# Cosmic Web Reconstruction through Density Ridges

Yen-Chi Chen

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February 12, 2015

# Outline

- Introduction to Cosmic Web
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- Filament Coverage and Uncertainty Measures
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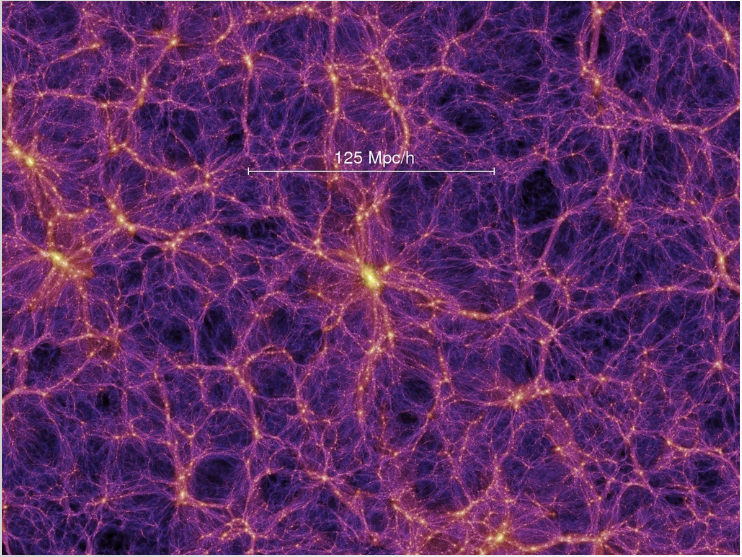


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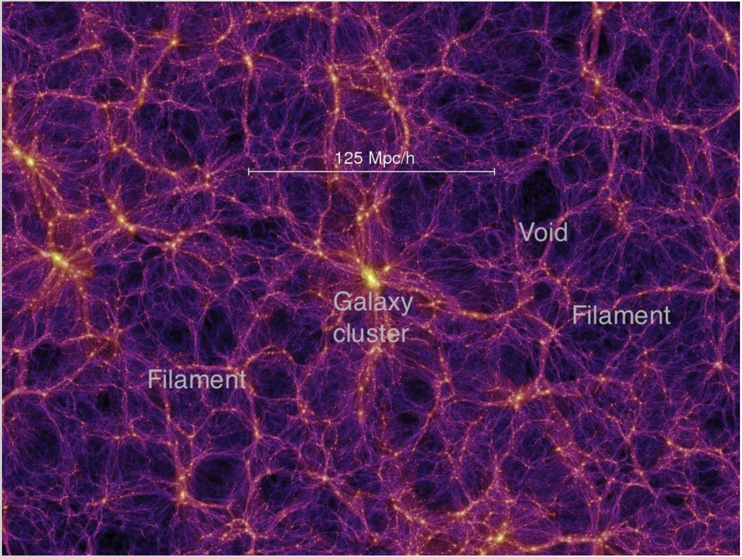


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# A Glance at our Universe

(Loading)

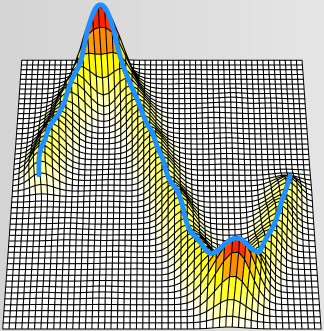
# Statistical Model for Filaments: Density Ridges

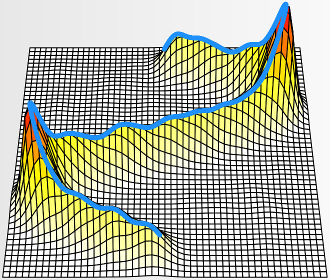
Formally, we define a filament to be a **ridge** of the density.

# Example: Ridges in Mountians


Credit: Google

# Example: Ridges in Smooth Functions





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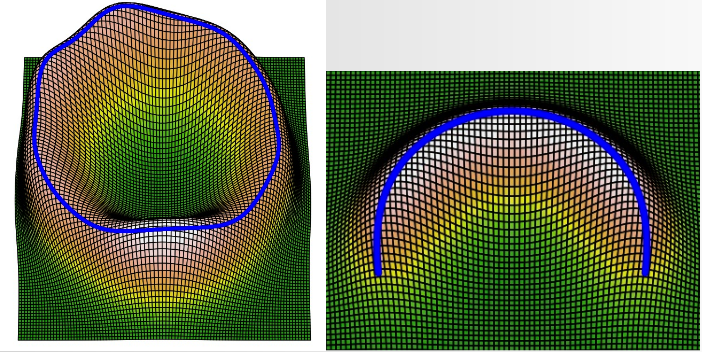
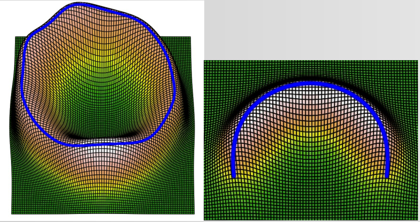


# Ridges: Local Modes in Subspace



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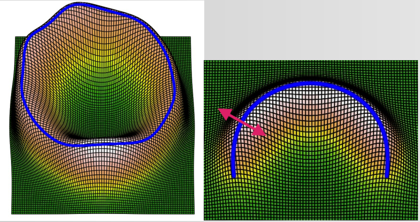

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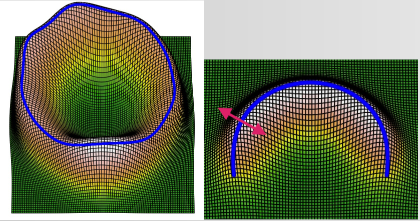

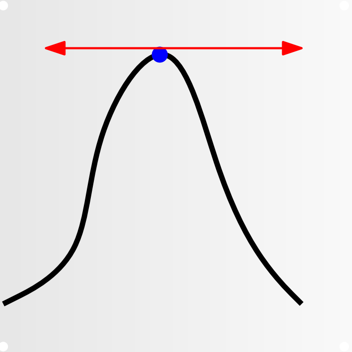

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- In practice, we estimate $p$ by the kernel density estimator $\widehat{p}_n$.

# Algorithm

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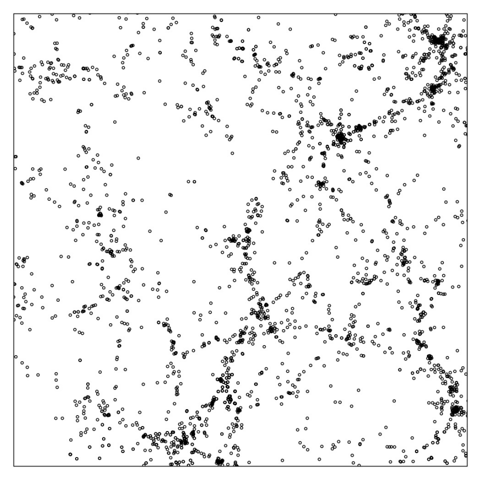

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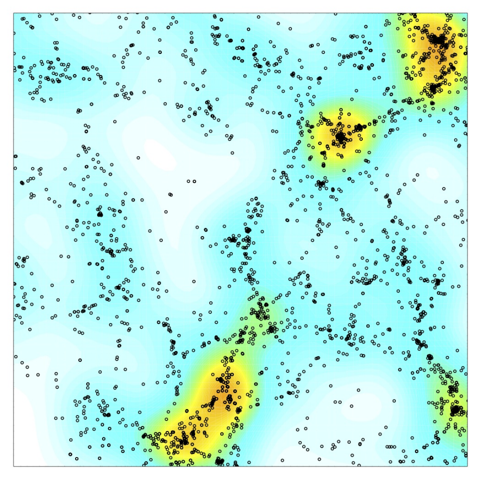

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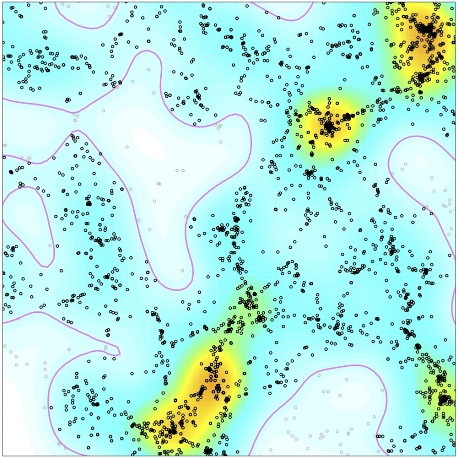

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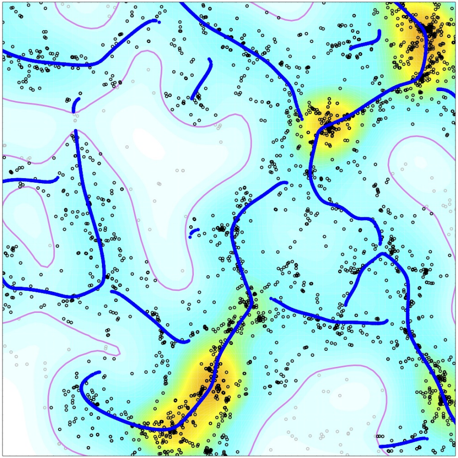

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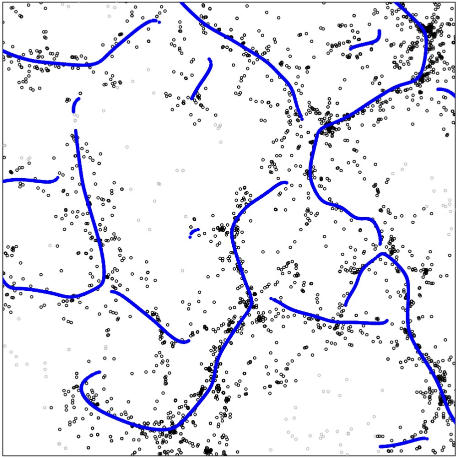

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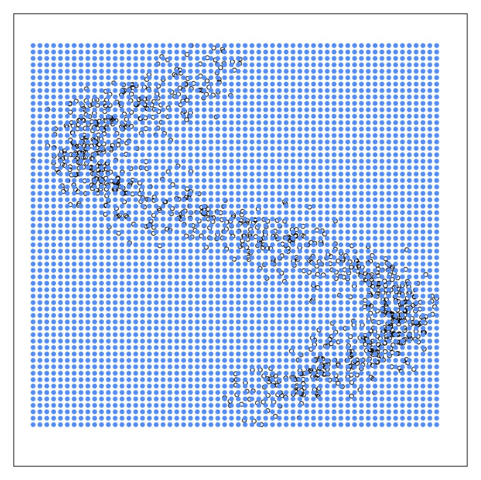

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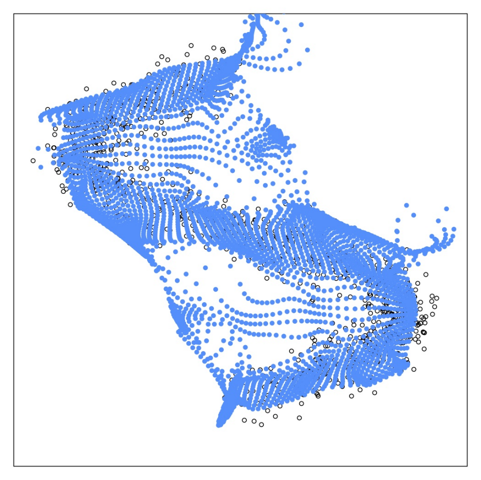

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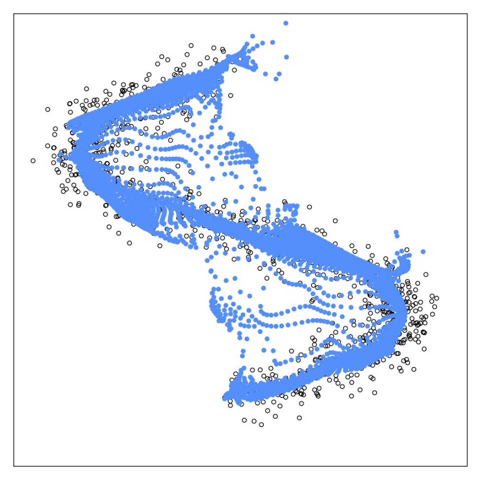

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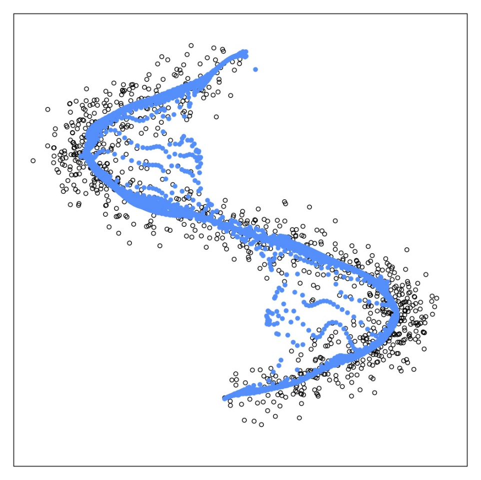

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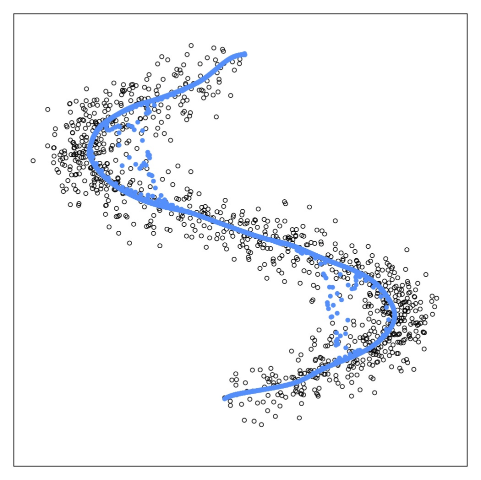

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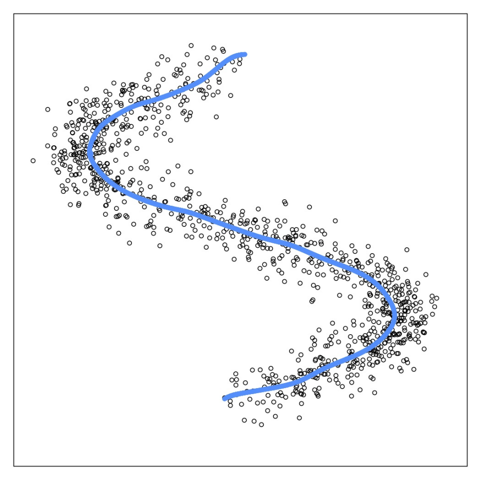
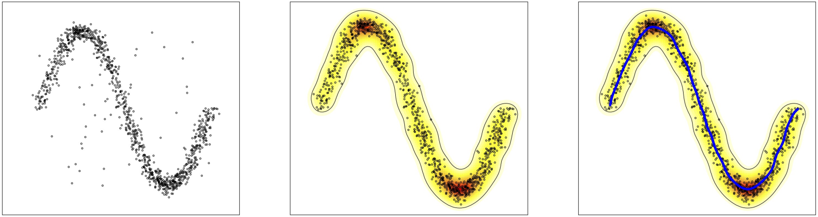


# Summary for the Algorithm



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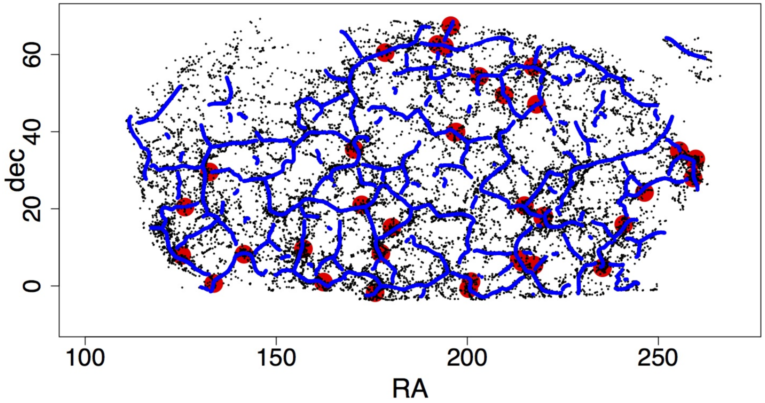

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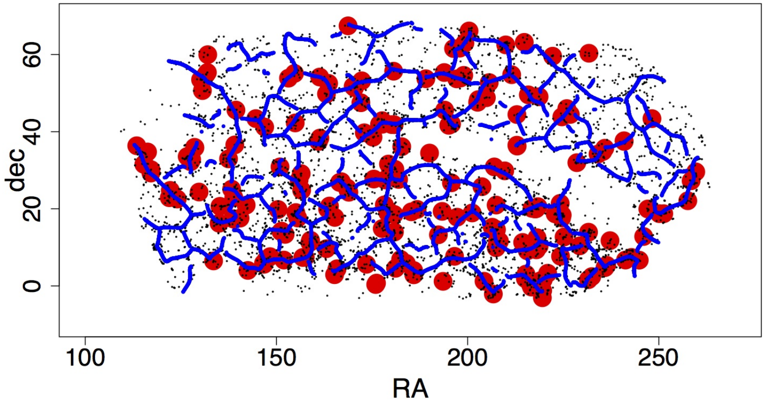

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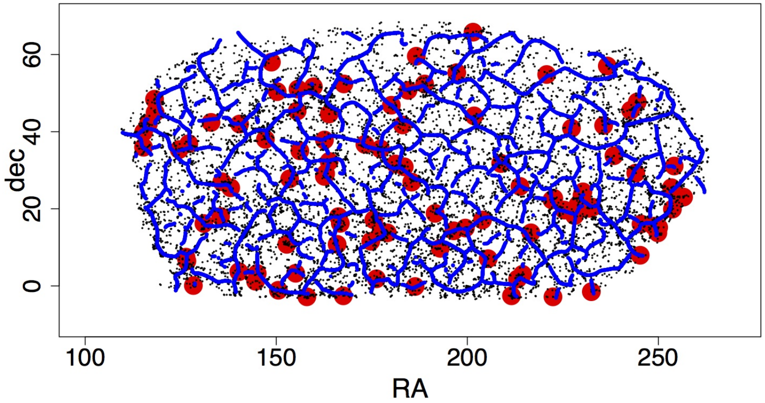

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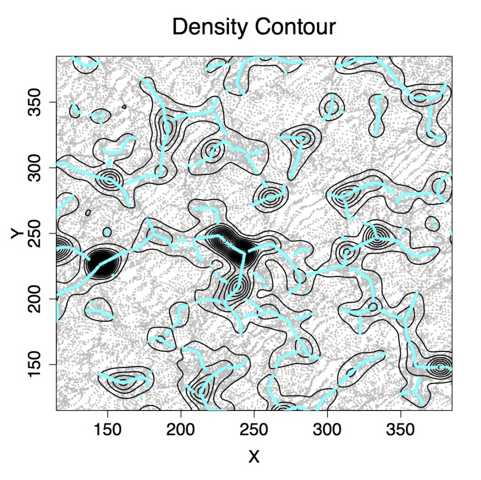
- We define ‘true’ filaments as applying our method to ‘all’ galaxies in the simulation.
- We subsample part of the galaxies from the simulation.

# Simulation: Consistency



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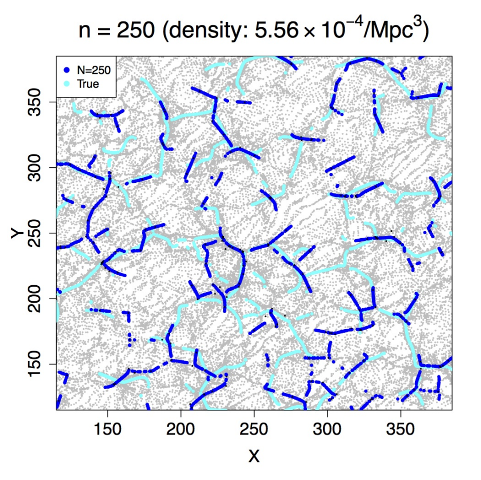

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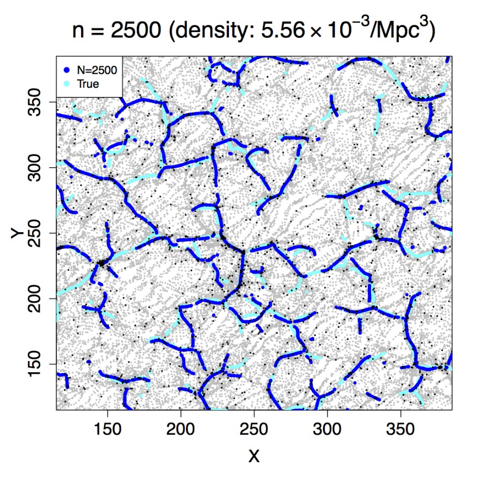

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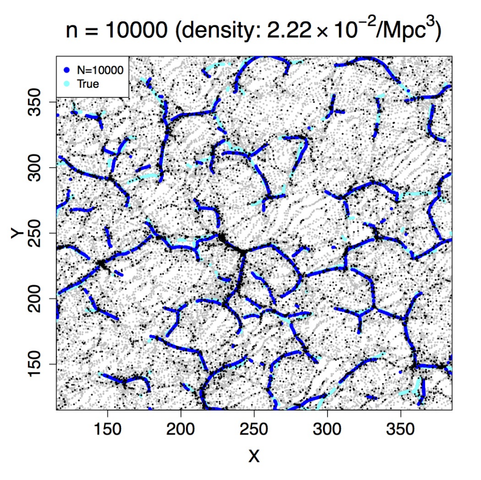

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- *Filament coverage* provides a simple way to attain this.
- **True positive coverage**:

$$TP(r) = \frac{\text{length}(R \cap \widehat{R}_n \oplus r)}{\text{length}(R)}.$$

- **False positive coverage**:

$$FP(r) = 1 - \frac{\text{length}(\widehat{R}_n \cap R \oplus r)}{\text{length}(\widehat{R}_n)}.$$

- $R$ and $\widehat{R}_n$ are the 'true' filaments and estimated filaments.

# Illustration: Filament Coverage

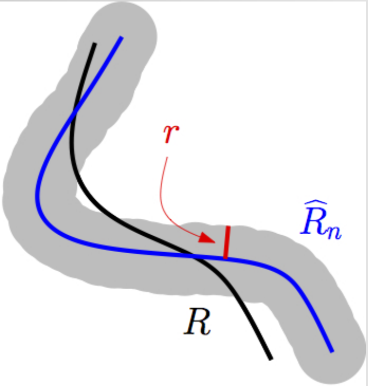


Figure: TP(r)

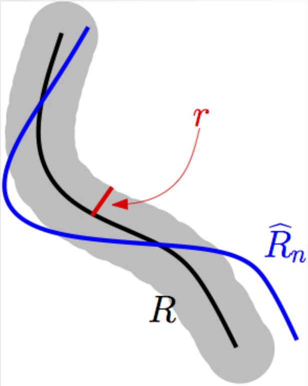


Figure: 1 - FP(r)

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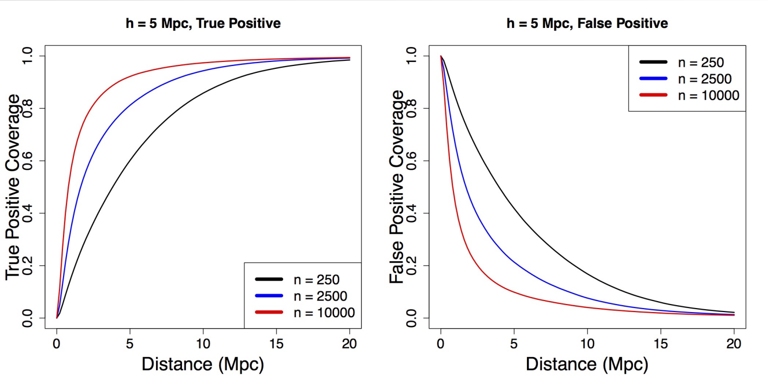


# The Need for Uncertainty Measure

- Filament coverage gives a (global) evaluation for filaments.
- We have no idea about the local uncertainty along filaments.
- Moreover, filament coverage requires the knowledge of truth.

# Uncertianty Measures

Let $R$ and $\widehat{R}_n$ be the true filaments and the estimated filaments. For each $x \in R$, we define the **(local) uncertainty measure** as

$$\rho_n^2(x) = \mathbb{E}\left(d^2(x, \widehat{R}_n)\right),$$

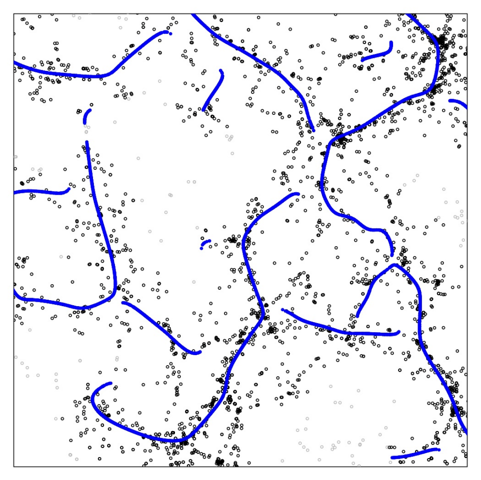
where $d(x, A)$ is the projection distance from point $x$ to a set $A$.
*Remark*:

- This is analogous to the mean square error.

# Estimating Uncertainty Measures

We apply the local uncertainty measure to our estimated filaments and use the *bootstrap* to evaluate the errors.

# Real Data Evaluation



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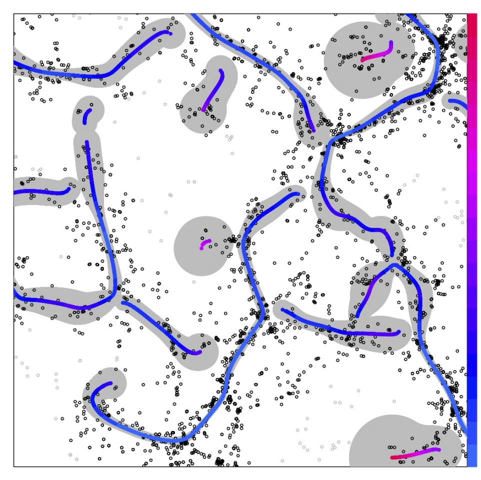

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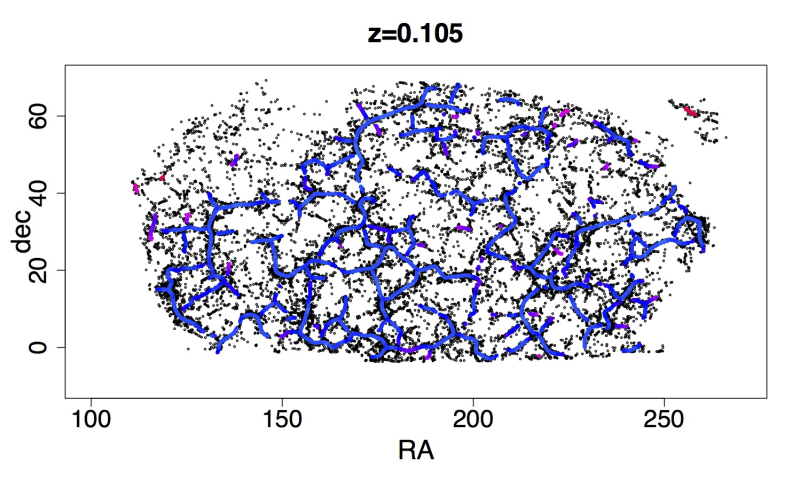

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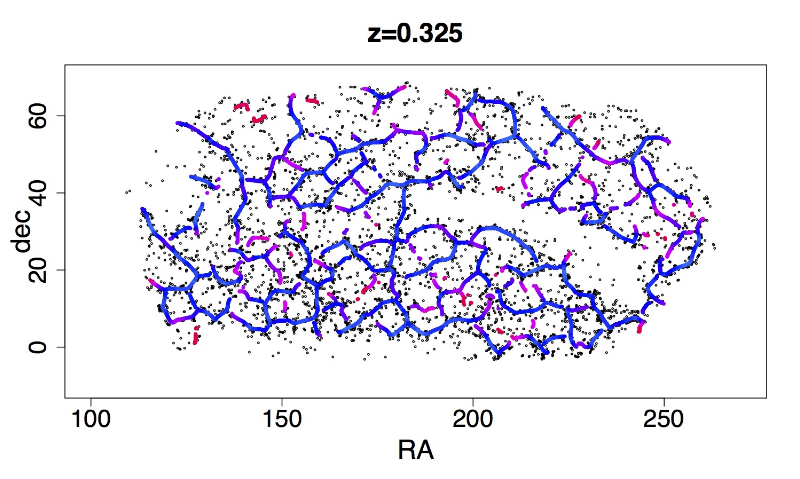

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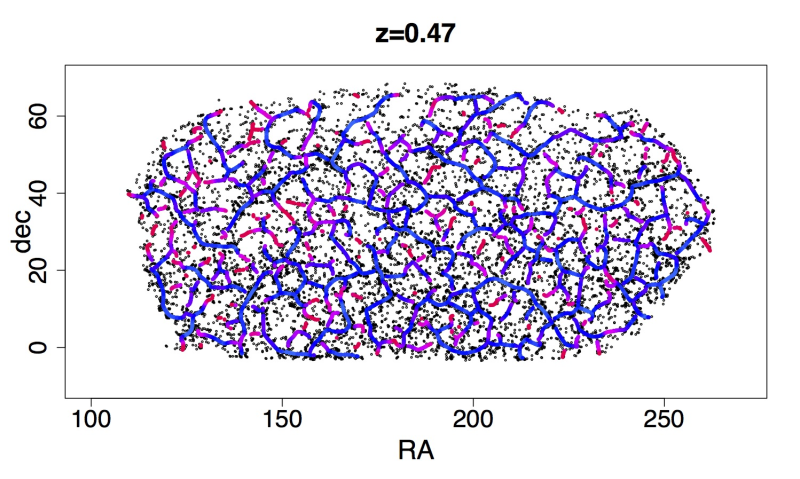

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- We analyze three datasets (at different ranges of redshifts).

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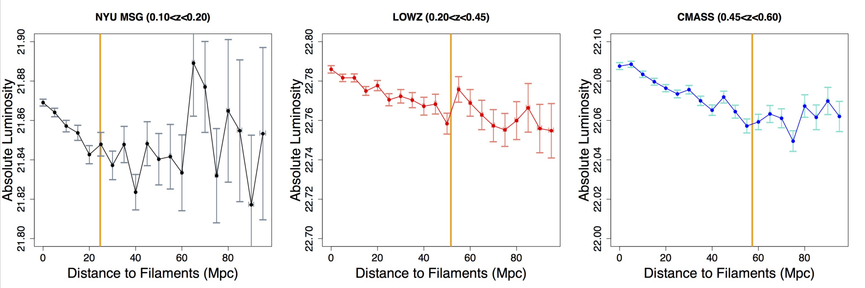

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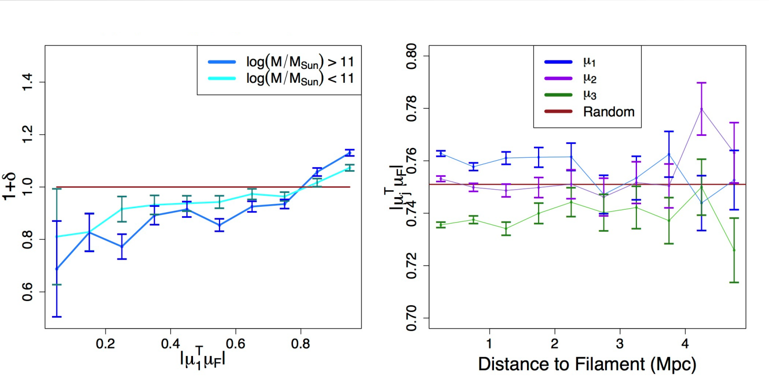

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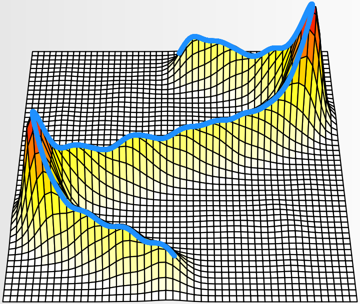

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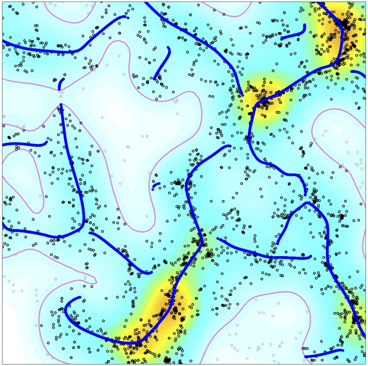

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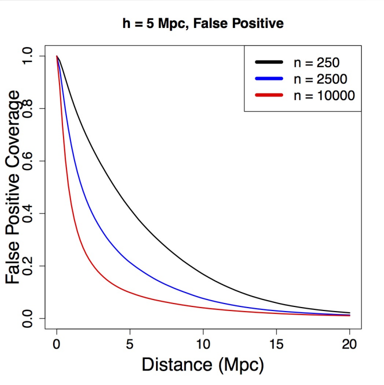

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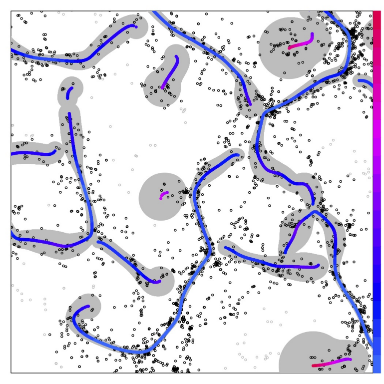

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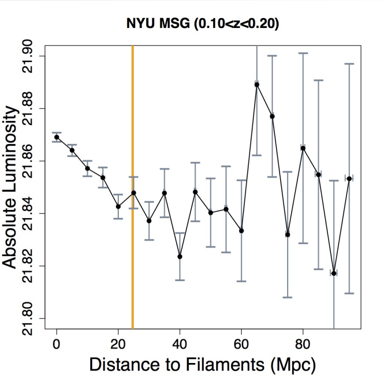
3. Consistency: filament coverage.
4. Errors: uncertainty measures.
5. Application: galaxy luminosity, alignment.



Thank you!

# reference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2. Chen, Yen-Chi, Christopher R. Genovese, and Larry Wasserman. "Asymptotic theory for density ridges." arXiv preprint arXiv:1406.5663 (2014).
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