1 [5 pts] Use the arc length formula to find the length of the curve y = 3x - 5 between x = 1 and x = 5.

$$\int_1^5 \sqrt{1+(f'(x))^2}\,dx = \int_1^5 \sqrt{1+(3)^2}\,dx \qquad (3x-5)' = 3$$

$$= \int_1^5 \sqrt{10}\,dx$$

$$= \left[\sqrt{10}\,x\right]_1^5 = \boxed{5\sqrt{10} - \sqrt{10}} = 4\sqrt{10}$$

2 [10 pts] Determine whether the improper integral $\int_0^\infty \frac{24x^2}{(4x^3+1)^{\frac{4}{3}}}dx$ converges and if it does evaluate it.

$$\int_0^\infty \frac{24x^2}{(4x^3+1)^{4/3}}dx = \lim_{t\to\infty} 24\int_0^t \frac{x^2}{(4x^3+1)^{4/3}}dx = \lim_{t\to\infty} 2\int_1^t \frac{1}{u^{4/3}}du$$

let $u = 4x^3+1$

$du = 12x^2\,dx$

$\frac{1}{12}du = x^2\,dx$

$$= \lim_{t\to\infty} 2\int_1^t u^{-\frac{4}{3}}du$$

$$= \lim_{t\to\infty} 2\left[-3u^{-\frac{1}{3}}\right]_1^t$$

$$= \lim_{t\to t} 2\left[\frac{-3}{\sqrt[3]{t}} + \frac{3}{\sqrt[3]{1}}\right]$$

$$\lim_{t\to\infty}\frac{-3}{\sqrt[3]{t}} = \frac{-3}{\infty} = 0 \qquad = 2[0+3] = \boxed{6}$$

Yes Converges

3. [10 pts] Suppose that at time t = 10 seconds an object is traveling at 30.0 meters per second. Its acceleration *a(t)* is measured at two-second intervals until time t=20, with the following results (the units of acceleration are meters per second^2):

t	10	12	14	16	18	20
a(t)	2.3	2.3	2.5	2.6	2.6	2.7

Use the trapezoidal rule to estimate the velocity of the object at time *t* = 20

$\Delta x = 2$ ✓

$\frac{\Delta x}{2} = 1$

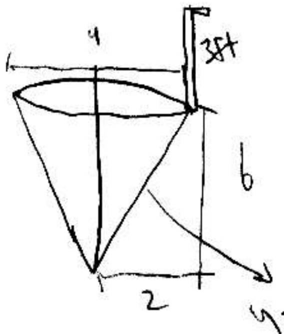
$[f(10) + 2f(12) + 2f(14) + 2f(16) + 2f(18) + f(20)]$

$= [2.3 + 4.6 + 5 + 5.2 + 5.2 + 2.7] = 25$

$25 + 30 = \boxed{55 \frac{m}{s}}$

ok

4 [10 pts] A conical tank 6 feet high and 4 feet in diameter at the top is filled with oil that weighs 60 lb/ft^3. The oil is being pumped out through a spout that rises 3 feet above the top of the tank. How much work is needed to pump out all the oil?



$60\ \text{lb}/\text{ft}^3$

F × dist mu = W

F = m·a

area = πr^2

$r = \frac{y}{3}$

$h = 9 - y$

$y = 3x$

$x = \frac{y}{3}$

$F = 60 \cdot \pi \left(\frac{y}{3}\right)^2 (9-y)$

$$60\pi \int_0^6 \frac{y^2}{9}(9-y)\,dy = 60\pi \int_0^6 y^2 - \frac{y^3}{9}\,dy = 60\pi\left[\frac{1}{3}y^3 - \frac{1}{36}y^4\right]_0^6$$

$$60\pi\left\{\left[\frac{1}{3}(6)^3 - \frac{1}{36}(6)^4\right] - 0\right\} = 60\pi[72 - 36] = \boxed{2160\pi \text{ ft-lbs}}$$

10

5 [10 pts] Evaluate $\int \frac{\sin(3t)\cos(3t)}{\cos^2(3t) - 3\cos(3t) + 2} dt$ $= \frac{1}{3}\int \frac{\sin u \cos u}{\cos^2 u - 3\cos u + 2} du$

10

let $u = 3t$

$du = 3dt$

$\frac{1}{3} du = dt$

let $v = \cos u$

$dv = -\sin u\, du$

$-dv = \sin u\, du$

$= -\frac{1}{3}\int \frac{v}{v^2 - 3v + 2} dv = -\frac{1}{3}\int \frac{v}{(v-1)(v-2)} dv$

$\frac{v}{(v-1)(v-2)} = \frac{A}{v-1} + \frac{B}{(v-2)}$

$\begin{array}{cc} 1 & 2 \end{array}$
$\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$
-3

$v = A(v-2) + B(v-1)$

let $v = 2$ $\quad$ $v = 1$

$2 = B$ $\quad$ $-1 = A$

$-\frac{1}{3}\int \frac{2}{(v-2)} - \frac{1}{(v-1)} dv$

$= -\frac{1}{3}\left[2\ln|v-2| - \ln|v-1|\right] + C$

$= \boxed{-\frac{1}{3}\left[2\ln|\cos(3t)-2| - \ln|\cos(3t)-1|\right] + C}$

6 [15 pts] For each of the following definite integrals either evaluate the integral or convince me that it is not possible to do so.

a) $\int_{-1}^{1} \frac{1}{x} dx$ $= \int_{-1}^{0} \frac{1}{x} dx + \int_{0}^{1} \frac{1}{x} dx$ let $0 = t$

$= \lim_{t \to 0} \int_{-1}^{t} \frac{1}{x} dx + \lim_{t \to 0} \int_{t}^{1} \frac{1}{x} dx$

$= \lim_{t \to 0} [\ln|x|]_{-1}^{t} + \lim_{t \to 0} [\ln|x|]_{t}^{1}$

$= \lim_{t \to 0} [\ln t -$

$\shortparallel$

undefined because

$\ln x$ is not defined when $x \leq 0$

$\boxed{\text{divergent}}$

b) $\int_{0}^{1} \frac{7}{\sqrt{4-x^2}} dx$ $= \int \frac{7(2\cos\theta)}{\sqrt{4-(2\sin\theta)^2}} d\theta = 7\int \frac{2\cos\theta}{\sqrt{4(1-\sin^2\theta)}} d\theta = 7\int \frac{2\cos\theta}{2\cos\theta} d\theta$

let $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sin\theta = \frac{x}{2}$

$\theta = \sin^{-1}\frac{x}{2}$

$= 7\int d\theta$

$= 7\theta = \left[7\sin^{-1}\frac{x}{2}\right]_{0}^{1}$

$= \boxed{7\sin^{-1}\frac{1}{2}} = \boxed{\frac{7\pi}{6}}$

$\int u\,dv = uv - \int v\,du$

c) $\int_{1}^{2} x^3 \ln x\, dx$ $= (\ln x)\left(\frac{x^4}{4}\right) - \int_{1}^{2} \frac{x^4}{4x} dx$

$u = \ln x$ $du = \frac{1}{x}$

$dv = x^3$ $v = \frac{x^4}{4}$

$= (\ln x)\left(\frac{x^4}{4}\right) - \int_{1}^{2} \frac{1}{4} x^3$

$\shortparallel$

$\left[(\ln x)\left(\frac{x^4}{4}\right) - \left[\frac{1}{16}x^4\right]\right]_{1}^{2}$

$\left[(\ln 2)\left(\frac{(2)^4}{4}\right) - \frac{1}{16}(2)^4\right] - \left[\ln(1)\left(\frac{1}{4}\right) - \frac{1}{16}(1)^4\right]$

$[4\ln 2 - 1] - \left[0 - \frac{1}{16}\right] = \boxed{4\ln 2 - \frac{15}{16}}$