Art of Fugue
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This manuscript started as Program Notes for a Lecture Without Words
(without spoken words, to be exact)
to accompany a performance of the complete Art of Fugue
at the 19th Annual Roland D. Pinkham M.D. Basic Science Lectureship on “Music and the Brain”
The music was performed on the Willis pipe organ
at the St. Joseph Church, Seattle, November 18, 2005
These Notes (last updated July 2015) are being expanded for a book
“From Bach to Einstein and Beyond: Variations on Music, Science and Society”

Brief Introduction to the fugue

Counterpoint is a musical idiom characterized by the balance between melody (or rather: melodies, as we shall see and hear) and harmony. Literally from the Latin “punctus contra punctum” (“point against a point”) the composition process is subject to restrictions of varying severity. The strictest example is that of a “canon”: two (or more) isomorphic voices singing at the same time. “Isomorphic” means to be related by a well-defined transformation, such as when the second voice is delayed by a fixed number of beats or bars, or mirrored about a fixed note (so that low notes become high and vice versa), or played at a different tempo. One of the four canons included in the published Art of Fugue has the second voice sing the melody inverted in pitch, played twice as slow and backwards in time! (the “sloth canon”). Needless to say, such contraptions are often more of a demonstration of the composer’s skill than an enjoyable pieces of music. In the baroque period, it was a favored pastime of “learned composers” to publish just the first voice, and then challenge the reader to discover the possible continuation(s). Good examples are the various canons on the Royal Theme in the Musical offering.

In order to make canons more interesting musically, the rules are often relaxed to some extent. One way of doing this is to allow one of the voices to be free, i.e. unrelated to the Theme. The beautiful canons in Bach's Goldberg Variations are of this type. The popular Canon in D by Pachelbel is a three-voice canon in the form of a passacaglia, with a 2-bar bass motive repeating itself (“ostinato” - i.e. obstinately) through the whole piece.

Perhaps the optimal combination of counterpoint restrictions and artistic freedom is manifested in the fugue. Here the Theme is short, and when it is over, the second voice sings an isomorphic version of the Theme, and then another voice comes in and so on (usually Soprano, Alto, Tenor and Bass). As in the canon, the Theme can undergo many modifications: delay, inversion, speed-up or slow-down etc. But whenever a voice does not sing the Theme, it is free to do whatever pleases the composer. This seems to be the optimum choice, enabling the listener to enjoy the recognition of the various modes in which the Theme appears, and at the same time providing enough freedom in the free voices to create a piece of Music with the whole range of emotions and feelings.
Fig. 1 shows the first page of the first and simplest fugue from Bach's *Art of Fugue*. You can easily see the Theme starting the piece in the first four bars in the Alto, followed by Soprano in bar 5, Bass in bar 9 and Tenor in bar 13. After that, there follows an “episode” with no Theme at all, then the Theme comes again in Alto (bar 23) and so it goes.
Fig. 2 (below) shows the first page of the much more complex Fugue #7. In an almost incomprehensibly intricate construction, and with an even more incomprehensible beauty, the Theme (modified by a mild syncopation) overlaps with its copies played upright or inverted, and twice as fast or twice as slow!

The structure becomes so complex that we have to invent a labeling scheme:

Let us label the Themes by bar/voice mode/tempo, with

- bar = bar # of the start of the Theme
- voice = S(oprano), A(lto), T(enor) or B(ass)
- mode = r(ectus) or i(nversus)
- tempo = N(ormal), F(ast) or S(low)

Then the structure of the first page of Fugue #7 (Fig. 2 above) can be represented by:

1TrF, 2SiN, 3AiF, 5BiS, 6TrF, 7ArF, 9TiF, 13½ SrF (continues on the next page of the score; not shown here)
All this becomes much easier to follow and appreciate if we change from the standard Western sheet music notation to a “piano-roll” notation. Fig. 3 (below) shows the entire Fugue #7 using software I developed with my UW students. The time is on the horizontal axis, and pitch on the vertical. The voices (S, A, T and B) are distinguished by different colors, and all appearances of the Theme are highlighted. Even readers with no previous training in music find it easy to follow the structure after just a few minutes of practice. And since the whole fugue fits on a single sheet, you can appreciate the whole design: in the slow mode, the Theme appears just once in each voice, and climbs majestically from Bass (inversus; in blue) to Tenor (rectus; in yellow) to Alto (inversus; in green) and finally Soprano (rectus; in red). And at the same time, the copies of the Theme in the normal and fast modes play (literally, like children play on a playground) above and below, and the free voices (the non-highlighted notes) provide the ultimate melodic and harmonic complexity.
Notice how beautifully Bach tempers the regularity of the structure with clever and beautiful “spur-of-the-moment” ideas: the “majestic” presentation of the slow Theme in the Alto is followed immediately by a “double flip” like in ice-skating: the fast Theme inverted and then upright, all without breaking stride!

After contemplating all this complexity, we must once again emphasize that it sounds heavenly. The harmonies are out of this world, and the braid of the fast, slower and very slow melodies is overwhelming. To listen to such art with full appreciation is a gift not given to many, but fortunately that ability (the “art of listening to a fugue” - see below) can be cultivated.

The Dissonance in Bach

We will argue often in this book that Bach was not a baroque composer – he is timeless. And his harmonies, and in particular his unique and daring use of the dissonances is a major reason for this claim. The dissonances are often hidden as passing tones, or as dissonance with the previous tone left sounding by a tie. In Bach's time, performers were often encouraged to minimize the most dramatic dissonances by shortening the tie, or by emphasizing the resolution of the passing tone. This may have placated the innocent ears of the 18th century parishioners – but our ears are now used to the sound of Wagner, Schoenberg or Prokofiev, and I believe that instead of minimizing Bach's dissonances we should maximize them. I go as far as re-striking the notes as needed, at least when I try to cultivate my (and my listeners') “art of listening to a fugue”. If you do this, you discover that the dissonances are not only beautiful, but even indispensable part of Bach's music in general, and of the KdF in particular. This also requires the proper choice of the instrument. This is discussed is some detail below, and in great detail in the expanded version of these Notes.

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1 To help the reader to see this complex trick, I have added a vertical green line marking the end of the slow Theme in Alto (end of the middle row on the Figure). The first note after the vertical green line is the last note of the slow Theme, and at the same time the first note of the fast Theme. A rectus version of the fast Theme follows immediately - the first note on the third row is simultaneously the last note of the inversus and the first note of the rectus.

And to make things really complex, Bach performs a similar trick in Soprano – see the vertical red line marking the transition from the normal Theme to the fast Theme. This time, the note on the left of the line is the first note of the normal Theme, and the first note on the right is the first note of the fast Theme. And as all this is going on, Tenor comes in twice, and Bass once (just before the start of the final slow Theme in Soprano).
After using couple of fugues from Johann Sebastian Bach's KdF (Kunst der Fuge = Art of Fugue) in our introduction to counterpoint, we will now discuss the KdF systematically and in some detail. It is a huge collection (16 fugues and 4 canons with a total of 40,000 notes) of fugal variations on a single, simple Theme, all in the key of D (recall the discussion in the Introduction). In a certain sense, it is a fugue of fugues, or a “superfugue”. The Theme undergoes many modifications (inversions, syncopation etc.). Fig. 4 shows the seven main forms of the KdF Theme, in the order of increasing complexity and/or “distance” from the original Theme:

In addition, the Themes can be inverted in pitch, or played at several different tempi at once. Also, several fugues are “double” – with one additional Theme, or even triple – with two additional Themes. But the main aspect differentiating individual fugues is the treatment of the free voices that determines the particular musical character and emotional “affekt” to each fugue.

It is my goal to convince you that the Art of Fugue is not a dry, boring exercise in counterpoint, but rather it is beautiful, glorious Music. I will try to do that with an interpretation which attempts to treat the Art of Fugue not as a baroque period composition, but as a work which is timeless and abstract, yet so human. And on the most fundamental level, this is just an excuse to play the Unfinished Fugue.

Summary of the evening Art of Fugue Lecture series, April 30 - May 14, 2004

[ ] The Art of Fugue (Kunst der Fuge = KdF) is in fact a triple art:

[ ] art of composing a fugue
[ ] art of playing a fugue
[ ] art of listening to a fugue

The main task of the art of listening to a fugue is to achieve the right balance between the perception of the melodies, and the perception of the harmony. It is a serious mistake to attempt to clearly identify each time the Theme comes up – and some misguided performers even try to “help” the listener to do

Note: The last theme shown on the Figure below is almost universally considered to be just the 3rd theme of the Unfinished fugue, to be followed by the “real” KdF Theme in the completion that has either been lost, or never written because of Bach's death. And many composers, with varying degrees of foolishness and sometimes even arrogance proceeded to write their own completions. My own “theory” of all this is discussed below.
just that, by excessive differences in the registration of the individual voices, thus damaging the fusion of the counterpoint into Music.

Until quite recently, I was convinced that the ideal instrument for the performance of KdF is the pipe organ. The arguments include the frequent and essential use of the pedal point, a necessity for voice fusion as well as voice separation, the clarity and simplicity of sound needed for the proper treatment of dissonances, and the intensely personal, meditative nature of many of the fugues. However, the registration must be chosen so that the organ does not sound “organy” – with the exception of Fugues 6 and 9, KdF is not well served by the “majestic” organ sound. In particular, the bass should not be “reinforced” by the 16' stops, as this amounts to dropping the pitch by an octave\(^3\), and thereby re-writing Bach’s work ….

Today, after decades of searching for the “right” pipe organ, I no longer hope I can find one. Most organs do sound “too organy”, so when you come to a hall where an organist plays KdF, you first say “ha! Pipe organ” and only afterwards: “Ha! It is the KdF”. Even more importantly, the scaling of most pipe organs produces weakening of the sound as you go from the upper register downwards, i.e. from Soprano to the Bass. This is fatal for a fugue: instead of a conversation between four equal voices, you often hear an “accompanied melody”. It can be alleviated by using a registration adding more stops, but then the sound is so rich that the perception of the dissonances is much damaged.

After much experimentation, I have come back to where I started some 20 years ago: the piano. It enables the cleanest treatment of the dissonances (much better than that provided by the harpsichord). Unfortunately, it cannot provide the sustained sounds so important in the “pedal tones” so important in several fugues of the KdF. Also, the absence of the pedalboard makes a proper rendition of some passages (by the two hands only) a very difficult proposition.

The resulting solution I am working on involves a judicious mix of the piano for some fugues, organ for others, and possibly even using MIDI and new soundfonts. Purists will be certainly complaining, but I believe that Bach would approve. He was quite a tinkerer himself (recall his invention of the “viola pomposa”) and he felt free to break just about any “rule” of counterpoint if it sounded good. When giving an invited lecture on all this, I played a recording I made of one of the fugues. Afterwards, the professor (a professional organist) asked me where I found an organ with such a lovely stopped pipe. He was incredulous when I told him it was a synthetic soundfont ...

[ ] The structure of the KDF

The KDF naturally splits into three distinct categories:
1. fugues 1 through 13
2. the Unfinished Fugue 14
3. the four canons.

Group 3 belongs to the category of "learned" scholarly works, and it has done much to give KdF its dreaded but undeserved reputation. As an additional exercise in reading the “piano roll notation (that will be used for every individual fugue later in the book) Figure 4 shows the famous “sloth canon”

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\(^3\) In the first approximation, our hearing system assigns pitch based on the lowest common denominator of a frequency spectrum we hear. Adding higher pitches (such as a 4' or 2' rank) to the nominal 8' pitch does not change the perceived pitch of the tone, as these are just partials of the original already present – with more of higher partials the sound becomes richer. However, there are no partials with halved frequency in the original tone. That is why pulling a 16' stop changes not just the timbre but also the pitch. Some organists double down on this crime by pulling a deep 32' rank, too. The listener's whole body then vibrates, but this is only acceptable when the piece was composed with this in mind. A fugue should be played “as written”, to preserve the delicate balance between the voices.
from KdF. The nickname reflects the fact that the “follower” is twice as slow, and upside down, relative to the “leader”. In the middle of the canon, the leader and the follower trade places.

In fact, it is a mystery why the four canons are included at all – most scholars believe that it is just a small fragment of intended collection, made incomplete by Bach's premature death. I believe that the performance of the “complete” Art of Fugue benefits if this group is omitted⁴.

Figure 3: The Sloth Canon from the KdF

[ ] At the lectures, I presented a new theory/hypothesis/explanation why the last fugue is unfinished. This will not fit in these Notes - see the www page above for details.

[ ] At the conclusion of the lecture series, I discussed the connection of Bach's music to our troubled times. Again, see the www page above.

⁴ Recently, I have changed my mind on this, at least to some extent – explanation coming up soon …
Overview of the fugues of KdF, with brief summaries.

In my performances the order of the fugues follows the commonly accepted sequence, with the exception of using Fugues 10 and 11 to separate the two mirror versions of Fugues 12 and 13 – the reasons for this are discussed below.

I. Lovely introduction to KdF; deceptively “simple”
II. Perhaps the most dissonant of the whole KdF
III. Evolves from a quiet chromaticism to a dramatic ending
IV. The “cuckoo” fugue
V. A dialogue in the form of a stretto fugue
VI. A fugue that must be played on a full organ
VII. This quiet but more complex sibling of Fugue VI must be played softly and pensively.
VIII. Lively mix of three Themes and several secondary motifs, with an overwhelmingly calming end.
IX. The Glenn Gould's (and Swingle Singers') “warhorse”.

INTERMISSION

X. Because of the richness of counterpoint writing, I call this fugue “A symphony with four voices and two Themes.
XII Rectus
XIII Rectus
XI. Fantastic partner of fugue 8, with four voices, 3 Themes and a spectacular chromatic free voice.
XIII Inversus
XII. Inversus

A Very brief Intermission

XIV The Unfinished Fugue. Glenn Gould thought it is “the most beautiful of all music”. The very idea of “completing” it is a blasphemy.

I The first fugue revisited.
The comprehensive list of the Bach resources is enormous.; here are some recommendations:

**Books:** every Bach lover should own *The New Bach Reader: A Life of Johann Sebastian Bach in Letters and Documents* (originally edited by Mendel and David, and updated by Wolff.) Parts I. and V.-VII. contain a very nice Introduction and essays. When you are done with these, you can join the afficionados in enjoying the parts II. - IV. - a veritable treasure of letters, facts and anecdotes about just everything known about Bach. And the Bibliography will point you to more reading.

A new and very useful book has been published since the “New Bach Reader”: “Johann Sebastian's Bach Art od Fugue: Performance Practice Based on German Eighteenth-Century Theory” by Ewald Demeyre. Regrettably, it is not accompanied by a recording that would illustrate how the approach advocated in the book actually sounds. In any case, as with any other scholarly treatises, one should study them, but one should not feel obliged to follow all the recommendations. As argued above, our ears may just now be able to fully appreciate the timeless harmonies written by Bach in that 18th century ...

**WWW:** A very interesting and useful resource contains the NPR Public Radio programs 2029 and 2030. All the fugues and canons of KdF are played, each by a different organist, and introduced by knowledgeable commentary. Some general issues are discussed in the accompanying essays.

http://pipedreams.publicradio.org/articles/artoffugue/index.shtml
Priceless!

Youtube is of course full of items related to Bach in general and to KdF in particular.

From the large number of available recordings, I own:

**Organ:**
Tachezi (http://www.youtube.com/watch?v=Lrb0dHKJBR4&list=TLaY832-2GdFg)
Gould (see below)
Walcha (http://www.youtube.com/watch?v=xzZJ8tDuC08)
   http://www.youtube.com/watch?v=WM995ITuZpw including completion
Fergusson
Roberta Gary
Tuma
Lippicot
Marie

**Harpsichord**
Koopman

**Piano:**
Sokolov http://www.youtube.com/watch?v=cOlQzoULv4E
Koroliov
1/6 http://www.youtube.com/watch?v=BELqzhnNd-Y
2/6 http://www.youtube.com/watch?v=tIKZtm-GfRE
3/6 http://www.youtube.com/watch?v=VCFzk-bp9Ao
Rangell
Nikolayeva  
Gryaznov  

PIERRE-LAURENT AIMARD

Chamber Orchestra
Musica Antiqua Koln (Reinhard Goebel)
Benedetti / Marcello Ruffini  

String Quartet
Fine Arts Quaret  
Jordi Savall  
Quartetto Italiano  

Misc.
The Art of the Fugue for 4 quartets  
Laibach

[ ] Herbert Tachezi. Apart from some minor complaints, I find this to be a very nice, clean recording. Tachezi ends the KdF proper - as I do - by Fugue #12 Inversus. But he does not play the Unfinished fugue at all - he does not like its Theme! (he cannot get over the first few bars - see below).

[ ] Michael Ferguson. This is a must-have, for his "completion" if for nothing else. Ferguson plays the Unfinished fugue with the full organ, and his Bach shouts his name (B.A.C.H.) for everyone to hear. The completion is correspondingly bombastic. It is so bad that it is good - it should be a sensation at any musico-logical party (after everyone has had a drink or two…). The rest of the CD is actually quite good. And quite unexpectedly, Fugue #9 is very subdued.

[ ] Several recent recordings (violating Glenn Gould’s maxim that each new recording must be different from all previous ones) are mostly academically “correct,” and therefore boring (Tuma, …. more discussion coming up)

[ ] Glenn Gould. The "Art of Fugue" CD contains Fugues 1 - 9 played on pipe organ [sic] - in a very Gouldian, detached, staccato way, followed by several fugues played on the piano. I play many of the Fugues (especially #3, 5 and 7) much slower and softer, but this CD is an ear-opener as to how a pipe

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Although Gould started as an organist, he became famous as a pianist, and this is his only available organ recording. Due to merciless reviews by ignorant music critics (example: “Gould's approach seems downright unmusical, and the image it evokes for me is of the trained seal who beeps out 'God Save the Queen' on a set of car horns”) we will never know how he played fugues 10-14 on the organ. That is much to be regretted.
organ can sound. And the video of Gould's interview with Bruno Monsaingeon (number XV ["An Art of Fugue"] of the Sony VHS "The Glenn Gould Collection") is a true jewel and a must-have for anyone interested in KdF. You will probably disagree with Gould’s harsh words about Beethoven, but his discussions and demonstrations of Bach Fugues are brilliant, and his account of the Unfinished Fugue is unforgettable. It is also a rare treat to actually see Gould’s fingers at work, and you will get to hear him sing as he would on his CDs if his recording engineers let him…

[ ] Just a few weeks ago, I discovered Andrew Rangell, at his memorable KdF lecture/recital at the University of Washington. His concept is very idiosyncratic – reminiscent of Glenn Gould but less harsh, less pointillistic – with all the respect to Glenn Gould I must say Rangell is more musical, or at least more pleasant to listen to. At the [website for his new CD](http://pipedreams.publicradio.org/articles/artoffugue/) there is a free sample from each of the fugues. For my taste he emphasizes the clarity of the voices too much at the expense of harmony – but after listening to him I concluded that maybe I emphasize harmony too much, at the expense of the soprano/alto/tenor and bass … I will work on it.

[ ] Other recordings: viols, string quartets, chamber orchestra, saxophones, Swingle Singers, … (more discussion coming up here)

[ ] Internet resources

As you may expect, there is a multitude of items related to the Art of Fugue on the web – descriptions and explanations, scholarly as well as “scholarly” essays on topics ranging from Bach's alleged numerology to various completions of the last, unfinished Fugue, and more. Perhaps the best single resource is from National Public Radio: [http://pipedreams.publicradio.org/articles/artoffugue/](http://pipedreams.publicradio.org/articles/artoffugue/) with additional links. NPR also has a remarkable three-hour discussion of the KdF with Michael Barone at [http://pipedreams.publicradio.org/listings/2000/2029/](http://pipedreams.publicradio.org/listings/2000/2029/) and [http://pipedreams.publicradio.org/listings/2000/2030/](http://pipedreams.publicradio.org/listings/2000/2030/). All the fugues of KdF are performed by well known organists, with a large diversity of their approaches and concepts of the work.

[ ] Note: it is quite easy for the first-time fugue listener to happen upon a dry and boring recording and be discouraged, or to become victim of an overly romantic and sentimental performance and be discouraged. One should be especially beware of the various “MIDI realizations” that are often nothing but results of a mechanical encoding of the fugues “by the mouse”. Unfortunately, this may extinguish any interest in counterpoint at the first attempt...
PROGRAM NOTES

PART I.

Fugue 1: This is the simplest of KdF; it is often included in “Best of Bach” collections. Yet, simplest does not mean simple – there are no simple, pedagogical fugues in KdF. Many connoisseurs consider the #1 as one of the most beautiful fugues, in KdF and out. The technical tricks of contrapuntal composition, waiting to be exploited in subsequent fugues, are not used at all here (there is just a hint or two of a stretto). This means that the essence of the piece is a free composition. Listen to how much is going on in addition to the Theme, which is presented in its fundamental form at the beginning. The characteristic feature of this fugue is the ubiquitous two-note motif which keeps popping in and out, below or above the legato voices. If I had to illustrate the phenomenon of particle-antiparticle pairs popping in and out of the vacuum to a Music major, this is what I would use as a metaphor. Note also the dramatic semi-close just before the end, apparently calling for a reverberation provided by church acoustics, and the subsequent pedal point against which the last quote of the Theme plays. And through the whole piece, note the marvelous dissonances (if the performer is not out to minimize them!).

Overall, this is a magnificent portal to KdF.

Fugue 2: The treatment of the Theme is still quite simple here. It is enlivened by syncopating the last bar – and Bach gets an idea: why not make the persistent syncopation in the free voices the defining feature of this fugue? And so he does. This compositional process, seemingly originating in a random idea, is used in many of the fugues, and gives them a fresh, improvisational character. One can distinctly hear the moment when the idea first occurred, and then developed naturally, like a plant sprouts from a seed. You would almost believe that Bach just sat down and played the fugue, and somebody listened and wrote it down. I should note that Bach was indeed quite capable of such a feat – there is a famous story of King Frederick giving Bach a difficult chromatic Theme, with Bach improvising a fugue on this Royal Theme on the spot. Incidentally, the King wrote that Theme himself. Imagine the chance of something like this happening today! But anyway: Art of Fugue only seems to be improvisational – every single note is worked out to absolute perfection. So going back to fugue #2, pure joy emanates from this work, and at the same time, one must be in awe of the perfection with which the piece is crafted. Note how a pipe organ is able to render the staccato of the accompaniment, as well as the legato, sustained tones of the Theme.

Fugue 3: This is one of only two fugues where the Theme appears in two different forms (not counting simple inversions or augmentations/diminutions). [The other fugue with two versions of the Theme is the mirror Fugue #12.] The countersubject beneath the (inverted) Theme makes this Fugue very chromatic, and quite melancholy. Listen to how the voices in the Manuals play against the pedal point at the end, with a flow of transitions between consonance and dissonance, until all is resolved.

Fugue 4: The last of the “simple” fugues, this is a pure joy again, but much more complex than #2. This time, the persistent pattern is derived from the last four notes of the Theme (again inverted) and becomes interwoven with a two-note “cuckoo” motif. The same comment on the seemingly improvisational nature of Bach’s work we made on Fugue #2 applies here, too. First one cuckoo is heard, and soon the forest is full of them – quite literally a “moveable feast”. And once again – the pedal point at the end clearly requires an instrument with sustained tones.

Fugue 5: The “stretto” fugue (a ‘stretto’ occurs when a voice starts to sing the Theme even before the previous voice is quite finished with it). The syncopated version of the Theme, and its inversion, combine in a series of strettos of varying overlap (the Theme takes 4 bars, and the overlaps used here are 3 bars, then ½ bar, 1 bar and 1 1/2 bars). At the end, the overlap reduces to zero – the upright and
inverted versions of the Theme are played at the same time, “in anti-parallel”. At a key point of the piece, the usually “minor” nature of the Theme changes to “Major”, with subtle but profound effect. In the middle, the fugue comes twice to a tentative close, followed in each case by a quadruple [sic] stretto-like episode. Over and above these technical details, to me this is a prayer in a form of a fugue.

**Fugue 6:** For all other KdF fugues, the use of pipe organ is arguable; for this one it seems imperative. The syncopated Theme and its inversion play against each other at two different tempi at the same time, accompanied by frequent tirades in “Stile francese”. Here I do use 16’ reeds in the pedal – but I also use 16’ stops in the Manuals, so that the balance of the voices is not distorted. A very powerful and optimistic piece; one of the few KdF fugues where the full “organy” sound is needed. When playing this fugue, I have the image of sun beams shining through a grove of giant sequoias.

**Fugue 7:** A quiet version of Fugue #6, and even more complex – the original tempo is combined both with an augmentation (twice as slow) as well as with a diminution (twice as fast). I play this as “normal, slow, and very slow”, so that the slowest version acts almost as a kind of pedal point. Most often you will hear it as “fast, normal and slow”, but this destroys the ubiquitous, contemplative counterpoint of the free voices. As the piece progresses, the slowest version of the Theme climbs from bass to tenor, to alto, and finally to the soprano. As discussed in the Introduction above, this is very nicely visible on the piano roll display, where you can also easily see which versions of the Themes are upright and which ones are inverted. But forgetting all technicalities, this is a fugue for meditation.

**Fugue 8:** Only 3 voices, but what a fugue! It is a triple fugue – those three voices are in charge of three different Themes, so when they all combine at the end (in five different versions) there is not a redundant note to be heard! The fugue begins with an exposition of a new Theme, accompanied by an 8-note figure that occurs 19 times in the 39 bars of the first part. Next, a second Theme is introduced as a countersubject and combined with the first one. Finally, at bar 94 (i.e. after many other fugues are already finished) a fragmented version of the main KdF Theme is introduced. As it first comes in alto, it is not all that easy to recognize. Then the rest of this huge fugue is spent on exploring all possible combinations. At the end, after the three Themes have been combined for the last time, the excitement and agitation of the fugue comes to a peaceful resolution, as a brook arriving into the ocean.

**Fugue 9:** This 4-voice fugue is double (i.e. it has two different Themes) but it is in fact quite simple, and a great deal of fun. Again, the new Theme is explored first, then joined by the original KdF Theme. The whole piece is nothing short of exuberant – play this to someone who claims that Art of Fugue is a dry, scholarly and indigestible work! This fugue has been a signature piece of Glenn Gould, both on the piano and on the organ, as well as a “piece de resistance” of the original Les Swingle Singers group (with emphasis on the “original”).

**INTERMESSION**

**PART II**

**Fugue 11:** With this fugue, we are leaving the realm of the ordinary. When I play it, I feel as if Bach invited me home and is walking me through all his chromatic and dissonant magic. In fact, I feel that I am fortunate just being able to appreciate this – it feels like a gift. The fugue is a more complex version of Fugue #8, with the same three Themes (but as it is a 4-voice fugue, there will be a free voice available even when all three Themes are combined – this accounts for most of the added complexity). The show begins with the exposition of the fragmented version of the main KdF Theme, followed by an inversion of the first Theme of Fugue #8, and then the second Theme of #8. After that, anything goes. Among the bewildering variety of subsidiary patterns which inhabit various parts of
this fugue in a manner reminiscent of the patterns on the Mandelbrot set, the most dramatic figures are
the explicitly chromatic runs extending for many bars at a time. I can imagine Bach's contemporaries
having great difficulty with this piece – but our modern ears, having been exposed to Bartok and
Hindemith, can take it now. We can play it slowly, and rejoice in the chromaticism and dissonance.
Just before the end, the main Theme and its inversion are combined with a zero offset, as in Fugue #5,
but what a difference in the intensity! Sometimes one must say about a particular piece of music: “…
sound and fury, signifying nothing.” In Bach, sound and fury mean a great deal!
The very end is subdued, as if after all this drama Bach said, “well, there you have it …”

Fugue 12 Rectus: In the two versions of a mirror fugue (rectus and inversus), the pitches of all the
voices are inverted as in a mirror. If you did this to an arbitrary piece, you obtain a cacophony, so the
original piece must be written with the inversion in mind. Even then the inversion is not purely
mathematical, but a liberty as to the accidentals is used to make both versions musical, and even
different in “affekt” (affekt was the term used in the 18th century to describe the effect, the impact of
music on the listener).

Mirroring the whole piece is obviously not a natural concept. You may ask what is the purpose of
such an obviously artificial construct, and it is a good question. This fugue is an example what this
technique can yield in the hands of a genius. Both the rectus and the inversus are breathtakingly
beautiful, and in spite of being more or less mathematically related, each have their own individuality.
Most performers play them one after the other, but I find that this diminishes rather than enhances the
experience – the immediate comparison dulls the senses, and creates the dreaded impression of a
“learned piece”. Because of this, I insert Fugue #10 between #12 rectus and #12 inversus. In this way,
the beauty of each can be fully appreciated, without losing the connections (especially on the second,
third and subsequent hearings 😊).

In the following, I will discuss both variants – points specific to the Inversus will be discussed
later. Please note that it is not at all clear if the Rectus or the Inversus were the primary composition.

The fugue appears exceedingly simple – a new version of the main Theme is introduced in the
most straightforward manner. And then, in another example of Bach’s compositional process, an idea
originates and gradually develops into a most beautiful modification of the Theme, based simply on a
run up and down the d-minor scale. This version is then explored in all four voices, accompanied by a
short fragment in a manner reminiscent of a Palestrina or Josquin Des Prez. The piece ends with a
cascade made of that short fragment at simple octave pitch increments. From the point of view of the
performer, these quiet fugues are very interesting, to say the least. There are a number of places where
the feet perform trills; in other places you get to play one voice with your right hand on one manual,
second voice with your left hand on another manual, 3rd voice with your left foot, and the 4th voice
with your right foot!

All recordings I know of which do include this pair of mirror fugues play the Inversus version
immediately after the Rectus. As mentioned above, I believe that this is a serious error. Rather than
appreciating the relationship between the two, the overall impression is that of a boring, academic
exercise. By interposing Fugue #10 between the two mirror fugues, the listener (and the player) gets a
full appreciation of the beauty of each fugue – so similar yet so different.

Fugue 13, Rectus and Inversus Until June 2012, I hated these fugues, and I included them in the
“infamous” category III, to be omitted in performance. Then, while recovering from a spine surgery, I
decided to give them one more chance. I discovered that with a simple but far-reaching change of
phrasing of the Theme, everything suddenly made sense, and these two fugues instantly became my
favorites. The treatment of the Theme is quite adventurous, and the change of phrasing mentioned
above consists of slightly extending the first and the third note, so that the opening 5th or 4th, upright or
inverted, become subtly but unmistakably identifiable. Another beneficial change is a breaking of the
incessant monotony of the dotted notes by going into triplets, when triplets are explicitly written in
another voice. The fugues then literally come alive, full of great inventiveness and complexity, despite
the texture limited to three voices. And as in the pair 12r and 12i, the 13r and 13i are at the same time similar as well as different – Sir Donald Tovey notwithstanding (a rant on Tovey coming up…).

Fugue 10: Johann Sebastian Bach never thought much about the various rules of counterpoint to be observed (note that term Baroque was originally derived from a Portuguese word meaning "a pearl of irregular shape"). In this fugue, he violates a great many of the rules – resulting in one of the most beautiful and spiritual fugues of Kunst der Fuge. It starts with an extremely compact exposition of a new Theme, beginning with neither a tonic nor a dominant, but a leading tone. The answer by the tenor, in a stretto already the first time, is not in a dominant but in the subdominant, and the next voice (bass) is inverted, then another close stretto follows – simply a mess right from the start. When things calm down, a syncopated version of the main KdF Theme is introduced, only to be accompanied by another version of itself which is aborted in the middle. And yet, the result is profound, contemplative and glorious at the same time. When they finally get played together, the two Themes turn out exceptionally well matched together. In addition, this fugue is written in “double counterpoint” at the 10-th, which means that one (or both) voices can be doubled at the third or at the sixth. Voice doubling has a striking effect on the recognizability of the two Themes, which normally is (and should be) quite difficult. It is important to note that separability of voices is not desirable as a rule, as it interferes with the perception of harmony. Again, as pointed above: it is a grave mistake committed by many organists and fugue arrangers to try and “help the listener” identify the individual voices (the ultimate degree of this is found on many MIDI realizations on the web, where different voices are separated even spatially (by exaggerated stereo effects) and the fugue is completely destroyed). It is just that every now and then, as in this happy case, a clear voice separation results in a passage of exquisite beauty. Here you must realize that although the pipe organ has a keyboard, it is after all still a wind instrument, and therefore it should sing!

Incredibly, this magnificent fugue is just a preparation for the miracle that follows.

Fugue 12 Inversus: We have already discussed this mirror fugue, and now we are equipped to appreciate the subtle but profound differences between Rectus and Inversus. From the opening expositions of the Theme, it is obvious that the Inversus is much more melancholy than the Rectus – a falling 5th sounds much more sad then a raising 5th. This is reinforced by Bach’s choice to change the nature of some key chords from Major to minor and vice versa. But the main change is of course the bass in one becoming the soprano in the other and vice versa. This transitions into a spectacular shower of the first five notes of the Theme just before the end. And then it leads to the miracle with which the piece ends – the low rumble of the 16-th notes which ends the Rectus becomes, in the conclusion of the Inversus, a kind wave of Bach's hand wishing us farewell. This cannot but remind me of Thomas Mann’s description of an analogous miracle in the ending of Beethoven’s piano sonata Op. 111, when the original Theme comes back, blessed with kindness, after all it went through in all the variations (we will find another parallel to Op. 111 at the very end of the recital). So irrespective of the details of the compositional history, the Inversus has to be played last. The last seven bars are perhaps the most touching moment in Music. So, as far I am are concerned, the Fugue #12 Inversus is the proper end of the Art of Fugue, if one wishes to include only fugues completed by Bach.

PART III.

Fugue 14: It is hard to believe that, after the superlatives employed in the discussion so far, there would be something even more profound, more beautiful and more moving. And yet I will argue that the giant, unfinished Fugue 14 is the pinnacle of Bach’s life work, if not of much of Music in general. That is why any attempt at a "completion" is preposterous. The problem is not a puzzle in counterpoint to be solved, and you cannot "complete" the life of a genius.
Technically, it is a triple fugue for 4 voices. It is a received wisdom among the scholars that this fugue is either a fragment of a quadruple fugue, or it does not belong to KdF, since “the KdF Theme is missing”. In fact, it seems obvious to me that the theme with which this fugue begins is in fact the KdF Theme, modified more substantially than the modifications in the most of the fugues, but less than the changes made to the Theme in Fugue #13 and in some of the canons. At the first sight (or rather, first hearing), the modified Theme appears awkward due to a definite, sudden and somewhat unnatural break in the middle of the Theme. It does take some patience to endure the first six bars until the second voice comes in. But then one realizes that the second, slower part of the Theme is designed to serve a harmonic, rather than melodic, purpose. After reaching back to medieval harmonies and progressions combined with quasi-New Age aimless wandering, the Theme gets combined with its inversions in a series of strettos, but what you hear is mostly just the initial interval of musical fourth or fifth, and the harmonies.

This goes on for 114 bars – longer than many other KdF fugues, but this is only a beginning. The second Theme, as different from the first one as imaginable, is presented and developed. For extended lengths of time, the texture reduces to just three musical voices of heavenly simplicity and beauty. Quoting Thomas Mann on Beethoven's Op. 111 again: "Here the language is no longer purified of the flourishes, but the flourishes of the appearance of their subjective domination. The appearance of art is thrown off. At last, art always throws off the appearance of art." The sweetness of the Music is out of this world, yet it is presented completely without any pretense, as the most natural of things.

And then, after an almost Beethovenesque transition, Bach finally signs his name as a third and final Theme (in the German musical notation, B.A.C.H. is playable as Bb.A.C.B.) The fugue on this very unique, symbolically charged theme contains harmonies that would not be out of place in a composition by a Schoenberg. And finally, a rather abrupt transition leads to the first combination of all three themes. This first combination is also the last one – as legend has it, the pen dropped out of Bach's hand… As mentioned above, I have my own theory of why this Fugue is unfinished. The key piece of material evidence is the peculiar sheet of paper Bach used for the last 20 or so bars he wrote, but the main argument follows from the music itself. Paraphrasing the famous Fermat Theorem tease, the margins of these Program Notes are too narrow to contain an exposition of my theory (so I am writing a book…).

The Unfinished Fugue is a monument to Bach's genius which is difficult to fully characterize without starting to feel somewhat theatrical. As Glenn Gould points out, in this single composition Bach looks back at least 150 years, and also anticipates music not to be written for another 150 years - and he does it with beauty and wisdom which takes multiple listening to even start to appreciate.

Fugue 1: It is not really satisfying to end the program with an unfinished work, even if it is a masterpiece. Apart from the dozen or so misguided attempts to write a "completion", most performers just play all the notes Bach wrote and stop abruptly, and often they follow with a beautiful organ chorale which was added to the first print edition as a "consolation piece" to compensate the buyers for the incomplete nature of the cycle. I do this somewhat differently. First, I stop not at the last note but at the last full 4-voice chord Bach wrote, and I do it in a way which I find dramatic but not violently abrupt.

More importantly, instead of the chorale which has nothing to do with Art of Fugue, I conclude by playing Fugue #1 again. As in Beethoven's Opus 111, where the Arietta Theme comes back at the end, in its simplest form, and as in Bach's Goldberg Variations, where the original Aria returns, verbatim, after all the wonders Bach subjected it to in the Variations, so it seems appropriate to repeat the simplest presentation of the Kunst der Fuge Theme at the end of the Art of Fugue, to fully realize that Bach indeed went much beyond Music in this work – he reflected and meditated, and makes us reflect and meditate, on the inner wisdom of the Heavens and Earth.
APPENDIX A: Notes for a String Quartet

During the re-opening of the remarkable Frye Art Museum in Seattle after its renovation, entertainment was provided by the “Anderson quartet”, and I was intrigued not only by their beautiful playing, but also by their names: Mary Anderson, Larissa Anderson, Jacob Anderson and Sam Anderson. An inquiry confirmed that they indeed were siblings, and this gave me the idea to suggest to them that perhaps they were uniquely positioned to play the Art of Fugue. I believe that the performance Notes I gave them may be of general interest, as they reflect details about the concept I am trying to achieve.

Preliminaries:

I suggest you do NOT listen to any recordings of the KdF (if you already have, forget about how they played it :-)
Following Glenn Gould's maxim: only make a new recording if it will be different from the existing ones. That is what I propose to do.
I (VC) humbly but firmly believe that I understand KdF better than anyone else (or, if not better, then at least differently)
Based on hearing the quartet at the Frye Art Museum, I believe that you have unique abilities for this project (more on this below and in person).

Summary of main points:

1. KdF is not a baroque composition – it is a timeless piece of art, not beholden to the period of its creation. In this respect, it is similar to Bach's Sonatas for various unaccompanied instruments, Sonatas for Violin and Harpsichord (as played by Jamie Laredo, with Glenn Gould on the piano[sic]) or Viola da Gamba (with Leonard Rose on cello and Glenn Gould again) and other selected masterpieces.
2. Consequently, KdF should be played in a non-period, abstract manner, minimizing embellishments (especially when not explicitly written out by Bach) and not hiding dissonances, with emphasis on thoughtfulness, even when exuberant in some fugues. Everything should be done in moderation (the celebrated Glenn Gould notwithstanding! He was famously opinionated, and many of his recordings suffer from exaggerated extremes … But when it worked out, he produced memorable masterpieces).
3. Don't even think about “how would Bach play it”. As with much great art, KdF is greater than the composer himself.
4. The acoustical balance of S/A/T/B is of utmost importance. In fact, it was the impossibility of appropriate balance at ALL pipe organs I tried in the last 20 years that led me on a search for a different/complementary performance.
5. At the same time, I always felt an important aspect of the organ (or piano/harps) performance was that it should be played by a single performer (will explain). The remarkable and effortless unity of Anderson Quartet (after all, you are siblings!) may deliver the impression of a single performer.
6. There should be (a subtle!) emphasis on secondary Theme(s), non-Theme counter-themes and repetitive segments (when Bach “gets and idea” and plays with it for a while).
7. About the voices: it is generally believed that all Bach voices are beautiful melodies. They are not. Bach was, above all, a harmonist, and he did not hesitate to distort a melody significantly for the sake of harmonic consistency. So some voices sound very strange indeed, when played separately. But the combined result is beautiful (and non-trivial!).
8. On the other hand, there are cases when an inner voice possesses an exquisite beauty that tends to get hidden in the ensemble. In such cases, a gentle emphasis is appropriate – I will show specific examples.

9. As on a pipe organ, dynamics should be non-romantic! Mostly by phrasing. In fact, when I play the KdF on organ, I have been “accused” of sounding “like a string quartet”. In a complementary spirit, a string quartet performance should try to sound like a pipe organ. Of course, neither attempt can quite succeed – the point is to try!

10. Under no circumstances should there be “bouncy dancing” in the performance. Although Bach was indeed a German, and a little heavy at times (not in KdF!) one should resist the temptation.

11. Same prohibition of any “romantic” rubato.

12. Lastly, and most importantly, the single most regrettable mistake in most KdF performances is an exaggerated emphasis on the melodic aspects, especially when “helping” the listener to identify each and every moment a Theme comes in. As I discuss in my lectures and Program Notes (above) the “Art of Listening to a Fugue” involves cultivation of a deliberate ambiguity in perception of melodies and perception of harmony, thus enabling a fusion of Counterpoint into Music.

**Epilogue:** Sadly, the string quartet project never materialized. I am afraid the rigorous instructions in these Notes frightened them ... 

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**Additional Reading:**

The role of KdF in my work is described in the outline of my book (in progress).

outline of my book