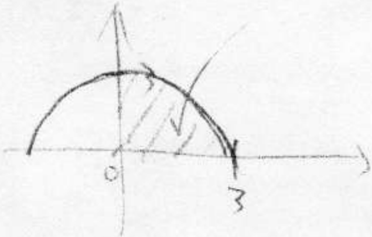
1. [5 pts] Evaluate $\int_0^3 \sqrt{9 - x^2}\ dx$



$3^2\pi$ $\quad 9\pi \times \frac{1}{4}$

Answer: $\frac{9}{4}\pi$

10

2. [10 pts] a) Little Johnny drops a rock (initial velocity zero) from a balcony 144 feet above the ground. Given that the acceleration due to gravity is -32 ft/sec^2, what is the vertical velocity of the rock at the moment it hits the ground?

$V_0 = 0$ $h_0 = 144ft$ $a(t) = -32\ ft/sec^2$

$s(t) = 0$

$v(t) = -32t$

$s(t) = -16t^2 + 144$

$0 = -16t^2 + 144$

$\sqrt{\frac{-144}{-16}} = \sqrt{t^2}$

$3sec = t \longrightarrow v(3) = -32(3)$

$\boxed{v(t) = -96\ ft/s}$ ✓

b) Johnny has a second rock that he decides to throw instead of dropping it. With what initial velocity would he have to throw it in order for it to get to the ground in a third the time it took the one he dropped?

$\frac{1}{3}(3) = 1\ sec$

$v(1) = ?$

$v(t) = -32t + V_0$

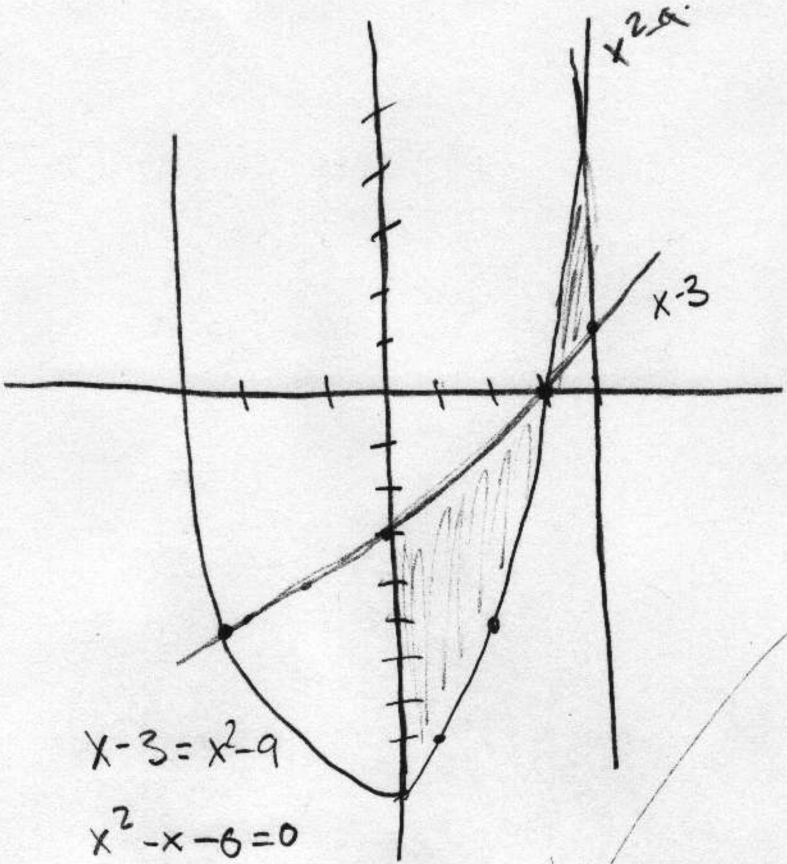
$s(t) = -16t^2 + V_0 + 144$

$0 = -16(1)^2 + V_0 + 144$

$-144 = -16 + V_0$

$\boxed{-128\ ft/sec} = V_0$ ✓

3 [10 pts] If $f(x) = x - 3$ and $g(x) = x^2 - 9$, find the area bounded by the graphs of f and g, the y-axis and the line $x = 4$.



$x-3 = x^2-9$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$3 \quad -2$

10

$$\int_0^3 x-3-(x^2-9)\,dx + \int_3^4 x^2-9-(x-3)\,dx$$

$$-x^2+9 \quad dx$$

$$\left(\frac{x^2}{2} - 3x - \frac{x^3}{3} + 9x\right)_0^3 + \left(\frac{x^3}{3} - 9x - \frac{x^2}{2} + 3x\right)_3^4$$

$$\left(\frac{9}{2} - 9 - 9 + 27\right) + \left(\left(\frac{64}{3} - 36 - 8 + 12\right) - \left(9 - 27 - \frac{9}{2} + 9\right)\right)$$

$$-10\tfrac{2}{3} - \left(-13\tfrac{1}{2}\right)$$

$$13\tfrac{1}{2} + 2\tfrac{5}{6} = \boxed{16\tfrac{1}{3}}$$

4 [5 pts] Two toy rocket-trains are shot off simultaneously from the same starting place on parallel tracks. The velocity of the first one after t minutes is A(t), and that of the second one is B(t). Both A(t) and B(t) are measured in meters per minute.

Consider the integral $\int_0^3 (A(t) - B(t))\,dt$

a) What quantity does this integral represent?

The distance covered by train B from 0 to 3 minutes subtracted from the distance covered by train A from 0 to 3 minutes. (The difference in distance between the two trains at 3 minutes)

b) What is the unit of measurement of the quantity?

meters

5

c) If the integral is negative, what can you deduce?

You can deduce that that train B traveled further than train A from 0 to 3 minutes.

5 [5 pts] Use the Midpoint Rule (taking y-values at the midpoints of the intervals) with n = 3 subdivisions to find the approximate value of

$$\int_0^6 x^2 + 5 \quad dx$$

$\Delta x = \frac{b-a}{n} = \frac{6-0}{3} = 2$ (subdivisions are 2 units wide), so the midpoints are at $x=1, x=3, x=5$

$\int x^2+5\,dx = \frac{1}{3}x^3+5x$

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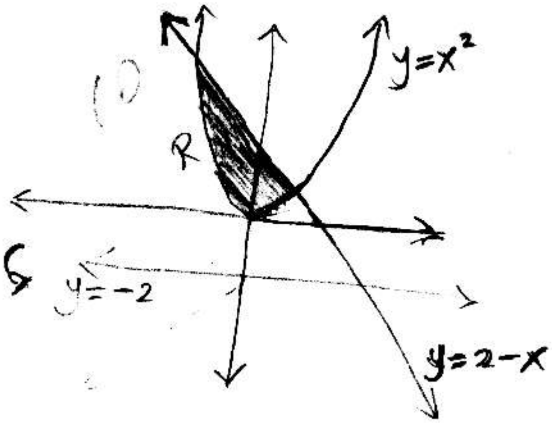
$$\int_0^6 x^2+5\,dx = \Delta x(f(\bar{x}_1) + \cdots f(\bar{x}_n))$$
$$= 2(f(1)+f(3)+f(5))$$
$$= 2(6+14+30)$$
$$= \boxed{100}$$

6 [10 pts] Let R be the region bounded by the curves $y = x^2$ and $y = 2 - x$. Find the volume of the solid obtained by rotating R around the line $y = -2$



$x^2 = 2 - x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \ \& \ 1$

$$\pi\int_{-2}^{1}(2 - x + 2)^2 dx - \pi\int_{-2}^{1}(x^2+2)^2 dx$$

$$= \pi\int_{-2}^{1}(4-x)^2 - (x^2+2)^2 dx = \pi\int_{-2}^{1}(16-8x+x^2) - (x^4+4x^2+4) dx$$

$$= \pi\int_{-2}^{1} 12 - 8x - 3x^2 - x^4 dx = \pi\left[12x - 4x^2 - x^3 - \frac{x^5}{5}\right]\Big|_{-2}^{1}$$

$$= \pi\left[(12 - 4 - 1 - 1/5) - (-24 - 16 + 8 + \frac{32}{5})\right] = \pi\left[(34/5) - (-\frac{128}{5})\right]$$

$$= \boxed{\pi\left(\frac{162}{5}\right)}$$

7 [15 pts] Find the following integrals

a) $$\int x e^{x^2} \sec^2(e^{x^2})\,dx$$

Let $u = x^2$. $du = 2x\,dx \Rightarrow dx = \frac{du}{2x}$

$\int x e^{x^2} \sec^2(e^{x^2})\,dx = \int x e^u \sec^2(e^u) \cdot \frac{du}{2x} = \frac{1}{2}\int e^u \sec^2(e^u)\,du$

Let $v = e^u$ $dv = e^u\,du$

$\frac{1}{2}\int e^u \sec^2(e^u)\,du = \frac{1}{2}\int \sec^2 v\,dv = \frac{1}{2}\tan v + C$

$= \frac{1}{2}\tan e^u + C = \frac{1}{2}\tan(e^{x^2}) + C$

b) $$\int (\sin t)^3\,dt$$ $= \int \sin^2 t \cdot \sin t\,dt = \int (1-\cos^2 t)\cdot \sin t\,dt$

Let $u = \cos t$. $du = -\sin t\,dt \Rightarrow \sin t\,dt = -du$

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$\int (1-\cos^2 t)\cdot \sin t\,dt = \int (1-u^2)\cdot -du = \int (-1+u^2)\,du$

$= -u + \frac{1}{3}u^3 + C = -\cos t + \frac{1}{3}\cos^3 t + C$

c) $$\int x\sqrt{4-x}\,dx$$

Let $u = 4-x$. $du = -dx$ $\Rightarrow dx = -du$ and $x = 4-u$

$\int x\sqrt{4-x}\,dx = \int (4-u)\sqrt{u}\cdot -du = -\int 4\sqrt{u} - u^{3/2}\,du$

$= -4\cdot\frac{2}{3}u^{3/2} + \frac{2}{5}u^{5/2} + C = \boxed{-\frac{8}{3}(4-x)^{3/2} + \frac{2}{5}(4-x)^{5/2} + C}$