HW Set 2 Solutions

Note: problems 1 and 6 are quite long, and therefore cannot come up on the Exam as such. But the many skills exercised in these problems will be needed for dealing with some related, less time consuming problems on the Exam. Therefore, please study the solutions with great care – it is supposed to be an additional learning experience.

Recall also that I do not necessarily append the units to all answers, if I stay within the standard metric system. But when I find it useful to append the unit, I do (for example to point out that the units of the absorption are $m^2$)

Problem 1

a) Construct the deCaus tuning scheme. Start with perfect major thirds F-A-C♯, then build three perfect fifths upward from each of these three notes. see Figure 1

b) determine the (main) beating rates of
   b1) the C Major chord
   There are no beats (all intervals are just, by construction). You can verify this explicitly by steps of b2)
   b2) the $B^b$ Major chord
   Assume the $B^b$ root at the $B^b_4$ (just above the A(440)).
   First determine the frequency of $D_5$ (i.e. the middle of the chord):

   $$f(D_5) = 440 \times 2^{5/12} = 587.3 \text{ Hz}$$

   Then the frequencies of the bottom and the top of the chord from Figure 1:

   $$F_4 : B^b_4 : D_5 : F_5 = 1 : 675/512 : 27/16 : 2$$
   therefore:

   $$f(B^b_4) = 587.3 \times 675/512 \times 16/27 = 458.8 \text{ Hz}$$

   $$f(F_5) = 587.3 \times 2 \times 16/27 = 696.1 \text{ Hz}$$

   and the beat rates are:

   $3^M : |458.8 \times 5 - 587.3 \times 4| = 55.2 \text{ Hz}$

   $3^m : |587.3 \times 6 - 696.1 \times 5| = 43.4 \text{ Hz}$

   $5^{th} : |458.8 \times 3 - 696.1 \times 2| = 15.8 \text{ Hz}$
c) You decide to improve the $B^b$ Major chord by moving the notes $B^b$ and $F$ to perfection. Which Major and minor chords will be affected, and by how much?

See Figure 2:

Columns B are the cents (relative to F) of the deCaus tuning from Figure 1.
Columns C and D are the defects $[\mathcal{f}(actual) - \mathcal{f}(just)]$ of the $3^M$ and the $5^{th}$ for the Major chord with root at column A.
Columns D and E are the defects $[\mathcal{f}(actual) - \mathcal{f}(just)]$ of the $5^{th}$ and $3^M$ for the minor chord with root at column A.

Note: the actual EXCEL calculations are very simple:

for the chords C major and C minor
$C_1 = B5 - B1 - 386.31$  $D_1 = B8 - B1 - 701.95$  $E_1 = B8 - B4 - 386.31$

for the chords $C^#$ Major and minor
$C_2 = B6 - B2 - 386.31$  $D_2 = B9 - B2 - 701.95$  $E_2 = B9 - B5 - 386.31$

etc for chords D, $E^b$ and so on

Then the column F is a copy of column B, with the cents of $B^b$ and $F$ changed to make the $B^b$ chord perfect.

And finally columns G,H,I show how the change affected all chords that contain the notes $B^b$ or $F$.

Note that for any Major or minor chord, in any tuning or temperament:

the defect($3^M$) + defect($3^m$) = defect($5^{th}$)

So the Figure 2 indeed gives the defects of all three intervals contained in any of our chords, Major or minor.

Note: consider only chords in their root position, i.e. not the inversions. The root position of the C major chord is C-E-G; the inversions are E-G-C and G-C-E. Which, if any, inversions are part of the Fourier series?

Answer: G-C-E is in the ratio 3:4:5 so it is a member of the Fourier series; E-G-C is not (for the same reason for which a minor chord, even when in root position, is not a member of the Fourier series.).

Problem 2

You are listening to your favored music at 90 phons at 60 Hz as well as at 3 kHz. When neighbors complain, you reduce the sound intensity by a factor of 1,000 (independent of frequency). Determine the resulting Intensity, sound intensity level, loudness level and loudness at the two frequencies, and compare with the original.

Reading from graph and using $L = 2^{(LL-40)/10}$, see Figure 3 for the results.
Problem 3

Compare two tones, with $f_1 = 120$ Hz, SIL1 = 75 dB and $f_2 = 4$ kHz, SIL2 = 70 dB. Which tone is louder, and by how much?

Reading from graph:
- tone 1 has 60 phons, 4 sones
- tone 2 has 72 phons, 8+ sones ($2^{(72-40)/10} = 9.2$ sones)

So tone 2 has lower intensity but sounds louder (to an average human).

Problem 4

Two incoherent sources (e.g. violinists) emit sound of same SIL at frequencies $f_1$ and $f_2$, respectively. What can you say about the resulting sound as compared to the two separate sounds, if

- a) $f_1 = 1000$ Hz , $f_2 = 1000$ Hz: add the intensities
- b) $f_1 = 1000$ Hz , $f_2 = 1500$ Hz: add the loudnesses
- c) $f_1 = 150$ Hz, $f_2 = 3500$ Hz: perceive two separate tones

What are the maximum and minimum values of SIL in case a) if those two violinists become coherent?

You get the maximum where the two waveforms are in phase:

$$A \rightarrow 2A, I \rightarrow 4I; SIL \rightarrow SIL + 6dB$$

Minimum where they are out of phase: $I \rightarrow 0$, SIL not defined

Problem 5

If one violin produces SIL of 75 dB a) what will you get from two violins (playing unison, incoherent)?

$$75 + 10 \log_{10} 2 = 78$$

b) How much from 10 violins?

$$75 + 10 \log_{10} 10 = 85$$

c) How many violins do you need to get to 95 dB?

$$10^{(95-75)/10} = 100$$
Problem 6

A hall has Width, Depth, Height = 22 x 35 x 12 m. Walls are painted concrete, floor is wood, ceiling is plywood paneling, air is 20°C 50% humidity. a) the source emitting acoustical power of 0.0001 Watt is at the floor in the middle of the 20 m wall; the listener is in the same position at the opposing wall (yes, this is a little funny for simplicity). Determine

a1) the time of the arrival of the direct sound and first and second reflections

\[ \text{v(sound)} = 331 + 20 \times 0.6 \sim 343 \text{ m/s} \]

Hint: now draw a simple sketch of the room, with the positions of the speaker and the listener, and recall Pythagoras!

\[ t_{\text{direct}} = \frac{35}{343} = 102 \text{ ms} \]

the reflection from the side wall:

\[ t_{\text{horiz.}} = \frac{2 \sqrt{(35/2)^2 + (22/2)^2}}{343} = 121 \text{ ms} \]

the reflection from the ceiling:

\[ t_{\text{vert.}} = \frac{2 \sqrt{(35/2)^2 + 12^2}}{343} = 124 \text{ ms} \]

So the first reflection comes at 121 ms, 19 seconds after the direct sound; the second reflection comes 3 ms later.

a2) the \( R_{T60} \) at 2 kHz and at 8 kHz. Does it matter much that you don't have the exact absorption coefficients at 8 kHz?

\[ V = 35 \times 22 \times 12 = 9240 \]

absorption:

walls 2*(35+22)*12*.09 + ceiling 35*22*.010 + floor 35*22*.06 + air 9240*.010 = 339 \( \text{ m}^2 \)

so \( R_{T60} = 0.161 \times 9240/339 = 4.4 \text{ s} \)

At 8kHz the air absorption becomes dominant (mV = 0.086*9240 = 795) so we can assume the same surface absorptions as tabulated for 2kHz (or 4 kHz if you want) and get \( R_{T60} = 1.4 \text{ s} \)

Now go back to 2 kHz for the rest of the problem.
a3) Determine the reverberant SIL and the critical distance \( r_c \) and plot the direct SIL vs. \( \log r \).

\[
I_{\text{reverb}} = 4 \times \left( \text{source power } W \right)/A = 4 \times .0001/339 = 1.2 \times 10^{-6}
\]

\[
SIL_{\text{reverb}} = 10 \log_{10} \frac{1.2 \times 10^6}{10^{-12}} = 60.8 \text{ dB}
\]

\[
r_c = \sqrt{\frac{Q_g A}{16\pi}} = \sqrt{\frac{4 \times 339}{16\pi}} = 5.2 \text{ m}
\]

(since the geometrical “projection” factor \( Q_g = 4 \) for a source at the boundary between floor and wall.)

The direct sound goes as \( I_{\text{direct}} = Q_g \times W/r^2 \) and therefore

\[
SIL_{\text{direct}} = 10 \log_{10} \frac{10 \left( \log(4 \times .0001/4\pi) - 2 \log r + 12 \right)}{10^{-12}} = 75 - 20 \log r
\]

For drawing the graph we will use the values of \( SIL_{\text{direct}} \) at \( r = 1 \text{ m} \) and \( r = 10 \text{ m} \), which are, from the above result, equal to 75 dB and 55 dB, respectively. We will also recall our construction of the logarithmic scale and we get Figure 4.

b) Something was put on the long (side) walls (floor to ceiling) and that reduced the \( RT_{60} \) to 2.0 seconds. What was the stuff they covered the long walls with?

The new absorption is \( A_{\text{new}} = .161V/RT_{\text{new}} = .161 \times 9240/2 = 744 \) and

\[
A_{\text{new}} - A_{\text{old}} = 744 - 339 = 405 = \text{surface}_{\text{longwalls}} \times (\alpha_{\text{new}} - \alpha_{\text{old}}) = 2 \times 35 \times 12 \times (\alpha_{\text{new}} - 0.09)
\]

so we get the final result \( \alpha_{\text{new}} = 405/(2 \times 35 \times 12) + .09 = 0.57 \).

The Table of materials in the Text reveals that they have covered the long walls with acoustical tiles.

c) And now 600 occupied chairs were put in, each occupied by a person. What is the resulting

\[\text{c1) } RT_{60}\]

The occupied chairs bring additional absorption of \( 600 \times .064 = 384 \) and this brings the \( RT_{60} \) to 1.3 s.

\[\text{c2) reverberant SIL and } r_c. \text{ Add these to plot under a3).}\]

\[
I_{\text{reverb}} = 4 \times W/A = 4 \times .0001/(744 + 384) = 3.5 \times 10^{-7}
\]

\[
SIL_{\text{reverb}} = 10 \log_{10} \frac{3.5 \times 10^{-7}}{10^{-12}} = 54.4 \text{ dB}
\]

\[
r_c = \sqrt{\frac{Q_g A}{16\pi}} = \sqrt{\frac{4 \times 1128}{16\pi}} = 9.5 \text{ m}
\]
Problem 7

a) In a church, you notice a pipe about as tall as you are, producing sound of fundamental frequency 100 Hz. Is it an open-open or open-closed pipe? Explain your reasoning.

if open/open then \( L = \frac{v}{2f} = \frac{340}{2 \times 100} = 1.7 \text{ m} \)

if open/closed the \( L = \frac{v}{4f} = \frac{340}{4 \times 100} = 0.85 \text{ m} \). You are not that short, so the pipe is open/open

b) If the organ builder now changes the end of the pipe from open to closed - or from closed to open - depending on what you found in a), what will now be the new fundamental frequency, and what will be the frequency of the first overtone?

After closing one end, the fundamental will drop to \( \frac{100}{2} = 50 \text{ Hz} \), and the first overtone will be \( 3 \times 50 = 150 \text{ Hz} \).
Concepts to explain (Part 2):

- mechanism of hearing:
  - (eardrum, Eustachian tube, ossicles, cochlea, basilar membrane, hair cells)
- how does the brain get information about the frequency?
- and about the loudness?
- loudness vs. intensity: critical band
- Sound Intensity, Sound Intensity Level, Loudness level, Loudness
- addition of sounds, masking
- vibrato / tremolo
- beats vs. difference tones
- larynx, vocal folds, formants
- what are “soft reeds” and “hard reeds”
- is the clarinet reed hard or soft?
- how about the vocal folds: hard or soft?
- explain the difference(s) between the sound of an open/open vs. an open/closed pipe
- discuss the behavior of an closed/open conical pipe (as opposed to the cylindrical pipe)
- give an example of both of the three major types of pipe/bore:
  - cylindrical open/open, cylindrical closed/open, conical closed/open
- explain the mechanism of the “stop” selection in a pipe organ
- what happens to the sound of the pipe organ if the room temperature changes?
- what are the consequences of the inharmonicity of real strings on the sound of
  - a) struck/plucked strings
  - b) bowed strings.
- What about real (as opposed to the “ideal”) pipes?
- discuss the relation between the localization of the stimulus and the sound spectrum
- room acoustics: $RT_{60}$
- reverberant and direct SIL, critical distance

Explain why all large enough rooms have the same RT at high frequencies, and on what parameter does this value depend?

**Note:**

The column with the values of the loudness in Table on p. 59 is incorrect. The values of L (sones) should double with every increase of LL (phons) by a factor of 10, as explained in lecture, contained in the equation on p. 59, and handwritten next to the contours of equal loudness on Figure 11.4
Figure 1: Determination of frequencies and cents for the deCaus tuning

<table>
<thead>
<tr>
<th>Note</th>
<th>Frequency</th>
<th>Cents</th>
</tr>
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<tbody>
<tr>
<td>C</td>
<td>702</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>905.86</td>
<td></td>
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<tr>
<td>E</td>
<td>1088.27</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>976.54</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>772.63</td>
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</tr>
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<td>A</td>
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</tr>
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<td>B</td>
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</tr>
<tr>
<td>C#</td>
<td>772.63</td>
<td></td>
</tr>
<tr>
<td>D#</td>
<td>590.22</td>
<td></td>
</tr>
<tr>
<td>E#</td>
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<tr>
<td>F#</td>
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<td>G#</td>
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<td>Bb</td>
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</tr>
<tr>
<td>C#5/4</td>
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<tr>
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<td>D#5/4</td>
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</tr>
<tr>
<td>A#5/4</td>
<td>478</td>
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</tr>
</tbody>
</table>

Note name / frequency ratio relative to F / cents relative to F

Figure 2: 
a) columns B-E: Determination of the chord quality of all 12 Major and 12 minor chords for the deCaus tuning

b) columns F-I: Impact of making the $B^b$ Major chord perfect. (see Text for details)
Figure 3: Impact of a uniform reduction of intensity: you lose the bass! Any decent amplifier accounts for this automatically, and often there is an additional, user-friendly equalizer.

<table>
<thead>
<tr>
<th></th>
<th>original</th>
<th>after reduction</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>60 Hz</td>
<td>3 kHz</td>
</tr>
<tr>
<td>$I$ (W/m$^2$)</td>
<td>$2.5 \times 10^{-2}$</td>
<td>$7.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>SIL (dB)</td>
<td>104</td>
<td>89</td>
</tr>
<tr>
<td>LL (phants)</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>L (sones)</td>
<td>32</td>
<td>32</td>
</tr>
</tbody>
</table>

Figure 4: The $r$-dependence of SIL for two different setups of the assigned room (see Text for details). The two values of the critical distance $r_c$ are indicated.