# Modeling Speed Disturbance Absorption Following State-Control ActionExpected Chains: Integrated Car-Following and Lane-Changing Scenarios 

A Paper Submitted to<br>The Transportation Research Board for Review for Presentation \& Publication at the TRB 84th Annual Meeting in Washington, D.C. January 9-13, 2005<br>By<br>Ruihua Tao, Ph.D.<br>Maryland Department of Transportation<br>Office of Traffic \& Safety, 7491 Connelley Drive<br>Hanover, MD 21076<br>Phone: (410)787-5863; Fax: (410)582-9469<br>Email: rtao@sha.state.md.us<br>Heng Wei, Ph.D., P.E.<br>Department of Civil \& Environmental Engineering<br>University of Cincinnati<br>Baldwin Hall, PO Box 210071<br>Cincinnati, OH 45221-0071<br>Fax: (513)556-2599<br>E-mail: hengwei99@yahoo.com<br>Yinhai Wang, Ph.D.<br>Department of Civil \& Environmental Engineering<br>University of Washington<br>121 More Hall, Box 352700<br>Seattle, WA 98195-2700<br>Tel: (206)616-2696; Fax: (206)543-5965<br>Email: yinhai@u.washington.edu<br>and<br>Virginia P. Sisiopiku, Ph.D.<br>University of Alabama at Birmingham<br>Hoehn 311, 1530 3rd Ave. South<br>Birmingham, AL 35294-4440<br>Tel: (205) 934-8430; Fax: (205) 934-9855<br>E-mail: vsisiopi@uab.edu

# Modeling Speed Disturbance Absorption Following State-Control Action-Expected Chains: Integrated Car-Following and Lane-Changing Scenarios 

Ruihua Tao, Heng Wei, Yinhai Wang, and Virginia P. Sisiopiku


#### Abstract

This paper explores driver behaviors in a paired car-following mode in response to a speed disturbance from the front vehicle. A Current State-Control Action-Expected State (SAS) Chains is developed to provide a framework for modeling hierarchy of expected actions incurred in the need of speed disturbance absorption. Three car-following scenarios and one lane-changing scenario are identified with defined perceptual informative variables to describe the process of speed disturbance absorption. Those variables include dynamic spacing vs. the follower's speed, disturbance-effecting/ending spacing, headway, acceleration/deceleration, speed recovery period, speed advantage, and lane-changing duration. A significant advance in car-following modeling that is reflected in the paper is the integration of carfollowing and lane-changing behaviors in the SAS chains, and critical values of perceptual informative variables are statistically developed as a function of the follower's speed, using observed vehicle trajectory data. Moreover, models for the probability of a lane-changing response to a speed disturbance identified by the speed advantage variable, and models for acceptable lane-changing decision-making conditions at the adjacent lanes, are developed based on the analysis of observed vehicle trajectory data. The work presented in this paper provides well understanding of speed disturbance and speed absorption phenomenon and car-following and lane-changing behaviors at the microscopic level. This establishes the foundation for further research on the multiple speed disturbance absorption and its impact on traffic stabilities at the macroscopic analysis level.


Key Words: Speed Disturbance Absorption, Car-following, Lane-Changing, Perceptual Informative Variables, Action Chains, Microscopic Simulation, Modeling.

## 1. INTRODUCTION

Single-lane car-following models basically address speed-spacing relationships as one vehicle follows another on a straight roadway where there is no passing (neglecting all other subsidiary tasks such as steering, routing, etc.) [1]. Six types of car-following models have been developed or discussed in the past more than a half of the century. These six types are: 1) safety-distance car-following models, 2) stimulusresponse car-following models, 3) action-point car-following models, 4) fuzzy logic-based car-following models, 5) human element-based car-following models, and recently emerged 6) perceptual-satisficing approach - a framework of a car-following model. The first four types of car-following models were historically reviewed by Brackstone and McDonald [2]. There has been the lack of information as to when and where the models would be successful [3]. Mehmood et al. [4] identify and discuss the assumptions involved in many existing car-following models that restrict the models' ability to explain and predict driver behaviors in actual traffic situations.

Boer's study [5] raises some important issues that have been ignored in engineering car-following models, including the following: 1) car following is only one of the many tasks that drivers perform simultaneously and receives therefore only intermittent attention and control; 2) drivers are satisfied with a range of conditions that extend beyond the boundaries imposed by perceptual and control limitation, and 3 ) in each driving task drivers use a set of highly informative perceptual variables to guide decision making and control. On the basis of these three issues, Boer proposes a perceptual-satisficing theory based framework that seeks to depart from determinism and presents two approaches to further develop the area, namely satisfying behavior and task scheduling, which are apparently conjunctive to traffic psychology progresses. The problem with this approach, as Brackstone and McDonald points out [2,3], is the difficulty in obtaining data for calibrating the model.

Van Winsum [6] establishes a human element-based model that describes the relations of psychological factors (time headway, distance headway, and deceleration) used by the driver in car following. His model is based upon three assumptions: 1) a drivers uses time headway as a safety margin; 2) if distance to the lead vehicle is larger than the preferred distance headway that he/she tries to maintain, there is no safety-related reason for the driver to decelerate until the preferred distance headway is reached; and 3) the deceleration initiated by the driver is a function of the Time-to-Collision (TTC) parameter, which is estimated by the driver. Similar to Boer's model, Van Winsum's model has not been tested with data.

Above studies and issues inspired the authors to conduct further research using video tapping observations, extracted vehicle trajectory data from the videotapes, and simulation data from running CORSIM as well. The authors' observations imply the congruency and consistency of Boer's taxonomy of driving tasks from high level to low level - strategic, tactical, and operational [7]. Strategic level of driving task includes the motives and the goals for traveling - determining origin and destination (O-D) and planning link-based route. Tactical level is concerned with seeking appropriate links that are connections of the planned route and lanes that access to the downstream en-route link. As a result, lanebased alternative routes are identified during en route driving. Operational level is viewed as the low level task handlers and skilled-based controllers [7]. At this level the driver executes control commands with dexterity, smoothness, and coordination, constantly relying on feedback from his own responses to the dynamics of the system's counterparts (lead vehicle and roadway traffic). It is noticeable that local and asymptotic stabilities occur at this level of driving tasks. Based on Boer and Hoedemaeker's perceptual-satisficing theory-based framework [5], the authors propose the modified framework of a driver's driving tasks at the above three levels, as shown in Figure 1.

This paper summarizes the authors' work [8, 9] in exploring driver behaviors in a paired car following mode in response to a speed disturbance from the front vehicle, and develops a concept of the SAS

Chains. Four scenarios of car-following and lane-changing behaviors are identified in response to a speed decrease of the front vehicle. The impact of the following vehicle driver's responding behavior on dynamic spacing between the paired vehicles is analyzed and minimum dynamic spacing for each scenario are obtained from the observed trajectory data. As one of the expected responses, decisionmaking for a lane change is integrated with the car-following action chains. Observed vehicle trajectory data are applied to identify the perceptual informative variables for lane-changing decisions and are analyzed to conclude applicable minimum safety spacing in car-following and lane-changing executions. Probability models of using lane-changing response as a solution to absorb speed disturbances from the front vehicle and acceptable conditions at the adjacent lane for lane changing decision-making are developed. The work presented in this paper advances the understanding of speed disturbance and speed absorption phenomenon and modeling a car-following mode at the microscopic level.

## 2. HIERARCHY OF STATE-CONTROL ACTION CHAINS TO SPEED DISTURBANCE

Drivers generally engage in multiple tasks on roads. When following another vehicle on a single lane, the driver is mostly involved in monitoring-assessing/decision making-controlling processes. Once a monitoring process is initiated by attention it is decomposed into four stages: 1) pay attention to the perceptual variables that characterize a particular task in the form of the present state; 2) use those variables to evaluate current and expected future states, 3) assess whether the present performance will most likely remain acceptable for a certain amount of time, and if so 4) determine when attention should be given to the task again. If the present performance is not satisfactory, then the driver initiates an appropriate control process that employs a suitable skill to bring the vehicle within a limited time into an acceptable state that is expected to remain acceptable for some time [6].

The logic of the driver's response to a speed disturbance is illustrated in Figure 2. When a speed disturbance occurs, this disturbance could either be recovered or not. A recovered speed disturbance normally exhibits two phases: speed reduction and recovery as shown in Figure 3 (1). The following vehicle may respond to it immediately or sometime later, or just keep its original speed until the speed disturbance is recovered. When the drive responds to a speed disturbance in front, he/she may choose to respond to it by controlling the vehicle speed or the vehicle path (lane changing).

Assuming a speed disturbance is initiated at $\mathrm{t}_{0}$ and recovered at $\mathrm{t}_{0}+\mathrm{T}$, i.e., the lead vehicles' speed decreases at $\mathrm{t}_{0}$ and gets back to its original speed after a period of time $T$ (recovery period), the following vehicle could respond in different ways, namely, three car-following scenarios and lane-changing scenario, which constitute the basic State-Control Action-Expected State chains (see Figure 3).

- If the initial spacing and lead vehicle's speed are acceptable to the follower at $t_{0}$, and the reduced spacing remains acceptable over $T$, no action is actually in need for the follower. In this case, $T$ is viewed as tolerable recovery period. This case is categorized into car-following scenario a), as illustrated in Figure 3 (2).
- If during $T$ the state of the spacing perceptual variable becomes unacceptable due to decreased spacing caused by the reduction of the lead vehicle's speed, the following driver initiates a control action of deceleration or lane-changing. If it is allowed to bring the unacceptable state of the spacing perceptual variable back to an acceptable level during $T$, the follower remains in the same lane while reducing the speed to control the spacing and then reinstate following the lead vehicle. This case is categorized into car-following scenario b), as illustrated in Figure 3 (3). It should be noted that the perception of "acceptable" or "unacceptable" perceptual spacing varies from driver to driver. The drivers who would like to use controlled deceleration may adopt a larger spacing as an acceptable threshold than drivers who choose to hit the brake pedals harder.

In order to avoid a possible collision, the braking action of the driver of the following vehicle can start as early as when the driver perceives the speed drop of the lead vehicle to as late as when he/she has reached the minimum braking distance to avoid the target risk. Moreover, the adoption of deceleration rates would differ among drivers. Some may just release the gas pedal to reduce the vehicular speed in response to the speed drop of the lead vehicle when a large enough spacing is available. Other drivers may apply "controlled braking" to control their speed and the spacing between the two vehicles, whereas some drivers may choose to brake at the last minute with "maximum braking force." Figure 3 (3) shows an area embraced by blue line where a control action may be taken.

- If at $t_{0}$ the initial state of the perceptual variables of the following vehicle is unacceptable and at target risk level, an appropriate control action is immediately taken to bring them away from the target risk. In this case the interactive speeds of the two vehicles are shown in car-following scenario c) of Figure 3 (4). If an adjacent lane allows higher speed, a lane change may be executed. This case is categorized into lane-changing scenario, as shown in Figure 3 (7). A perceptual variable, speed advantage that will be discussed later, is employed to identify the condition of gaining speed from the adjacent lane.

If the initial spacing and lead vehicle's speed are acceptable to the follower at $t_{0}$ and the recovery period is intolerable to the follower, a lane change may be executed when an adjacent lane allows higher speed. If no adjacent lane is perceived advantageous in speed, the follower acts with car-following scenario b).

Models for three car-following scenarios and the lane-changing scenario are discussed in details the following sections.

## 3. SCENARIOS OF SPEED DISTURBANCE ABSORPTION

### 3.1 Car-following Scenarios

If the driver decides to reduce the vehicle speed in response to the speed disturbance, the speeds of the two vehicles interact and the dynamic spacing between the two vehicles can vary greatly but will follow the following law:

$$
\begin{array}{ll}
S(t)=S\left(t_{0}\right)+\int_{t_{0}}^{t} \dot{x}_{n}(t) d t-\int_{t_{0}}^{t} \dot{x}_{n+1}(t) d t  \tag{1}\\
\text { Where, } n & =\text { front vehicle } \\
n+1 & =\text { the first following vehicle } \\
t_{0} & =\text { beginning time point of the scenario, } \\
t & =\text { time variable, } \\
\dot{x}_{n}(t) & =\text { speed of vehicle } \mathrm{n} \text { (lead vehicle) at time } \mathrm{t} \\
\dot{x}_{n+1}(t) & =\text { speed of vehicle } \mathrm{n}+1 \text { (following vehicle) at time } \mathrm{t} \\
S(t) & =\text { distance headway between vehicles } \mathrm{n}, \text { and } \mathrm{n}+1 \text { at time } t .
\end{array}
$$

In each time interval, the spacing changes dynamically. Figure 2 illustrates the basic state-control actionexpected chains that are presented with three car-following scenarios and lane-changing scenario. Table 1 presents a summary of variables in minimum spacing models for all study scenarios and cases. Table 2 gives observed maximum acceleration or deceleration values for each $10-\mathrm{mph}$ increment for determining
thresholds of acceleration as simulating a car-following scenario. Two control variables are used in the following sections: 1) The spacing when the follower begins response to speed disturbance is called "Disturbance-Effecting Spacing" (EfS); and 2) The spacing when the follower begins speed recovery from speed disturbance is called "Disturbance-Ending Spacing" (EnS). The values of these two variables are determined by equations that are derived from statistic analysis of observed vehicle trajectory data, as shown in Table 1.

### 3.1.1 Car-Following Scenario a)

In this scenario as shown in Figure 3 (2), the following vehicle does not respond to the speed disturbance from the lead vehicle and accepts a reduced spacing between them. During the time interval $\left[t_{0}, t_{1}\right]$ vehicle $n$ reduces its speed and a speed disturbance is developed. According to Rothery's conclusion [10] from reexamining the data obtained in the studies by Todosiev [11] and Rothery [12], the deceleration rate of an individual vehicle can be assumed as almost constant. Thus, Equation (1) becomes:

$$
\begin{align*}
& S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t-t_{0}\right)^{2} \text { where, } \ddot{x}_{n}^{-}<0, t \in\left[t_{0}, t_{1}\right], \text { and } \\
& \operatorname{Min} S(t)=S\left(t_{1}\right)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}, \text { at } t=t_{1} \tag{2}
\end{align*}
$$

Where, $\ddot{x}_{n}^{-}=$deceleration rate of vehicle $n$,
In the time interval $\left(t_{1}, t_{0}+T\right]$ the dynamic spacing between the two vehicles becomes:

$$
\begin{align*}
& S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right) \times\left(t-t_{1}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(t-t_{1}\right)^{2}, \text { and } \\
& \operatorname{Min} S(t)=S\left(t_{0}+T\right)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2} \times\left[1-\frac{\ddot{x}_{n}^{-}}{\ddot{x}_{n}^{+}}\right], \text {at } t=t_{0}+T \tag{3}
\end{align*}
$$

In the case of scenario a) during the entire period of $\left[t_{0}, t_{0}+T\right], S(t)$ has a minimum value of

$$
\begin{equation*}
\operatorname{Min} S(t)=S\left(t_{0}+T\right)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2} \times\left[1-\frac{\ddot{x}_{n}^{-}}{\ddot{x}_{n}^{+}}\right], \text {at } t=t_{0}+T \tag{4}
\end{equation*}
$$

Where, $\ddot{x}_{n}^{+}=$acceleration rate of vehicle $n$.

### 3.1.2 Car-Following Scenario b)

Under this scenario, a control action can be taken at time as late as reaching the minimum EfS, when a driver of the following vehicle meets the minimum braking distance requirement for reducing its speed to avoid a collision. The deceleration rate can vary and as a result the control action may end earlier or late. Thus, there exist three possible cases determined by the beginning time and g force applied by a control action: 1) $\delta<t_{1}$ and $\delta^{\prime}>t_{1}$; 2) $\delta<t_{1}$ and $\delta^{\prime}<t_{1}$; and 3) $\delta>t_{1}$ and $\delta^{\prime}>t_{1}$, shown in Scenario b) of Figure 3.

Case 1: $\delta<t_{1}$ and $\delta^{\prime}>t_{1}$

In $\left[t_{0}, \delta\right]$, the following vehicle keeps its original speed and the dynamic spacing and minimum spacing during this interval is expressed in the same way as described in Equation 2.

$$
\begin{align*}
& S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t-t_{0}\right)^{2} \quad \text { where, } \ddot{x}_{n}^{-}<0, t \in\left[t_{0}, t_{1}\right], \text { and } \\
& \operatorname{Min} S(t)=S\left(t_{1}\right)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2} \geq E f S_{\min }, \text { at } t=\delta
\end{align*}
$$

In $\left(\delta, t_{1}\right]$, the following vehicle reduces its speed to keep a safe distance from the vehicle in front. The dynamic spacing between them and its minimum value are:

$$
\begin{align*}
& S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t-t_{0}\right)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-} \times(t-\delta)^{2}, \text { and } \\
& \operatorname{Min} S(t)=S\left(t_{1}\right)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-}\left(t_{1}-t_{0}\right)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-}\left(t_{1}-\delta\right)^{2}, \text { at } t=t_{1} \tag{6}
\end{align*}
$$

Where, $\ddot{x}_{n+1}^{-}=$deceleration rate of vehicle $\mathrm{n}+1$.
In $\left(t_{1}, \delta^{\prime}\right]$, the following vehicle $\mathrm{n}+1$ keeps reducing its speed and the lead vehicle n recovers its speed. The dynamic spacing between them and its minimum value is:

$$
\begin{align*}
& S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right) \times\left(t-t_{1}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(t-t_{1}\right)^{2} \\
& \quad-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(t_{1}-\delta\right)^{2}-\ddot{x}_{n+1}^{-} \times\left(t_{1}-\delta\right) \times\left(t-t_{1}\right)-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(t-t_{1}\right)^{2}, \text { and } \\
& \text { Min } S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)\left(\delta^{\prime}-t_{0}\right)-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\delta\right)\left(t_{1}-\delta\right), \text { at } t=\delta^{\prime}  \tag{7}\\
& \text { Min } S(t)=S\left(\delta^{\prime}\right), \text {, i.e., minimum disturbance-ending spacing. }
\end{align*}
$$

Case 2: $\delta<t_{1}$ and $\delta^{\prime}<t_{1}$
In $\left[t_{0}, \delta\right]$, the dynamic spacing and its minimum value are as described in Equation (5). In ( $\left.\delta, \delta^{\prime}\right]$, the speed disturbance is continuously developed and vehicle $\mathrm{n}+1$ reduces its speed starting at $t=\delta$ to keep a safe distance space from the vehicle in front until $t=\delta^{\prime}$. At that time, the speed of the following vehicle reaches the speed of vehicle $n$. The dynamic spacing and its minimum value are:

$$
\begin{align*}
& S(t)=S(\delta)+\ddot{x}_{n}^{-} \times\left(\delta-t_{0}\right) \times(t-\delta)+\frac{1}{2} \ddot{x}_{n}^{-} \times(t-\delta)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-} \times(t-\delta)^{2}, \text { and } \\
& \operatorname{Min} S(t)=S\left(\delta^{\prime}\right)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(\delta^{\prime}-t_{0}\right)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\delta\right)^{2} \geq E n S_{\text {min }}, \text { at } t=\delta^{\prime} \tag{8}
\end{align*}
$$

In $\left(\delta^{\prime}, t_{0}+T\right]$, the following vehicle keeps roughly the same speed profile as that of the vehicle directly in front. The dynamic spacing between them can be regarded as constant. Therefore, the minimum value of dynamic spacing in this case is $\operatorname{Min} S(t)=S\left(\delta^{\prime}\right)$.

Case 3: $\delta>t_{1}$ and $\delta^{\prime}>t_{1}$

In $\left[t_{0}, t_{1}\right]$, the following vehicle keeps its original speed and the dynamic spacing and its minimum value are the same as in Equation 2.

In $\left[t_{1}, \delta\right]$, vehicle n recovers its speed. The perceptual state of vehicle $\mathrm{n}+1$ has been kept acceptable and the following vehicle travels at its original speed until it becomes unacceptable at $t=\delta$. The dynamic spacing between the two vehicles and the minimum value of this spacing are:

$$
\begin{align*}
& S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)\left(t-t_{1}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(t-t_{1}\right)^{2}, \text { and } \\
& \text { Min } S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)\left(\delta-t_{1}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(\delta-t_{1}\right)^{2} \text { at } t=\delta \tag{9}
\end{align*}
$$

In $\left(\delta, \delta^{\prime}\right]$, vehicle n continuously recovers its speed and vehicle $\mathrm{n}+1$ reduces its speed to keep a safe distance headway from the vehicle in front. The dynamic spacing and its minimum value are:

$$
\begin{align*}
& S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(t-t_{1}\right)^{2}+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)\left(t-t_{1}\right)-\frac{1}{2} \ddot{x}_{n+1}^{-} \times(t-\delta)^{2}, \text { and } \\
& \operatorname{Min} S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(\delta^{\prime}-t_{1}\right)^{2}+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)\left(\delta^{\prime}-t_{1}\right)-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\delta\right)^{2} \text { at } t=\delta^{\prime}, \tag{10}
\end{align*}
$$

In this scenario, $\operatorname{Min} S(t)=S\left(\delta^{\prime}\right)$.

### 3.1.3 Car-Following Scenario c)

In scenario c), a control action is taken at the time of $\tau$, but the deceleration rate may vary, from small or large, which in turn results in the control action ending earlier or later. Thus, there exist two possible cases, i.e. 1) $\delta^{\prime}>t_{1}$; and 2) $\delta^{\prime}<t_{1}$; as shown in Scenario c) of Figure 3.

Case 1: $\delta^{\prime}>t_{1}$
This case is similar to case 1 in Scenario b) except for the beginning time of a control action of the following vehicle. Therefore:

$$
\begin{equation*}
\operatorname{Min} S(t)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right) \times\left(\delta^{\prime}-t_{1}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(\delta^{\prime}-t_{1}\right)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\tau\right)^{2}, \text { at } t=\delta^{\prime} \tag{11}
\end{equation*}
$$

Case 2: $\delta^{\prime}<t_{1}$
Case 2 is similar to the case 2 of Scenario b) with

$$
\begin{equation*}
\operatorname{Min} S(t)=S\left(\delta^{\prime}\right)=S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(\delta^{\prime}-t_{0}\right)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\tau\right)^{2}, \text { at } t=\delta^{\prime} \tag{12}
\end{equation*}
$$

In scenario a), during the entire period of the speed disturbance of the lead vehicle the following vehicle does not reduce its speed but the spacing between the two vehicles is reduced by
$\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2} \times\left[1-\frac{\ddot{x}_{n}^{-}}{\ddot{x}_{n}^{+}}\right]$. This spacing reduction absorbs a speed reduction in the amount of $\Delta \dot{x}_{n}=$ $\dot{x}_{n}\left(t_{0}\right)-\dot{x}_{n}\left(t_{1}\right)$.

For a given spacing $S\left(t_{0}\right), \Delta \dot{x}_{n}$ must be

$$
\begin{equation*}
\Delta \dot{x}_{n} \leq \sqrt{\frac{2\left[S\left(t_{0}\right)-S_{\text {safe }}\right]}{-\frac{1}{\ddot{x}_{n}^{-}}+\frac{1}{\ddot{x}_{n}^{+}}}} \tag{13}
\end{equation*}
$$

Otherwise, $\Delta \dot{x}_{n}$ must propagate backwards through the speed reduction of the following vehicle. This propagated speed reduction has an impact on the vehicle following the next following vehicle as shown in Figure 4.

If the spacing between vehicles $n+1$ and $n+2$ is large enough to absorb the propagated speed reduction, then $\Delta \dot{x}_{n}$ must be:

$$
\begin{equation*}
\left(\Delta \dot{x}_{n}\right)^{2} \leq \frac{2\left[S\left(t_{0}\right)-S_{\text {safe }}\right]}{\frac{1}{\ddot{x}_{n}^{+}}-\frac{1}{\ddot{x}_{n}^{-}}}+\frac{2\left[S_{n+1}\left(t_{0}\right)-S_{\text {safe }}\right]}{\frac{1}{\ddot{x}_{n}^{+}}-\frac{1}{\ddot{x}_{n+1}^{-}}} \tag{14}
\end{equation*}
$$

Otherwise, the initial speed disturbance will be continuously transmitted backward through the line of vehicles.

Therefore, Equation (13) gives the magnitude of the speed disturbance that a single spacing can absorb, and Equation (14) gives the magnitude of the speed disturbance that two consecutive spacings can absorb in the situation where the speed disturbance is propagated.

Figure 5 shows the distribution of spacing and headway between pairs of vehicles for all scenarios executed as a response to car-following speed disturbance. Observed minimum spacing at the control action times and handbook-recommended minimum safety spacing are demonstrated in Figure 5, which will be discussed in detail in section 3.2.3.

### 3.2 Lane-Changing Scenario for Response to Speed Disturbance

Two types of lane-changing behaviors, mandatory and discretionary, were traditionally defined [13, 14]. A Driver's decision to make a lane change is dependent on his route plan, lane status (whether the current lane is in the route-link), and traffic conditions in the current and adjacent lanes. It has been commonly recognized that drivers perform mandatory lane change to connect the next link on their path, bypass a downstream lane blockage, obey lane-use regulations, and respond to lane-use signs and variable message signs. A decision to perform discretionary lane change is based on whether a driver can tolerate the leading vehicle's speed and the flow speed in the adjacent lane(s). Other reason for discretionary lane change also includes obtaining queue advantage, which is very likely to occur when an adjacent lane has a shorter queue [13]. As shown in Figure 2, one phenomenon that has been previously drawn little attention is the case when a lane change may be executed in responding to a car-following speed disturbance from the front vehicle. In this case the lane change falls into the category of discretionary
lane change. Figure 3 (7) illustrates the process when a lane change occurs at the time $\delta$ in response to a speed disturbance. Minimum spacing variables for lane-changing scenario is also summarized in Table 1.

### 3.2.1 Probability of Lane-Changing Response Perpetuated via Speed Advantage

Gipps [15] proposed the concept of speed advantage to identify a need of a (discretionary) lane change and Wei et al. [16] proposed a quantitative definition of speed disadvantage and speed advantage to describe decisional conditions for a discretionary lane change on urban streets. In the authors' study of the speed absorption phenomenon using observed vehicle trajectory data, speed advantage is redefined as a perceptual variable for a driver to compare speed performance with the adjacent lane(s). Speed advantage (SA) is described in the following mathematical equation.

$$
\begin{equation*}
S A=\frac{V_{L D}-V_{F T}}{V_{L D}} \tag{15}
\end{equation*}
$$

Where, $S A=$ speed advantage compared to an adjacent lane; 0 if $V_{L D} \leq V_{F T}$
$V_{L D}=$ the lead vehicle's speed in an adjacent lane, $\mathrm{ft} / \mathrm{s}$ or mph ;
$V_{F T}=$ the front vehicle's speed, $\mathrm{ft} / \mathrm{s}$ or mph ;
Using the SA variable, the probability of motivating a lane change incurred from a speed disturbance by the front vehicle could be estimated by the following equations, which were derived from statistical analysis of vehicle trajectory data obtained in Kansas City, Missouri [9]:

For two-way streets:


$$
\begin{equation*}
P(x=S A)=100 \%\left[0.0149+0.2097 x+1.1081 x^{2}+1.2074 x^{3}-1.6351 x^{4}\right] \tag{16}
\end{equation*}
$$

For one-way street:

$$
\begin{equation*}
P(x=S A)=100 \%\left[0.145+0.1592 x+0.637 x^{2}+0.3837 x^{3}+0.3197 x^{4}\right] \tag{17}
\end{equation*}
$$

Where $\mathrm{P}_{\mathrm{A}}(\mathrm{x}=\mathrm{SA})=$ probability of generating a motivation to change to the speed-advantaged lane when $x=\mathrm{SA}$. The correlation coefficient $\mathrm{R}^{2}$ value for equations (3) and (4) is 0.9757 and 0.9812 , respectively, at the confidence level of $95 \%$. For example, if the front vehicle speed reduces to 30 mph while the adjacent lane's speed remains at $50 \mathrm{mph}, 31.1 \%$ of followers would possibly change to the adjacent lane in a two-way street. Similarly, $32.7 \%$ of follows do so in a one-way street.

### 3.2.2 Acceptable Conditions at the Adjacent Lane to Execute A Lane Change

Both the gap between the lead (in the adjacent lane) and the prospective lane changer (termed as "lead gap") and the gap between the prospective lane changer and the lag vehicle that is in the adjacent lane (termed as "lag gap") are major factors affecting the driver's lane-changing decisions. More significantly, these acceptable gaps are coherent with the speed of the prospective lane changer and the lag vehicle. It is assumed that the acceptable lead gap for a lane-changing decision could be expressed as a function of the subject vehicle's speed and the lag gap as a function of the lag vehicle's speed.

Figure 6 visualizes the distribution of observed lead gaps versus lane changer's speeds and lag gaps versus the lag vehicle's speeds. It presents the concerned gap and speed values at the moment as the maneuver of a lane change is started. Comparisons between the observed gap values and the
recommended minimum safe gaps (minimum safety braking distances plus reacting distances at varied speeds) recommended in Kansas Driving Handbook are also presented in Figures 6. It is obviously seen from Figures 6 that a portion of the sampling drivers, as perform discretionary lane changes, actually accepts smaller gaps than the handbook-recommended values. Those drivers accepting no less than the handbook-recommended values are viewed as conservative drivers, otherwise as regular drivers. The critical acceptable gaps are modeled using a curve representing the lower boundary of the observed data, as shown by the lowest curve in Figure 6. Acceptable gap conditions to execute a discretionary lane change can be described by the following equation.

$$
\text { Lead/Lag Gap }> \begin{cases}\geq 0.3336 V_{\text {sub }} .1 .6398 & \text { (conservative drivers) }  \tag{19}\\ \geq 7.979 e^{0.0556 V_{\text {sub }}} \text { and }<0.3336 V_{\text {sub }} & 1.6398 \\ \text { (regular drivers) }\end{cases}
$$

### 3.2.3 Disturbance-Effecting Spacing Variable

The author's findings from vehicle trajectory data [9] indicate that $90 \%$ of observed discretionary lane changes happen before the lane-changers' speeds are actually affected by the speed disturbance from the front vehicles. In other words, as both an adjacent lane is one of route-lane and provides acceptable gap condition, a driver who is potentially trapped in the car-following speed disturbance tends to switch onto an adjacent lane before reducing its speed. Such a probability is $90 \%$ on the observation basis. In summary, a driver tends to consider a discretionary lane change only when the following two conditions exist: 1) The driver perceives that the speed of the head vehicle is less than his or her desired speed; 2) The driver perceives that he could remain or increase speed by changing to another lane. Refereeing to Figure 2 and Figure 3 (7), the driver will commence a lane change at $\mathrm{t} \leq \delta_{\mathrm{n}+1}$ before reaching the minimum safety spacing. The spacing at this moment is called the disturbance-effecting spacing, or EfS. Figure 5 shows the distribution of spacing and headway between pairs of vehicles when a lane change is executed as a response to car-following speed disturbance. Equations for estimating minimum spacing and headway are derived from the statistical analysis as shown in Figure 5. They are described as follows. Those minimum spacing and headway models are also recommended as thresholds control for car-following scenarios, as summarized in Table 1.

Minimum disturbance-effecting spacing vs. speed $V$ :

$$
S_{\min }=\left\{\begin{array}{lc}
16.484 e^{0.0197 V} & \text { (Regular Driver) }  \tag{20}\\
0.3336 V^{-1.6398 V} & \text { (Conservative Driver) }
\end{array}\right.
$$

Minimum headway $H_{\text {min }}$ vs. speed $V$ (regular Driver):

$$
H_{\text {min }}=\left\{\begin{array}{lc}
0.0006 V^{2}-0.049 \mathrm{~V}+1.83 & (\mathrm{~V} \geq 30 \mathrm{ft} / \mathrm{s})  \tag{21}\\
3.0668 e^{-0.0409 V} & (\mathrm{~V}<30 \mathrm{ft} / \mathrm{s})
\end{array}\right.
$$

### 3.2.4 Acceleration, Speed and Lane-Changing Duration

A lane changer may accelerate or decelerate when manipulating a lane-changing maneuver so as to keep a safe headway to the lead vehicle or to the lag. Acceleration varies with the individual speeds of the lane changer and the lead vehicle, as well as the differences between them. Table 2 gives observed maximum acceleration or deceleration values for each $10-\mathrm{mph}$ increment for determining thresholds of acceleration as simulating a lane-changing maneuver.

The authors' observations also indicate that lane-changing duration has little coherent relationship with speeds of over $10 \mathrm{ft} / \mathrm{sec}(3 \mathrm{~m} / \mathrm{s})$. For discretionary lane change, the average duration of lane changing ranges from 2.33 seconds to 2.51 seconds with standard deviations from 0.56 to 0.93 seconds. Therefore, $2.3 \sim 2.5$ seconds may be recommended as a general description of the lane-changing duration.

## 4. CONCLUSIONS

The contributions that have been made through the studies presented in this paper include the following.

- New findings from observations and vehicle trajectory data are instilled into the current knowledge of understanding the car-following behaviors in response to a speed disturbance and the process of a speed disturbance absorption and/or propagation.
- Heuristic models for State-Control Action-Expected Chains are developed to integrate carfollowing and lane-changing behaviors in response to speed disturbance in a car-following mode
- Threshold values of major perceptual variables are derived from vehicle trajectory data, namely, minimum disturbance-effecting/ending spacing (i.e., EfS and EnS), maximum acceleration/deceleration at a control action time, and lane-changing duration.
- Models for probability of lane-changing response to speed disturbance via speed advantage variable, and models for acceptable lane-changing decision-making conditions at the adjacent lanes, are developed based analysis of observed vehicle trajectory data.
- As a result of above findings, a new heuristic driving modeling framework is proposed based on Boer's Perceptual-Satisficing Theory-Based framework. It provides a simplified, easily understandable architecture for modeling travel and driving behaviors.

The work presented here sheds some light on the effect that a single speed disturbance of a front vehicle has on the spacing and speed of the following vehicle in a car-following mode at the microscopic level. This establishes the foundation for further research on multiple speed disturbance absorption and its impact on traffic stability at the macroscopic analysis level.

## REFERENCES

1. Richard W. Rothery. Car Following Models. Revised Monograph on Traffic Flow Theory. October 2000
2. Brackstone M. and M. McDonald. What Is the Answer? And Come to That, What Are the Questions? Transportation Research Part F, Vol. 2, No. 4 (1999), pp. 221-224.
3. Brackstone M. and M. McDonald. Car-Following: A Historical Review. Transportation Research Part F, Vol. 2, No. 4 (1999), pp. 181-196.
4. Mehmood?
5. Edwin R. Boer and Marika Hoedemaeker. Modeling Driving Behavior with Different Degrees of Automation: A Hierarchical Decision Framework of Interacting Mental Models. $17^{\text {th }}$ European Annual Conference on Human Decision Making and Manual Control, December 14-16, 1998. Valenciennes, France.
6. Van Winsum, W. The Human Element in Car Following Models. Transportation Research Part F, Vol. 2, No. 4 (1999), pp. 207-211.
7. Boer, E. R. Car Following from the Driver's Perspective. Transportation Research Part F, Vol. 2, No. 4 (1999), pp. 201-206
8. Tao, Ruihua. Speed Disturbance, Absorption and Traffic Stability. Ph.D. Dissertation. Michigan State University. 2003
9. Wei, Heng. Observed Lane-Choice and Lane-Changing Behaviors on an Urban Street Network using Video-Capture-Based Approach and Suggested Structures of Their Models. Ph.D. Dissertation. University of Kansas. 1999
10. Gartner, N., G. C. Messer, and A. K. Rathi. Traffic Flow Theory. Chapter 4 of R. W. Rothery, Car Following Models. Transportation Research Board Special Report 165, Transportation Research Board, 2000, Page 4-6.
11. Todosiev, E. P.. The Action Point Model of the Driver Vehicle System. Report No. 202A-3. Ohio State University, Engineering Experiment Station, Columbus, Ohio, 1963
12. Rothery, R. W., R. Silver, R. Herman and C. Torner(1968), Analysis of Experiments on Single Lane Bus Flow. Operation Research 12, 1968, pp 913
13. Tomer Toledo, H. N. Houtsopoulos, and Ben-Akiva, M.E. "Modeling Integrated Lane-changing Behavior." In preprints of $82^{\text {nd }}$ Transportation Research Board Annual Meeting, Washington D.C., January 12-16, 2003
14. Yunlong Zhang, Larry E. Owen, and James E. Clark. A Multi-Regime Approach for Microscopic Traffic Simulation. A paper presented at Transportation Research Board $77^{\text {th }}$ Annual Meeting in Washington, D.C. January 11-15, 1998.
15. Gipps, P.G., A Model for the Structure of Lane Changing Decisions. Transportation Research, 20B, pp. 403-414
16. Heng Wei, Eric Meyer, Joe Lee, and Chuen Feng. Characterizing and Modeling Observed LaneChanging Behavior for Lane-Vehicle-Based Microscopic Simulation on an Urban Street Network. Transportation Research Record No. 1710, 2000. pp 104-113

## TABLE OF TABLES AND FIGURES

TABLE 1: Summary of Variables in Minimum Spacing Models for All Study Scenarios and Cases ....... 13
TABLE 2: Observed Maximum Accelerations for 10-mph Increments in Urban Streets ......................... 14
FIGURE 1: Heuristic Driver Modeling Framework ............................................................................... 15
FIGURE 2: State-Control Action-Expected Chains Responding to Speed Disturbance ........................... 16
FIGURE 3: Illustration of Speed Disturbance and Car-Following Scenarios .......................................... 17
FIGURE 3 (cont.): Illustration of Speed Disturbance and Car-Following Scenarios................................ 18
FIGURE 3 (cont.): Illustration of Speed Disturbance and Car-Following Scenarios................................ 19
FIGURE 4: Process of a Speed Disturbance Propagated through a Line of Vehicles ............................... 20
FIGURE 5: Observed Spacing and Headway between Pairs of Vehicles at Control Action Times*......... 21
FIGURE 6: Acceptable Gap Conditions in the Adjacent Lane for Lang-Changing Decision-Making...... 22

Nひ

Tao, Wei and Wang

TABLE 1: Summary of Variables in Minimum Spacing Models for All Study Scenarios and Cases

| Scenarios and Cases |  | Disturbance-Effecting Spacing (EfS), ft |  | Disturbance-Ending Spacing (DnS), ft |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Minim EfS | Control Action Time | Minimum EnS | Control Action Time |
| CarFollowing $\underline{\text { Scenario a) }}$ | All cases | N/A | N/A | $S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2} \times\left[1-\frac{\ddot{x}_{n}^{-}}{\ddot{x}_{n}^{+}}\right]$ | $t=t_{0}+T$ |
| Car- <br> Following Scenario b) | Case 1: $\delta<t_{1}$ and $\delta^{\prime}>t_{1}$ | $\left\{\begin{array}{r} 0.3336 \dot{x}_{n+1} 1.6398 \\ 16.484 e^{0.0197 \dot{x}_{n+1}} \end{array} \text { (conservative) }\right. \text { (regular) }$ | $t=\delta$ | $S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right) \times\left(\delta^{\prime}-t_{0}\right)-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\delta\right) \times\left(t_{1}-\delta\right)$ | $t=\delta^{\prime}$ |
|  | Case 2: $\delta<t_{1}$ and $\delta^{\prime}<t_{1}$ |  | $t=\delta$ | $S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(\delta^{\prime}-t_{0}\right)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\delta\right)^{2}$ | $t=\delta^{\prime}$ |
|  | Case 3: $\begin{gathered} \delta>t_{1} \text { and } \\ \delta^{\prime}>t_{1} \end{gathered}$ |  | $t=\delta$ | $S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(\delta-t_{1}\right)^{2}+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right) \times\left(\delta-t_{1}\right)-\frac{1}{2} \ddot{x}_{n+1} \times(\delta-\delta)^{2}$ | $t=\delta^{\prime}$ |
| Car- <br> Following Scenario c) | Case 1: $\delta^{\prime}>t_{1}$ |  | $t=\delta$ | $\begin{gathered} S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right) \times\left(\delta^{\prime}-t_{1}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(\delta-t_{1}\right)^{2} \\ -\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\tau\right)^{2} \end{gathered}$ | $t=\delta^{\prime}$ |
|  | Case 2: $\delta^{\prime}<t_{1}$ |  | $t=\delta$ | $S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(\delta^{\prime}-t_{0}\right)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\tau\right)^{2}$ | $t=\delta^{\prime}$ |
| LaneChanging Scenario | Case 1: $\delta<t_{1}$ | $\begin{aligned} & \left\{\begin{array}{c} 0.3336 \dot{x}_{n+1}{ }^{1.6398} \text { (conservative) } \\ 16.484 e^{0.0197 \dot{x}_{n+1}} \\ \text { (regular) } \end{array}\right. \\ & t_{l}=\text { duration of lane change } \end{aligned}$ | $t=\delta$ | $S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(\delta^{\prime}-t_{0}\right)^{2}-\frac{1}{2} \ddot{x}_{n+1}^{-} \times\left(\delta^{\prime}-\delta\right)^{2}$ | $t=\delta+t_{l}$ |
|  | Case 2: $\delta>t_{1}$ |  | $t=\delta$ | $S\left(t_{0}\right)+\frac{1}{2} \ddot{x}_{n}^{+} \times\left(\delta-t_{1}\right)^{2}+\frac{1}{2} \ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right)^{2}+\ddot{x}_{n}^{-} \times\left(t_{1}-t_{0}\right) \times\left(\delta-t_{1}\right)-\frac{1}{2} \ddot{x}_{n+1} \times(\delta-\delta)^{2}$ | $t=\delta+t_{l}$ |

Notes: 1) The spacing when the follower begins response to speed disturbance is called "Disturbance-Effecting Spacing" (EfS)
2) The spacing when the follower begins speed recovery from speed disturbance is called "Disturbance-Ending Spacing" (EnS)

TABLE 2: Observed Maximum Accelerations for 10-mph Increments in Urban Streets

| Type | Typical Maximum Acceleration Rate on Level Road (ft/sec$\left.{ }^{2}\right)$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Starting | $0-15 \mathrm{ft} / \mathrm{s}$ | $15-29 \mathrm{ft} / \mathrm{s}$ | $29-44 \mathrm{ft} / \mathrm{s}$ | $44-59 \mathrm{ft} / \mathrm{s}$ |  |
| $(0-10 \mathrm{mph})$ | $59-73 \mathrm{ft} / \mathrm{s}$ <br> $(10-20 \mathrm{mph})$ | $(20-30 \mathrm{mph})$ | $(30-40 \mathrm{mph})$ | $(40-50 \mathrm{mph})$ |  |
| Speed |  |  |  |  |  |

Data Source: vehicle trajectory data observed in Kansas City, Missouri, 1998

FIGURE 1: Heuristic Driver Modeling Framework


FIGURE 2: State-Control Action-Expected Chains Responding to Speed Disturbance


FIGURE 3: Illustration of Speed Disturbance and Car-Following Scenarios
(1) A Recovered Speed Disturbance:


Where, $\quad \dot{x}_{n}\left(t_{0}\right)=$ speed of vehicle n at time $t_{0}$,
$\dot{x}_{n}\left(t_{1}\right)=$ speed of vehicle n at time $t_{1}$,
$T \quad=$ time period of a speed disturbance development and recovery,

## (2) Car-Following Scenario a):


(3) Car-Following Scenario b):

## Speed $\dot{x}$



FIGURE 3 (cont.): Illustration of Speed Disturbance and Car-Following Scenarios

## (4) Car-Following Scenario c):



Where, $\dot{x}_{n}(t)=$ speed of the lead vehicle n at time $t$,
$\dot{x}_{n+1}(t)=$ speed of the following vehicle at time $t$,
$\dot{x}_{n+1}\left(t_{0}\right)=$ speed of the following vehicle at time $t_{1}$,
$\tau \quad=$ perception-reaction time of the following driver,
$\delta_{n+1}=$ time point before which the following vehicle travels at its original speed, and
$\delta_{n+1}^{\prime}=$ time point at which the following vehicle completes it speed reduction.
(5) Possible Cases in Scenario b):


FIGURE 3 (cont.): Illustration of Speed Disturbance and Car-Following Scenarios
(6) Possible Cases in Scenario c):


1: Case 1
2: Case 2

Where, $\delta=$ time point before that the following vehicle travels at its original speed, and $\delta^{\prime} \quad=$ time point at which the following vehicle completes it speed reduction.

## (7) Lane-Changing Scenario:




Where, $\delta=$ time point before that the following vehicle begins lane changing, and $\delta^{\prime} \quad=$ time point at which the following vehicle completes it speed reductio

FIGURE 4: Process of a Speed Disturbance Propagated through a Line of Vehicles


Speed disturbance propagation Speed $\dot{x}$


Where, $\quad S_{n+1}\left(t_{0}\right)=$ spacing between vehicles $\mathrm{n}+1$ and $\mathrm{n}+2$ at time $t_{0}$,
$S_{n}\left(t_{0}\right)=$ spacing between vehicles n and $\mathrm{n}+1$ at time $t_{0}$,
$\delta \quad=$ time point before that the following vehicle travels at its original speed, and
$\delta^{\prime} \quad=$ time point at which the following vehicle completes it speed reduction.

FIGURE 5: Observed Spacing and Headway between Pairs of Vehicles at Control Action Times*


Observed Headway of Target \& Ahead Vehicles Vs. the Target Vehicle's Speed


- Observed Headway of Target and Ahead Vehicles as A Lane Chance Begins
-     -         - ' Expon. (Minmum Headway Line)
-     -         - . Poly. (Minmum Headway Line)

[^0]FIGURE 6: Acceptable Gap Conditions in the Adjacent Lane for Lang-Changing Decision-Making



[^0]:    * Control Action Time: $\delta$ as the follower begins reducing speed due to the speed disturbance by the front vehicle.

