iMM} = \frac{M}{T-L} \left( l_2 + l_3 \right) \left( 1 - v_i \right) \left( P_{iM} - \alpha S_i^2 \right) \delta^{l_1+l_2} \]

\[ \pi_{iMD} = \pi_{jMD} = 0 \]
Duopoly market shares of firms \(i\) and \(j\) are as shown in Figure 2b. Thus, the profit functions, \(\pi_{iDD}\) and \(\pi_{jDD}\) are (31) and (32). Firm i’s profit functions we get by combining scenario 6 and the duopoly period constraints are the same as the firm i’s profit functions we get by market configuration obtained by combining scenario 4 and the duopoly market constraints for. Thus, the proof of scenario 4 not generating a market configuration also proves that scenario 6 does not lead us to a feasible market configuration.

**Proof of Scenario 9 not generating a market configuration:** Let’s analyze scenario 9 monopoly condition \(\max\{v_1, v_2, v_3\} > 1\) under three cases:

Case 1: \(v_1 > v_2, v_1 \geq v_3\), and \(v_1 > 1\)

Case 2: \(v_2 \geq v_1, v_2 \geq v_3\), and \(v_2 > 1\)

Case 3: \(v_3 > v_1, v_3 > v_2\), and \(v_3 > 1\)

Combining case 1 with the duopoly market constraints \(\frac{P_{jD}}{S_i} < \frac{P_{jD} - P_{iD}}{S_j - S_i}\) will give us an infeasible set. When we work on the conditions, we get the relationship \(1 \leq v_1 < \frac{P_{iD}}{S_i} < \frac{P_{jD} - P_{iD}}{S_j - S_i}\) that conflicts with \(\frac{P_{jD} - P_{iD}}{S_j - S_i} < 1\) in the duopoly market condition set. Thus, case 1 does not lead us to a feasible market configuration.

Combining case 2 with the duopoly market constraints \(\frac{P_{iD}}{S_i} < \frac{P_{jD}}{S_j}\) and \(\frac{P_{jD} - P_{iD}}{S_j - S_i} < 1\) will give us an infeasible set. When we work on the conditions, we get the relationship \(1 < v_2 < \frac{P_{jD} - P_{iD}}{S_j - S_i}\) that conflicts with \(\frac{P_{jD} - P_{iD}}{S_j - S_i} < 1\) in the duopoly market condition set. Thus, case 2 does not lead us to a feasible market configuration.
Combining case 3 constraints with the duopoly market constraints will give us the following set of constraints:

\[
\begin{align*}
v_3 &> v_1 \left( \frac{P_{id}}{S_i} > \frac{P_{jd}}{S_j} \right) \\
v_3 &> v_2 \left( \frac{P_{id}}{S_i} > \frac{P_{jd}}{S_j} \left(1 - \delta^h - \delta^l \right) - P_{id} \left( S_j - S_i \right) \delta_3 \right) \\
v_3 &> 1 \\
\frac{P_{id}}{S_i} &< \frac{P_{jd}}{S_j} \\
\frac{P_{jd} - P_{id}}{S_j - S_i} &< 1 \\
P_{id} &\geq \alpha S_i^2 \\
P_{jd} &\geq \alpha S_j^2 \\
P_{id} &\geq \alpha S_i^2 \\
P_{jd} &\geq \alpha S_j^2
\end{align*}
\]

In this market configuration, nobody would like to buy product i in the monopoly period. The profit functions for firms i and j will be as follows respectively:

\[
\begin{align*}
\pi_i &= \frac{M}{T-L} \left( T-L \right) \left( \frac{P_{jd} - P_{id}}{S_j - S_i} - \frac{P_{id}}{S_j} \right) \left( P_{id} - \alpha S_i^2 \right) \delta^{h+l+L} \\
\pi_i &= \frac{M}{T-L} \left( T-L \right) \left( \frac{P_{jd} - P_{id}}{S_j - S_i} \right) \left( P_{jd} - \alpha S_j^2 \right) \delta^{h+l+L}
\end{align*}
\]

Let’s call MC-S9 to the market configuration we could get by combining scenario 9-case 3 and the duopoly conditions. Let’s also call MC-S8 to the market configuration got by combining scenario 8 and the duopoly conditions. The only difference between MC-S8 and MC-S9 in terms of constraints is \( v_3 > 1 \) (\( P_{im} > S_i - S_j - \delta^h + P_{jd} \delta^l \)) is one of the constraints in MC-S9 and \( v_3 < 1 \) (\( P_{im} < S_i - S_j - \delta^h + P_{jd} \delta^l \)) is one of the constraints in MC-S8. The rest of the constraints are the
same for both MC-S9 and MC-S8. Let \( P_{iM}^{MC-S8} \) and \( P_{iM}^{MC-S9} \) be the optimal monopoly price for firm \( i \) in MC-S8 and MC-S9 respectively. Let \( \pi_i^{MC-S8} \) and \( \pi_i^{MC-S9} \) be the profit functions for firm \( i \) in MC-S8 and MC-S9 respectively. \( \pi_i^{MC-S8}\left( P_{iM}^{MC-S8}, P_{iD}, P_{jD} \right) \) is greater than \( \pi_i^{MC-S9}\left( P_{iM}^{MC-S9}, P_{iD}, P_{jD} \right) \) for fixed \( P_{iD} \) and \( P_{jD} \) since the difference between them is a positive number under the condition that \( v_3 < 1 \). Let \( \{P_{iD}^1, P_{jD}^1\} \) and \( \{P_{iD}^2, P_{jD}^2\} \) be the optimal prices for \( P_{iM}^{MC-S8} \) and \( P_{iM}^{MC-S9} \). Thus, \( \pi_i^{MC-S8}\left( P_{iM}^{MC-S8}, P_{iD}^1, P_{jD}^1 \right) > \pi_i^{MC-S8}\left( P_{iM}^{MC-S8}, P_{iD}^2, P_{jD}^2 \right) \) and \( \pi_i^{MC-S8}\left( P_{iM}^{MC-S9}, P_{iD}^2, P_{jD}^2 \right) > \pi_i^{MC-S9}\left( P_{iM}^{MC-S9}, P_{iD}^1, P_{jD}^1 \right). \) Therefore, the market configuration created as a result of the scenario 9-case 3 is dominated by the market configuration created as a result of scenario 8. Therefore, scenario 9 does not lead us to a market configuration. \( \square \)

**Proof of scenario 10 not generating a market configuration:** Combining scenario 10 conditions with the monopoly constraints will give us the following constraints:

\[
\begin{align*}
    v_1 &> v_2 \left( P_{iM} < P_{iD} \right)
\end{align*}
\]

\[
\begin{align*}
    v_1 &\geq v_3 \left( \frac{P_{iM}}{S_i} \leq \frac{P_{jD}}{S_j} \right)
\end{align*}
\]

\[
\begin{align*}
    v_1 &\leq 1
\end{align*}
\]

\[
\begin{align*}
    \frac{P_{jD}}{S_j} &< \frac{P_{iD}}{S_i}
\end{align*}
\]

\[
\begin{align*}
    \frac{P_{iD} - P_{jD}}{S_i - S_j} &< 1
\end{align*}
\]

\[
\begin{align*}
    P_{iD} &\geq \alpha S_i^2
\end{align*}
\]

\[
\begin{align*}
    P_{iM} &\geq \alpha S_i^2
\end{align*}
\]

\[
\begin{align*}
    P_{jD} &\geq \alpha S_j^2
\end{align*}
\]

*Insert Figure 14 Here*
Group M consumers follow the customer segmentation depicted above in Figure 14. Thus, the firms’ profit functions for the monopoly period will be as follows:

\[ \pi_{iMM} = \frac{M}{T-L} (l_2 + l_3) (1-v_i) \left( P_{iM} - \alpha S_i^2 \right) \delta^{t+l_2} \]

\[ \pi_{iMD} = \pi_{jMD} = 0 \]

Similar to (31) and (32), the profit functions, \( \pi_{iDD} \) and \( \pi_{jDD} \) should be:

\[ \pi_{iDD} = \frac{M}{T-L} l_4 \left( 1 - \frac{P_{iD} - P_{jD}}{S_i - S_j} \right) \left( P_{iM} - \alpha S_i^2 \right) \delta^{t+l_2+l_d} \]

\[ \pi_{jDD} = \frac{M}{T-L} l_4 \left( \frac{P_{iD} - P_{jD}}{S_i - S_j} - \frac{P_{jD}}{S_j} \right) \left( P_{jD} - \alpha S_j^2 \right) \delta^{t+l_2+l_d} \]

Let’s define a firm’s myopic monopoly policy such that a firm maximizes its profit in each period only. Denote \( P_{iMM} \) as firm \( i \)’s myopic optimal monopoly price and \( P_{iMD} \) as firm \( i \)’s myopic duopoly price.

Firm \( i \)’s myopic monopoly profit is

\[ \frac{M}{T-L} (l_2 + l_3) (1-v_i) \left( P_{iM} - \alpha S_i^2 \right) \delta^{t+l_2} \]

and firm \( i \)’s myopic duopoly profit is

\[ \frac{M}{T-L} l_4 \left( 1 - \frac{P_{iD} - P_{jD}}{S_i - S_j} \right) \left( P_{iD} - \alpha S_i^2 \right) \delta^{t+l_2+l_d} \]. From the FOCs,

\[ P_{iMM} = \frac{\alpha S_i^2 + S_i}{2} \]

and \( P_{iMD} = \frac{P_{jD} - S_j + S_i + \alpha S_i^2}{2} < P_{iMM} \).

Next, we prove that \( P_{iM}^* \geq P_{iMM} \). We do this by contradiction. Now, assume \( P_{iM}^* \) is the optimal monopoly price and \( P_{iM}^* < P_{iMM} \). Firm \( i \)’s total profits using \( P_{iM}^* \) and \( P_{iMM} \) for all fixed \( P_{iD} \) and \( P_{jD} \) are respectively as follows:
\[ \pi_1(P_{IM}^*, P_{iD}^*, P_{jD}) = \frac{M}{T-L} \left(l_2 + l_3\right)(1-v_1)(P_{IM}^* - \alpha S_i^2)\delta^{l_1+l_2} + \frac{M}{T-L} \left(l_2 + l_3\right) \left(v_1 - \frac{P_{iD}^* - P_{jD}}{S_i - S_j}\right)(P_{iD}^* - \alpha S_i^2)\delta^{l_1+l_2} + \frac{M}{T-L} l_4 \left(1 - \frac{P_{iD}^* - P_{jD}}{S_i - S_j}\right)(P_{iD}^* - \alpha S_i^2)\delta^{l_1+l_2} \]

\[ \pi_1(P_{IMM}^*, P_{iD}^*, P_{jD}) = \frac{M}{T-L} \left(l_2 + l_3\right)(1-v_1)(P_{IMM}^* - \alpha S_i^2)\delta^{l_1+l_2} + \frac{M}{T-L} \left(l_2 + l_3\right) \left(v_1 - \frac{P_{iD}^* - P_{jD}}{S_i - S_j}\right)(P_{iD}^* - \alpha S_i^2)\delta^{l_1+l_2} + \frac{M}{T-L} (T-L - \beta S_L) \left(1 - \frac{P_{iD}^* - P_{jD}}{S_i - S_j}\right)(P_{iD}^* - \alpha S_i^2)\delta^{l_1+l_2} \]

Similar to proof of scenario 3 not generating a market configuration, \( \pi_1(P_{IMM}^*, P_{iD}^*, P_{jD}) \) is larger than \( \pi_1(P_{IM}^*, P_{iD}^*, P_{jD}) \) for both fixed \( P_{iD}, P_{iD} \) and optimal duopoly prices price for \( P_{IM}^* \) and \( P_{IMM}^* \). Thus, \( P_{IMM}^* \leq P_{iM}^* \). Up to now, we have shown that \( P_{iMD}^* \leq P_{iMM}^* \leq P_{iM}^* \).

Now, assume \( P_{iD}^* \) is the optimal duopoly price and \( P_{iD}^* > P_{iM}^* \). Therefore, \( P_{iMD}^* < P_{iD}^* \). We will show that \( P_{iMD}^* \) cannot be less than \( P_{iD}^* \), thus \( P_{iD}^* \leq P_{iMD}^* \) by contradiction. Firm \( i \)'s total profits by using \( P_{iD}^* \) and \( P_{iMD}^* \) are respectively as follows:

\[ \pi_1(P_{iM}^*, P_{iD}^*, P_{jD}) = \frac{M}{T-L} \left(l_2 + l_3\right)(1-v_1)(P_{iM}^* - \alpha S_i^2)\delta^{l_1+l_2} + \frac{M}{T-L} \left(l_2 + l_3\right) \left(v_1 - \frac{P_{iD}^* - P_{jD}}{S_i - S_j}\right)(P_{iD}^* - \alpha S_i^2)\delta^{l_1+l_2} + \frac{M}{T-L} l_4 \left(1 - \frac{P_{iD}^* - P_{jD}}{S_i - S_j}\right)(P_{iD}^* - \alpha S_i^2)\delta^{l_1+l_2} \]

\[ \pi_1(P_{iM}^*, P_{iD}^*, P_{jD}) = \frac{M}{T-L} \left(l_2 + l_3\right)(1-v_1)(P_{iM}^* - \alpha S_i^2)\delta^{l_1+l_2} + \frac{M}{T-L} \left(l_2 + l_3\right) \left(v_1 - \frac{P_{iMD}^* - P_{jD}}{S_i - S_j}\right)(P_{iMD}^* - \alpha S_i^2)\delta^{l_1+l_2} + \frac{M}{T-L} l_4 \left(1 - \frac{P_{iMD}^* - P_{jD}}{S_i - S_j}\right)(P_{iMD}^* - \alpha S_i^2)\delta^{l_1+l_2} \]
Similar to proof of scenario 3 not generating a market configuration, $\pi_i(P_{iM}^*, P_{iMD}^*, P_{iD}^*)$ is larger than $\pi_i(P_{iM}^*, P_{iD}^*, P_{iD}^*)$ for both fixed $P_{iD}$ and optimal $P_{iD}$ for $P_{iM}^*$ and $P_{iMM}^*$. Thus, $P_{iMD}$ cannot be less than $P_{iD}^*$. Therefore, $P_{iD}^* < P_{iMD} < P_{iMM} < P_{iM}^*$. That conflicts with one of the constraints, $P_{iM} < P_{iD}$. Therefore, this scenario does not lead us to a market configuration.

\[\square\]

**Proof of scenario 13 not generating a market configuration:** Let’s analyze scenario 9 monopoly condition $\max\{v_1, v_2, v_3\} > 1$ under three cases:

Case 1: $v_1 > v_2$, $v_1 \geq v_3$, and $v_1 > 1$

Case 2: $v_2 \geq v_1$, $v_2 \geq v_3$, and $v_2 > 1$

Case 3: $v_3 > v_1$, $v_3 > v_2$, and $v_3 > 1$

Combining case 1 with the duopoly market conditions $\frac{P_{iD}}{S_j} < \frac{P_{iD}}{S_i}$ and $\frac{P_{iD} - P_{iD}}{S_i - S_j} < 1$ will gives us an infeasible set. When we work on the conditions $1 < v_i < \frac{P_{iD}}{S_j} < \frac{P_{iD} - P_{iD}}{S_i - S_j}$ that conflicts with duopoly condition $\frac{P_{iD} - P_{iD}}{S_j} < 1$. Therefore, case 1 does not lead us to a feasible market configuration.

Combining case 2 conditions with the duopoly market conditions will give us the following constraint set:

\[
v_2 \geq v_1 \left( P_{iM} \geq P_{iD} \right)
\]

\[
v_2 > v_3 \left( P_{iM} > \frac{P_{iD} \left( S_i - S_j \delta^i \right) - P_{iD} S_j \left(1 - \delta^i \right)}{S_i - S_j} \right)
\]

\[
v_2 > 1
\]
\[
\frac{P_{jD}}{S_j} < \frac{P_{iD}}{S_i}
\]

\[
\frac{P_{iD} - P_{jD}}{S_i - S_j} < 1
\]

\[
P_{iD} \geq \alpha S_i^2
\]

\[
P_{jM} \geq \alpha S_i^2
\]

\[
P_{jD} \geq \alpha S_j^2
\]

*Insert Figure 15 Here*

Market share diagram of consumers in Group M is shown in Figure 15 above. The monopoly profit functions will look like as follows:

\[
\pi_{iMM} = 0
\]

\[
\pi_{iMD} = \frac{M}{T - L} (l_2 + l_3) \left( 1 - \frac{P_{iD} - P_{jD}}{S_i - S_j} \right) \left( P_{iD} - \alpha S_i^2 \right) \delta_{i}^{l_2 + l_3}
\]

\[
\pi_{jMD} = \frac{M}{T - L} (l_2 + l_3) \left( \frac{P_{iD} - P_{jD}}{S_i - S_j} - \frac{P_{jD}}{S_j} \right) \left( P_{jD} - \alpha S_j^2 \right) \delta_{j}^{l_2 + l_3}
\]

Let’s call MC-S13 to the market configuration we could get by combining scenario 13-case 2 and the duopoly conditions. Let’s also call MC-S11 to the market configuration got by combining scenario 11 and the duopoly conditions. The only difference between MC-S11 and MC-S13 in terms of constraints is \(v_2 > 1 (P_{iM} > \delta^6 P_{iD} + S_i (1 - \delta^6))\) is one of the constraints in MC-S13 and \(v_2 < 1 (P_{iM} < \delta^6 P_{iD} + S_i (1 - \delta^6))\) is one of the constraints in MC-S11. The rest of the constraints are the same for both MC-S13 and MC-S11. Let \(P_{iM}^{MC-S11}\) and \(P_{iM}^{MC-S13}\) be the optimal monopoly price for firm \(i\) in MC-S11 and MC-S13 respectively. The difference between \(\pi_{i}^{MC-S11}(P_{iM}^{MC-S11}, P_{iD}, P_{jD})\) and \(\pi_{i}^{MC-S13}(P_{iM}^{MC-S13}, P_{iD}, P_{jD})\) for fixed \(P_{iD}\) and \(P_{jD}\) is as follows:
$$\frac{M}{T-L}(l_2 + l_3) \left(1 - \frac{P_{iM}^{MC-S11} - \delta^i_i P_{pD}}{S_i(1-\delta^i_i)} \right) \left( P_{iM}^{MC-S11} - \alpha S_i^2 \right) \delta^{l_i + l_2} - \left( P_{pD} - \alpha S_i^2 \right) \delta^{l_i + l_2}$$

1 - $$\frac{P_{iM}^{MC-S11} - \delta^i_i P_{pD}}{S_i(1-\delta^i_i)}$$ is positive since $$v_2 < 1$$ in MC-S11. $$\left( P_{iM}^{MC-S11} - \alpha S_i^2 \right) \delta^{l_i + l_2}$$ is greater than $$\left( P_{pD} - \alpha S_i^2 \right) \delta^{l_i + l_2}$$ since $$P_{iM}^{MC-S11}$$ is greater than $$P_{pD}$$ based on $$v_2 \geq v_1$$ and $$\delta^{l_i + l_2}$$ is greater than $$\delta^{l_1 + l_2}$$ (since $$0 < \delta < 1$$ and $$l_1 + l_2 + l_3 > l_1 + l_2$$).

Thus, $$\pi_{i}^{MC-S11} \left( P_{iM}^{MC-S11}, P_{iD}, P_{pD} \right) > \text{Profit}_i^{MC-S13} \left( P_{iM}^{MC-S13}, P_{iD}, P_{pD} \right)$$.

Let \( \{P_{iD}^1, P_{pD}^1\} \) and \( \{P_{iD}^2, P_{pD}^2\} \) be the optimal prices for $$P_{iM}^{MC-S11}$$ and $$P_{iM}^{MC-S13}$$. Thus, $$\pi_{i}^{MC-S11} \left( P_{iM}^{MC-S11}, P_{iD}^1, P_{pD}^1 \right) > \pi_{i}^{MC-S11} \left( P_{iM}^{MC-S11}, P_{iD}^2, P_{pD}^2 \right) > \pi_{i}^{MC-S13} \left( P_{iM}^{MC-S13}, P_{iD}^1, P_{pD}^1 \right)$$.

Therefore, the market configuration created as a result of the scenario 13-case 2 is dominated by the market configuration got by combining scenario 11 conditions with the duopoly conditions.

Now, let’s look at the last case, case 3. Combining case 3 with the duopoly market conditions $$\frac{P_{iD}}{S_j} < \frac{P_{iD}}{S_i}$$ and $$\frac{P_{iD} - P_{pD}}{S_i - S_j} < 1$$ will give us an infeasible set. When we work on the conditions

$$\frac{P_{iD}}{S_j} < 1 < \frac{P_{iD} - P_{pD}}{S_i - S_j}$$

that conflicts with the duopoly condition $$\frac{P_{iD} - P_{pD}}{S_i - S_j} < 1$$. Therefore, case 3 does not lead us to a feasible market configuration.

---

**Figure 11** Market share diagram of consumers in Group M

<table>
<thead>
<tr>
<th>Consumer valuation, v</th>
<th>Buy nothing</th>
<th>Buy X in monopoly</th>
<th>Buy Y in duopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>P_{iD}</td>
<td>P_{pD}</td>
</tr>
<tr>
<td>v_1</td>
<td></td>
<td>P_{pD} - P_{iD}</td>
<td>S_i - S_j</td>
</tr>
<tr>
<td>v_2</td>
<td></td>
<td>1</td>
<td>( P_{iD} - P_{pD} )</td>
</tr>
</tbody>
</table>

**Figure 12** Market share diagram of consumers in Group M

<table>
<thead>
<tr>
<th>Consumer valuation, v</th>
<th>Buy nothing</th>
<th>Buy X in monopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>P_{iD}</td>
</tr>
<tr>
<td>v_1</td>
<td></td>
<td>58</td>
</tr>
<tr>
<td>v_2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>v_3</td>
<td></td>
<td>( P_{pD} - P_{iD} )</td>
</tr>
</tbody>
</table>
Figure 13  Market share diagram of consumers in Group M

Figure 14  Market share diagram of consumers in Group M

Figure 15  Market share diagram of consumers in Group M