A simple model of Saturn's moon Enceladus to find internal temperature

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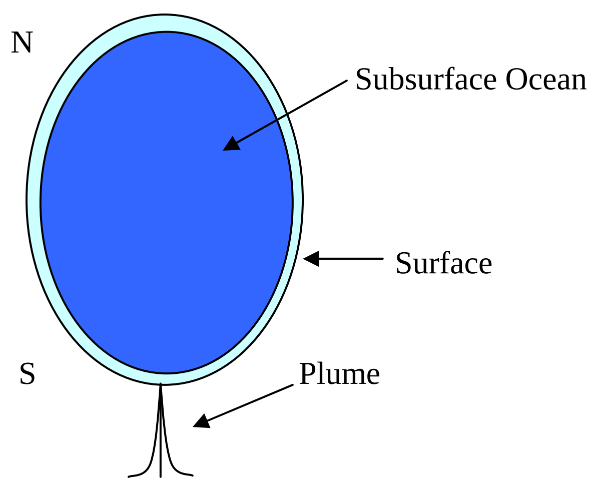
Enceladus is one of Saturn's 60 moons. It is very small and yet it is also very remarkable. A few years ago it was discovered that Enceladus is spewing water vapor from its surface; primarily from the South Pole. This water vapor also includes other organic materials and gases such as carbon dioxide, nitrogen, and methane[1,6].

The plumes are emerging from what scientists call "tiger stripes" because of the orientation of the cracks from which the plumes emerge[2]. Other models that have been presented include one similar to the one used here and another model discusses the decomposition of clathrates as a theory[3]. Other models describe the process of the water vapor getting to the surface as a result of tidal forces caused by the forces between Saturn and Enceladus[3]. These tidal forces cause the cracks where the plumes are exiting to open and close depending on where Enceladus is in its orbit[4]. The model used here does not include tidal forces and assumes that the subsurface ocean is infinitesimally close to the surface. What happens to the water vapor after it leaves the surface is also ignored. The water vapor could return to the surface to refreeze and add to the icy surface or escape the gravitational pull to fuel Saturn's e-ring[2,5]. These scenarios are discussed in other papers.

The goal was to find the internal temperature of the moon, assuming a liquid core model. The simplified model is drawn below:



The model used here follows that of a subsurface ocean very close to the surface. There are other models with more complex structures however, the hope of simplification is to see if the internal temperature that is calculated using this model and simple equations will be similar to those calculated in more complex models with more complex equations. To find the internal temperature, a simple calorimetry equation was used:

$$\Delta Q = mC_v \Delta T = \Delta E$$

Phase changes are neglected and it is assumed that the vapor remains as vapor as it moves from the interior of the moon out into space. For this problem two approaches or cases are used. By using two cases, each an extreme, we hope to find a reasonable range of temperatures that matches the temperatures found from other models. In the first case the final velocity is zero for the packet of water vapor, meaning that none of the packet of water vapor escapes the gravitational pull of Enceladus. The second case explores the other extreme. In Case 2 the final velocity of the water vapor packet is the escape velocity meaning that entire packet of vapor escapes the gravitational pull of the moon. In reality, the vapor is both leaving the surface and returning to it[5].

For Case 1, I found my equation for T_i following the steps below.

$\Delta Q = \Delta E$

$\Delta Q = mC_v\Delta T$

$\therefore \Delta E = mC_v\Delta T$

$\Delta E = mgh_f$

$\therefore \cancel{m}C_v\Delta T = \cancel{m}gh_f$

$C_v\Delta T = gh_f$

Solving for T_i:

$$T_i = T_f + \frac{gh}{C_v}$$

The equation for T_i for Case 2 is similar, but here the change in energy is equal to the final Kinetic energy.

$v_f = v_e$

$\Delta Q = \Delta E$

$\Delta Q = mC_v\Delta T$

$\Delta E = \frac{1}{2}mv_e^{2}$

$\cancel{m}C_v\Delta T = \frac{1}{2}\cancel{m}v_e^{2}$

$C_v\Delta T = \frac{1}{2}v_e^{2}$

Solving for T_i:

$$T_i = T_f + \frac{1}{2C_v}v_e^{2}$$

The majority of the Enceladus' surface is at a temperature of 72K. At the plumes, however, the temperature is at least 180K[1]. Therefore the temperature used for the final temperature in both cases is 180K. For Case 1 the internal temperature is approximately 203.2K. For Case 2, the internal temperature is 186.8K. Using this primitive model, the range of internal temperature for Enceladus is approximately between 186.8K and 203.2K.

This model, however, is lacking many factors. The other materials found in the plumes are ignored and each of these materials contributes a certain percentage of mass and different specific heat capacity to the calorimetry equation that is used. In order to modify these equations to take the other materials into account, one needs to know the approximate amount of each material present. With these figures one can easily substitute them into the equations. The total mass will still cancel but each specific heat capacity will have a specific percentage attached to it that relates to the percentage of the material found in the water vapor.

Other observations and calculations have also shown results for the internal temperature of Enceladus. C.C. Proco, *et al.* suggests that “even 200K can accelerate micrometer-sized particles to escape velocity” which shows that her prediction fits into the range of temperatures found here[5]. This paper however also suggests that temperatures of 273K could be localized and very near the surface due to frictional heating[5]. Linda Spilker in an early paper on Enceladus discusses that given the right circumstances, a liquid water source can be found in temperatures ranging from 170 to 273K[6]. A clathrates model assumed a temperature of 190K for an internal temperature[7]. Each of the papers mentioned above, using more complex methods, suggest an internal temperature close to or within the range that has been given here.

[1](2008, March, 26). Cassini Tastes Organic Material at Saturn's Geyser Moon. *NASA/JPL*, Retrieved August 18, 2008, from http://saturn.jpl.nasa.gov/news/press-release-details.cfm?newsID=827

[2](2008, August, 11). Cassini Begins Transmitting Data From Enceladus Flyby. *NASA/JPL*, Retrieved August 18, 2008, from http://saturn.jpl.nasa.gov/news/press-release-details.cfm?newsID=863

[3]F. Nimmo, J.R. Spencer, R.T. Pappalardo and M.E. Mullen, *Nature* , **447**, 289-291, 2007

[5]C. C. Porco, et al. "Cassini Observes the Active South Pole of Enceladus", *Science*, vol. 311, no. 5766, 2006, pp. 1393-1401

[6]Lakdawalla , Emily (2005, July, 30). Enceladus' South Polar Stripes Spew "Warm" Water. *The Planetary Society*, Retrieved August 20, 2008, from http://www.planetary.org/news/2005/0730_Enceladus_South_Polar_Stripes_Spew.html

[7]Susan W. Kieffer, Xinli Lu, Craig M. Bethke, John R. Spencer, Stephen Marshak, Alexandra Navrotsky. “A Clathrate Reservoir Hypothesis for Enceladus' South Polar Plume”, *Science*, vol. 314. no. 5806, pp. 1764 - 1766

[4]Steigerwald , Bill (2007, May, 16). Cracks on Enceladus Open and Close under Saturn's Pull. *NASA/JPL*, Retrieved August 20, 2008, from http://www.nasa.gov/mission_pages/cassini/media/enceladus_cracks.ht