From Bach to Einstein and Beyond
Variations on Science, Music and Society

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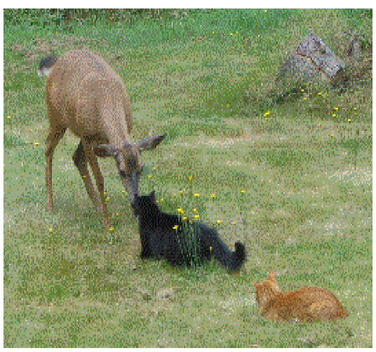
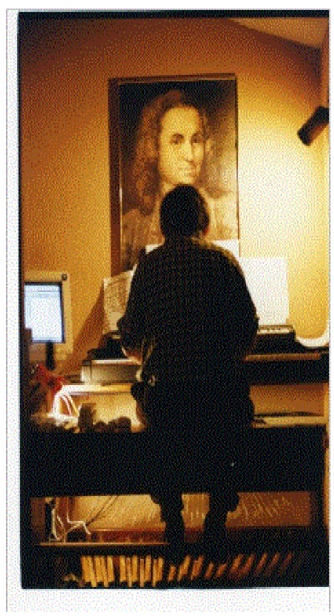
April 8, 2015

Introduction - Preface - Foreword

Vladi Chaloupka was born in what is now the Czech Republic, when it was under Nazi occupation. He grew up in the country subsequently dominated by the Soviet Union. In 1968 he experienced a shock and awe invasion, and escaped to Switzerland. There he obtained his PhD in Physics from the University of Geneva, and worked as a particle physicist at CERN. In 1975 he moved to the Stanford Linear Accelerator Center in California, and in 1981 he came to the University of Washington. After a career in experimental elementary particle physics, he is now working on merging his life experience with his three passions: science, music and human affairs, into one coherent whole.

At the University of Washington, Dr. Chaloupka is Emeritus Professor of Physics, originally specializing in the Experimental Particle Physics, and lately in the Foundations of Quantum Mechanics. For many years, he has also taught Physics of Music for the UW School of Music, and Science and Society for the Henry M. Jackson School of International Studies. He lives in a cabin on a forest glade 25 miles east of Seattle, where he can play organ at night without disturbing any (human) neighbors.


An organ recital at Duvall. The deer is playing with the cats, but at least the bear pays attention (or maybe he thinks that the organist might be a good dinner).

This book is for anyone curious about the Universe and about the affairs of human beings who play such a fascinating role - both breathtaking and negligible, compared to the scales of time and space. Its discussions of Modern Science should be accessible to readers who never took High School physics, yet interesting to scientists after spending the day in their offices and laboratories. An investigation of why Bachs last fugue is unfinished may, conceivably, intrigue a Professor of Music, while entertaining someone not trained in deciphering the intricacies of sheet music. And I hope that the difficulties and the urgency of a deep rethinking of some very fundamental aspects of how we arrange our affairs will be appreciated by some readers.

Writing for such a diverse audience is admittedly a tall order for a writer. The author, being a (recovering) experimental physicist, has conducted many empirical tests of his ability to accomplish all this, by teaching University courses for mixed audiences of science- and non-science majors, and he hopes this is reflected in the book you have in front of you.

Status of the Manuscript

The outline and the first chapter are included below. The individual fugues from Bachs *Kunst der Fuge* serve as a connecting thread[1] between discussions of issues of science, culture and society. The rest of the book is in various stages of completion; I am now looking for a publisher. This, I hope, will force me to finally finish the project. Any comments and suggestions will be much appreciated (to vladi@uw.edu).

[1]Recently, I decided to broaden the Interludes to include other musical topics in addition to the KdF fugues. This is not yet reflected in the Outline below, but some of the future Interludes are drafted under the Music chapter.

OUTLINE OF THE BOOK

Prelude
Outlining the background of the author and the motivation for the book

Bach's Kunst der Fuge
Elementary Introduction to Counterpoint

Physics is Different.
Speakable and Unspeakable in Quantum Mechanics

Contrapunctus I
J.S.Bach as an Amadeus phenomenon

Molecular Biology is Different, too.
but differently different: the awesome, overwhelming complexity of the Cell, and the promises, and the dangers ...

Contrapunctus II
Improvisation, creativity and (im)perfection.

Listening to the Gravitational Symphony of the Universe
From Pythagoras to the Laser Interferometer Space Antenna

Contrapunctus III
Bach in the melancholy mood.

Bach, Beethoven, Goethe, Einstein and Hitler
Reflections on science, culture, and on the nature of evil.

Contrapunctus IV
Moveable feast in four voices

How did we do with Nuclear Physics, and what did we learn?
Condemned to repeat the history but the stakes are much higher

Contrapunctus V
A prayer in the form of a fugue

Intelligent Design, and Climate Change: who are the Dodos?
Scientists as educators, advocates and activists: failing?
Contrapunctus VI
Sun beams shining through a redwood grove.

From the Big Bang to Grand Cosmic Recycling
Insights into the Human Condition from Science.

Contrapunctus VII
Quiet, yet the most complex.

Teaching PHIS216
Science and non-science students in the same classroom, as a microcosm of Society

Contrapunctus VIII
Complex minimalism: triple fugue for three voices

Grand Tour of the Universe and Human Affairs
The potent mix of the modern and ancient worlds.

Contrapunctus IX
The strange case of Glenn Gould.

INTERMISSION (before delving into the really serious stuff)

Contrapunctus XI
for ears used to Hindemith and Schonberg

The Basic Problem and the Big Gap
The upcoming phase transition in society, with the inevitability of a law of nature.

Contrapunctus XII Rectus
The case of Sir Donald Francis Tovey

What Is To Be Done?
not according to my infamous namesake

Contrapunctus XIII Rectus
How I almost missed this fugue.

Choreographers of Nature
An End of Darwin, for better or worse, is contemplated

Contrapunctus X
The many faces of Bach.

Freedom of Will in the Quantum Universe.
What I learned in life, and what I did not.

Contrapunctus XIII Inversus
Beauty of imperfect reflections

What About God?
On Science, and Religion vs. Spirituality.

Contrapunctus XII Inversus
Kunst der Fuge completed, but not everyone agrees.

World View inspired by Science and Music
or (modestly): The Meaning of It All

Contrapunctus XIV
Bach meditates on the inner wisdom of the Heaven and Earth

Postlude: Summing Up
with Exuberance and Humility

Contents

Chapter 1

PRELUDE

Outlining the background of the author and the motivation for the book

Shortly after I arrived at the Stanford Linear Accelerator Center in 1975, with a fresh PhD in Physics and somewhat broken English, I was invited to give a lecture at Caltech. The subject of my talk was the last experiment I worked on in Europe before coming to America. The experiment was somewhat unusual - we investigated the annihilations of antiprotons of very low energies in a bubble chamber. In order to get such slow antiprotons into the chamber, we had to use a superconducting tube which freezes out the magnetic field which would otherwise deflect the beam of particles.

So I was looking forward to giving the talk. The only thing which worried me was Richard Feynman. I had heard that - as wise and as kind as he was in general – he was capable of utterly demolishing a speaker. My only hope was that this being an experimental talk and Feynman a Nobel prize winning theorist, he simply wouldnt be there. My heart sank very low when I entered the room. Feynman was already there, in the first row.

Somehow I managed to get started, and to my growing astonishment and eventually delight, Feynman seemed to be on his best behavior. He was genuinely interested in the subject, his questions were friendly, and after the talk, after everyone else was gone, he spent nearly an hour with me at the blackboard, exploring the implications of the experiment.

Needless to say, I was overwhelmed. This was my first seminar at Caltech, and I was well treated by the great Feynman! I cannot deny that my ego kept me awake for most of the night. I, Vladimir – having come from a small Slavic country, with an unfortunate first name (*"Oh nice meeting you, Vladimir, how are things in Russia"? "Excuse me, but I am not a Russian - on the contrary, I am a Czech!"* – note that this was shortly after 1968 and I had left the country because of the Soviet-led invasion) – I have made it in America!

Upon return to Stanford, I mentioned the story to an older colleague. And he told me that as far as he knew, Feynman was mercilessly critical only to people at or near his own level; to all others he was very kind, provided they were not conceited fools.

I only discovered the full importance of the concepts of exuberance and humility rather late in life, but it may be that this episode played a role in my development from an awe-struck experimental physicist to an old College Professor – reading, writing, researching and teaching on topics from Quantum Computing to Bachs Art of Fugue, on the issues of exuberance and humility, and on the value of doubt.

1.1 A Worldview inspired by Science

Unfortunately, one can only imagine what Richard Feynman would have thought about the recent article in the NYTimes about Taking Science on Faith". Paul Davies, Professor of Physics and recipient of the 1995 Templeton prize, argues that *science has its own faith-based belief system* and that *until science comes up with a testable theory of the laws of the universe, its claim to be free of faith is manifestly bogus."*

Fortunately, Feynman left us an extensive record of writings and lecture transcripts dealing not just with physics, but with the "philosophical" questions as well. There will be much discussion of his views in this book, and this is just a Prelude, so here is a quote from another thoughtful physicist who disposes of Davies' claim in one sentence. In his "Dreams of a Final Theory", Steven Weinberg (Physics Nobel prize 1979) writes:

"the only way that any sort of science can proceed is to assume that there is no divine intervention and to see how far one can get with this assumption."

And indeed, scientists dont have to believe that nature is knowable - they just assume it is, and are prepared to give up if it no longer works. This does not seem anywhere in sight, but if it does happen one day, or rather - if we find the going more and more difficult, with ever diminishing returns – we will simply and humbly say: we seem to have reached our limits.

In essence, science is a very pragmatic, almost opportunistic enterprise, and it might appear surprising that it may serve as an inspiration for such a lofty concept as a persons worldview. In fact, most practicing, productive scientists are much too busy to think about such things. But in those who do find the time, science can produce a powerful impact, ranging from Weinbergs (in)famous (and misunderstood) view that *"the more the universe seems comprehensible, the more it seems pointless"*, through the hubris (and more) of James Watson, all the way to Carl Sagan rhapsodizing:

"In some respects, science has far surpassed religion in delivering awe. How is it that hardly any major religion has looked at science and concluded: "This is better than what we thought! The Universe is much bigger that our prophets said, grander, more subtle, more elegant. God must be even greater than we dreamed."

As we shall see, the intersection of modern science and technology with society, with its ancient wisdom and follies, grievances and hopes, makes for a potent mix.

1.2 Life experience

Some Notes from my Life Experience for what it may be worth

This book is a very personal account of just about everything I know and believe, so perhaps it is not inappropriate to include a few personal recollections.

I find it hard to contemplate that I was born in what is today the Czech Republic when it was under Nazi German occupation (it was called "Protectorat" then). I dont directly remember any of this, as I was only 2 years old when the liberation from the Germans was followed by the Soviet-inspired "dictatorship of the proletariat". When I was growing up in Prague, the family would gather around the radio (TV was an unknown concept then) and listen to Sundays broadcast of fairy tales for children which were of an artistic quality I have never encountered subsequently, anywhere. This was of course compensated by the crude Communist propaganda in the news segment which followed the "real" fairy tales

Even in a Communist country you cannot stop progress. Eventually, TV appeared, with similarly superb early evening program for children, followed by similarly crude propaganda in the Evening News. Later I became a student at the Charles University: it is a venerable institution, founded in 1348 by Charles IV, then Holy Roman Emperor and King of Bohemia. The wisdom of the centuries was hardly compatible with the requirement that to graduate as a physicist, I had to pass an exam from the theory of "Scientific Communism". Shortly before my graduation in 1965, we received our first mainframe computer, made in the Soviet Union. It came without any software whatsoever - not even with symbolic addressing. To this day I remember that the codes for addition, subtraction, multiplication and division were 11, 21, 31 and 41. The machine filled a large hall with many heavy cabinets of vacuum tubes and relays, with a performance which was a fraction of todays pocket calculator (and it was a state secret, too - once I was prevented by the secret police from taking my night shift at the computer, because the day before I was careless in my criticism of the Party).

Speaking of life with (or under) the Party: when I reflect on my youth, I sometimes miss the simplicity of it. The old joke is actually true: everything which was not explicitly allowed was forbidden. Still: when you broke the rules and spoke out, it did have an impact. Today, both in the United States as well as in the Czech Republic, anyone can say anything he or she wants, but the impact is necessarily diluted and therefore limited. In the not-so-distant past, every word of Alexander Solzhenitsyn or Vaclav Havel was clandestinely copied in Samizdat and passed around. Today, they can say, and publish, whatever they want, and they do – and their names draw blank stares, at best.

I am not an Alexander Solzhenitsyn or a Vaclav Havel. I never had the "privilege" which my former compatriot Havel shared with Nelson Mandela - to be imprisoned by the "old regime (and they both eventually became Presidents of the new regime). And yet: perhaps just one anecdote from my own experience will illustrate my point about the impact of a free speech in a society where there was none.

In 1964 I was President of the Physics Department student Discussion Club. Nationally, the writers, artists and scientists begun writing and talking more and more freely – the

Prague Spring was already getting under way – and in our Club we started to discuss the need for other political parties besides The Party. The word got around, and the authorities decided to organize a town meeting, with the participation of well known communist apparatchiks. The goal was to explain to the students how mistaken such ideas are. At the key point of the meeting, the editor of the main Communist newspaper took the floor and said: *"Comrades, let me cut to the chase. Things are in fact very simple. We have one Party, the Communist Party, and the Party leads our people one way. And now comes here comrade Chaloupka, and he wants to have another party which would lead our people another way. So will it be this way or that way? That would be terrible."* I summoned all the courage I could muster, I stood up and said: *"Well, comrades, maybe that would be really bad, but I can imagine something even worse. Imagine having only one Party, leading our people one way, and that is the wrong way."*

That was the end of the meeting. The hall erupted with students laughing, cheering, stamping the floor. The apparatchiks left, and the next day I was saved from expulsion from the University by - of all people - the chair of the Department of Marxism-Leninism. Remember – Prague Spring was already in the air, to come to full bloom in only four more years, and many of our Professors of Marxism were to lead significant roles in that precocious version of Gorbachevs perestroika and glasnost which came to the Soviet Union twenty years later [1]. What came to *us* in 1968 were of course Soviet soldiers and this was why and when I decided to leave.

And now, after intermediate stops at CERN[2] in Geneva, and at the Stanford Linear Accelerator Center in California, I ended up as Professor of Physics at the University of Washington in Seattle, with occasional dabbling in Music and in International Studies. As improbable as this collection of professional and semi-professional activities may seem, it does reflect my lifelong interests which I have been pursuing in my research and teaching, and I believe I have achieved some degree of synthesis of these topics into one coherent whole.

1.3 The book structure

The book has the form of Bachs Art of Fugue, Well Tempered Clavier or Goldberg Variations or Chopins Preludes or Etudes: a collection of heterogeneous pieces which nevertheless possess (or so I hope) an inner integrity and a common thread which make it into the "coherent whole" mentioned above. The overall attitude of the book reflects my lectures at UW and elsewhere with topics ranging from "Science, and Music, with Exuberance and

[1] At that time – in 1988 – Gorbachev did not have a Brezhnev above him as Dubcek did in 1968, so the perestroika was not snuffed out by an intervention, and the results were dramatic. However, the optimistic expectations now seem seriously downgraded by the attitude of Mr. Vladimir Vladimirovitch Putin ... – see the Brief Tour section in the book.

[2] CERN is a (French) acronym for European Center for Nuclear Research. It is a sprawling complex of accelerators and research buildings located across the Swiss-French border. Its fictional Director plays a major role in Dan Browns Angels and Demons (the description of CERN is wildly exaggerated, but I think it is still a much better book that the Da Vinci Code.)

Humility" to "What Is To Be Done About the Basic Problem: Challenge for International Studies" and "From Bach to Einstein and Beyond".

The chapters on music are interspersed with chapters on science and human affairs through the whole book. Many of the musical interludes[3] deal with one or more of the fugues from the Art of Fugue, often with additional musico-logical[4] observations which the kind reader might find interesting.

Two potential trouble spots should perhaps be mentioned, if only to assure the reader that the author is aware of them:

First, when writing about science, a major difficulty is to avoid over-explaining things that really are quite easy, while under-explaining concepts that are truly difficult. Many books on science for popular audiences go to great lengths to avoid any mathematical formulas - even the simplest arithmetics. Some authors seem to believe that the effort to learn the "scientific notation" (e.g. age of the Universe is 4.10^{26} nanoseconds) is just too much to ask for, and use instead the "plain" English: age of the Universe is four hundred million billion billion nanoseconds. And yet, in the same book you find sentences which many *scientists* might find difficult to parse and understand.

The approach chosen here is different. It is my experience that even for a student with no mathematics background at all, it is well worth his or her effort to really understand a few simple symbols and equations. Chapter XX describes student reaction to such an experiment, and the very first chapter guides the reader through a real understanding of the famous $E = mc^2$, yielding – after a few easy steps - some rather dramatic insights. Along the same lines, Appendix B introduces the scientific notation and invites the reader to appreciate its full power, and Appendix F reminds the reader (or introduces for the first time) the simplest properties of the complex numbers necessary to truly understand the Mandelbrot set (and Quantum Mechanics!).

I fully appreciate the risk of this approach – it has been said that a single equation in a book reduces the readership to one half. But my goal is not to break the sales records – perhaps quixotically, I imagine an audience of curious readers not afraid of expending some real effort to learn things which might be worth the effort.

The second potential difficulty is to write a book with fourteen chapters on music - and not just any music, but the most difficult and intricate counterpoint imaginable. In order to enable everyone to follow the music as it is being played, the "physicists piano roll music notation" is introduced in Appendix XX - it only takes a few minutes to learn how to use it (instead of the years it might take to learn reading the regular sheet music, with its staffs, clefs, sharps and flats). Most importantly, Bachs music is taken as a paradigm of beauty, complexity and creativity, and the discussion is general enough to allow the readers to substitute their own music or even other forms of art.

[3] see footnote 1

[4] The hyphen in "musico-logical" is one of the many priceless gems to be found in Hofstadters Goedel Escher Bach. In many ways, that book has been an inspiration for much of my thinking and writing.

In general: even more than in lectures, a book treating a broad variety of topics faces a problem: how to treat any particular issue without boring the expert while losing everyone else. Much effort went into dealing with this problem. Many technical aspects about science and music are dealt with in one of the Appendices. To overcome the "academic" tendency of using too many footnotes, only those thought to be useful during the first reading are contained within the text itself. Many other references and notes (often more substantial and/or longer, to be consulted at second reading - if any) are then collected in a separate Appendix. And all of this has been extensively tested on captive audience (my students) as well as with the help of genuine volunteers (see the Acknowledgements).

1.4 Notes on Science

My professional research background has been Experimental Particle Physics (antiprotons, quarks, neutrinos and such). As experiments have become so large that a single experimental group can (or rather must) have several hundreds or even several thousands of participating scientists, with spokespersons, committees and subcommittees, I gradually switched my activity to Physics of Music. This (not my pipe organ playing) also led to my adjunct[5] appointment at the School of Music.

In my physics teaching, I have been fortunate to teach Quantum Mechanics, at the undergraduate as well as the graduate level, as well as Physics of Music and Musical Acoustics. Recently I have been teaching a new graduate course on Quantum Information and Quantum Computing - a fascinating field in which physicists took the old puzzles of Quantum Mechanics that Einstein and Bohr were so fond of discussing, and transformed those mysteries into a resource! We will have a chance to discuss this remarkable development later in the book.

For my interdisciplinary courses on Science and Society I had to acquire a fair knowledge of Molecular Biology. It was a lesson in humility to go "back to school", and the result was a mixture of exuberance and awe which should be more than obvious through this book. It seems to me that the professional biologists are often somewhat jaded or blase, and it may take an "amateur" or a "dilettante"[6] from a different discipline to feel the extreme wonder

[5] At a research university, the adjunct appointment has a meaning different from that in, say, a community college. The pay is not just low, as is a general complaint elsewhere - at a university, the pay is zero. The professor is supposed to be paid by his or her primary department (Physics, in my case) . The Adjunct appointment recognizes an expertise in another field, and entitles the professor to teach appropriate courses and supervise the research of students in that field (including graduate studies leading to a doctorate). And the professors accept such an appointment partly from idealism, and partly because it is an open secret that most research work is done by good graduate or even undergraduate students, driven – at least initially – by professor's ideas.

[6] Good dictionaries still carry the original definition of a "dilettante" as "a lover of the fine arts"; a "connoisseur", and also point out that the word "amateur" originated from "amare" - to love. So to say about someone that he or she is a dilettante or an amateur is really a compliment (but this is generally not known, so be careful before using it in this way).

of all those Marvelous Molecular Machines. Similar attitude seems to exist among physicists towards Physics, too - but I believe I have been quite resistant to such a “professional fatigue”, and able to retain much of the childlike wonder when contemplating the mysteries of Relativity, Quantum Physics or Cosmology.

In general, I must admit to having neglected some of the technical, professional physics research lately, and spending instead much time reading and thinking about topics ranging from abstract mathematical logic to the issues of the origin of life and future of the human species. I humbly see myself as going in the footsteps of Carl Sagan (and, with even more humility, as attempting to guess what would Albert Einstein say today) with their reverence for the awe science can inspire when properly understood and taught.

1.5 Human Affairs

Notes on International Studies, and Human Affairs in general.

In a singular epiphany-like event in 1999, I realized (in parallel with and independently of Bill Joy) that, for the first time in human history, the capability of causing extreme harm is, or will soon be, in the hands of individuals or small groups. A great, irreversible process of “democratization of science” is under way: microbiology experiments such as the Polymerase Chain Reaction (Nobel prize in 1993) are now available as high school science kits, and the speed is quickening, exponentially. Todays “hackers” tinker with computer codes, creating annoying worms and viruses. Tomorrows hackers are bound to tinker with the DNA.

We will discuss these issues in great detail in this book - for now it should suffice to imagine the consequences of an accidental release of the newly reconstructed 1918 Spanish Flu virus, or an intentional release of a strain of smallpox engineered to be resistant to the existing vaccine.

This is what I call the “Basic Problem”. The actual manifestation of the problem will come as an intentional or accidental misuse of our new powers.

And the Basic Problem is a reflection of the “Big Gap”: the ever-increasing gap between the cumulative, exponential progress in science and technology on the one hand, and on the other hand, the lack of comparable progress in our ability to use our new technological tools thoughtfully and responsibly. To illustrate the ever-increasing gap I am talking about, just think of Oedipus by Sophocles, Medea by Euripides, or other ancient Greek comedies and tragedies, written more then two thousand years ago. Since then we have made mind-boggling progress in science and technology, but very little progress, if any, on the problems of love and hate, loyalty and betrayal, wisdom and foolishness they dealt with then.

All of this would be difficult enough in an ideal world, but we are not living in an ideal world. There is a multitude of aspects of human affairs that greatly aggravates the Basic Problem:

[ ] finiteness of natural resources, with oil as an obvious example, and water as an even more important one

[ ] the prospect of a significant climate change, possibly resulting in a large-scale demographic upheavals

[ ] continuing extreme inequalities between the "first World" and the "third World" countries

[ ] continuing ideological clashes, illustrated by (but not limited to) the conflict between Western secular democracy and the militant, fundamentalist form of Islam

[ ]

The extreme diversity of humans is a great blessing for our species, but also perhaps the most intractable problem. Every human being is unique, precious and valuable, but the variation along any one of the many dimensions of talents and moral attitudes is very large. Up to now, brilliant but careless, or psychologically unbalanced individuals could only cause a limited amount of harm. This is no longer the case. In addition, the specific problems listed above have the potential to aggravate countless individuals, newly empowered by the democratization of science, and motivate them to address their grievances by far more efficient ways then using explosive belts attached to their bodies.

The Basic Problem is fundamental, urgent and very much non-partisan - the ability (or lack of it) to think about the long term consequences of our actions and inactions is more or less evenly distributed across the political spectrum. This realization resulted in a transformation of my long-standing interest in international affairs into a desire to be able to contribute professionally. I will argue that a substantial difficulty is the need for a rethinking of some rather fundamental concepts such as national sovereignty. In fact, I will argue what may need rethinking are the very principles of the Enlightenment. The question is whether society is ready for this (and we will see that, from this point of view, 9/11 came at the worst possible time).

You may think that a serious, intense attention is being paid to these issues. In fact, most of our "hard" scientists are much too busy within the competitive environments of their laboratories to worry about any of this. And many social scientists are finding topics such as "Disciplinary Pluralism in Science Studies" or "The Postmodern Critique of Science" much more worthy of their scholarship. We will talk about this in detail, so for now just a note to put this in a quantitative context. The multimillion-dollar Human Genome project has set aside about 3% of their budget to study the Ethical, Legal and Social Issues (ELSI). Not only is 3% quite a small fraction, but it is not difficult to imagine in which of the E, L and S directions most of that fraction actually went.

In general, the connection of the Basic Problem to Science, Technology and International Affairs is obvious. As we shall see, the connection to Music comes from the need to reconcile the two somewhat incompatible attitudes of Exuberance and Humility, where music provides a precious guide, as well as a way to lighten up the otherwise grave and somewhat depressing considerations.

1.6 Religion and Science

An issue of a particular interest to me, with a close relationship to the three main subjects of this book, is the issue of Science and Religion. I share with Steven Weinberg ("Dreams of a Final Theory") his need for a thoughtful discussion of this difficult issue, but not his resolutely atheistic conclusion about the pointlessness of the Universe (alas, another thing I do not share with him is his Nobel Prize for Physics). I also do not feel the need for a militant atheism that seems to motivate Richard Dawkins and other "new atheists". On the other hand, I do not subscribe to the apologetical attempts of scientists like Gerald Schroeder ("The Hidden Face of God"), Kenneth Miller ("Searching for Darwins God") or Francis Collins ("The Language of God") to show that existing religion(s) are in fact consistent with modern science. My own position will be discussed at length - and with reference to Music, of course. I will argue that we can make a substantial progress by abandoning the old-fashioned dichotomy of materialism vs. spirituality and replacing it a view inspired by modern science (and especially by modern physics, cosmology and biology).

A specific subcategory of Science and Religion treated in the book is the issue of Intelligent Design, and again, my position seems to fall outside of existing camps. On the one hand, I argue (and I am tempted to say "of course, as a scientist") against granting the proponents of ID the status of a scientific theory, to be considered as a possible alternative to the "Standard Model of Biology". On the other hand, I do not share the irritation which most of the mainstream scientists seem to feel at the mere mention of Intelligent Design.

1.7 Music

I often say that music is an excellent playground for a physicist, in more ways than one. And I must quickly admit to a very definite bias: for me, the music of Johann Sebastian Bach represents one of the pinnacles of Civilization, and I believe that the very tip of that pinnacle consists of the 14 fugues of his Kunst der Fuge (Art of Fugue). As I have some rather strong views on this magnificent work, and as many of my views stem from my interpretation, there is a CD enclosed with the book, with my recording of the complete cycle on the pipe organ. (I will explain why the Art of Fugue should be performed on the organ, but it should not sound "organy"). There is also an accompanying web site featuring graphical and audio illustrations of various aspects of counterpoint in general, and of Kunst der Fuge in particular. The text attempts to debunk a number of Bach (and Einstein) myths[7] , explains my "Bach as Amadeus" concept, and introduces other musico-logical discussions.

An important part of the book is an exposition of my theory about the Unfinished Fugue which is – to my knowledge - novel. My hypothesis takes into account all the known "facts" (especially the issue of the faulty music paper Bach used for the last page), but the main clue comes from an analysis of Bachs religious, ethical and artistic views and attitudes, and

[7]Typical Bach myth is the claim - in the words of an eminent scholar - that Bach was a "Learned Musician". A typical Einstein story would make you believe that his violin playing was mediocre.

- above all - from the music itself when properly understood and interpreted. Of course, as in the case of my previous, similar remark about science, it will be up to the reader to judge if my understanding and interpretation is the "proper" one.

And a book on Bach and counterpoint cannot fail to include a discussion of Glenn Gould - his opinionated writings and pronouncements, his mannerism and exaggerations, but above all his permanent, immortal contributions to the musical treasure box of our Civilization.

1.8 Physics of Music

I have been researching topics in Musical Acoustics, and teaching PHYS207 (Physics of Music) since I came to the UW in 1981 (alternating with a Quantum Physics class, and a course on Science and Society - I have been very fortunate in teaching assignments). At the beginning, PHYS207 covered just the "regular" Physics of Music topics - vibrations, properties of sound, various music instruments etc.

Gradually, as I was becoming older (and, perhaps, wiser) the syllabus broadened to include interdisciplinary topics such as issues of intersections between "nature" and "nurture" (for example: "why does an octave have frequency ratio of 2:1?" versus "why is an octave divided (in Western music) into 12 semitones?"). Other burning issues include "How important[8] is Sound for Music?" or "What is the role of Imperfections in creating a Perception of Perfections?". We also talk about the general role of Music in society (which goes well beyond entertainment!) and we do engage in modest but meaningful Music Appreciation activities (culminating with a field trip to a pipe organ, where my captive audience is exposed to all the fugal counterpoint they ever wanted to hear (and more :-)

This year, I am changing the course title from "Physics of Music" to a more appropriate one: "Physics and Music" (or even maybe "Science and Music"). With a transition to a partial retirement, where I will be teaching just one course per year, I have chosen, not without some agony, to sacrifice teaching Quantum Physics and Science and Society, and I will teach a "Science, Music and Society" course. Perhaps the sacrifice is not too bad - even Quantum Physics has a close connection to Physics of Music, as a good part of Quantum Wave Mechanics can be illustrated using the classical concepts of vibrations and sound wave propagation.

1.9 Teaching

Some time ago, I was invited to give a talk at the annual Bach festival at Lake Chelan in the Eastern Washington desert (yes: a desert. The moist Seattle climate changes dramatically when you cross the Cascade mountains). The audience was a mix of vacationing Seattleites,

[8]When I asked this question to a Music doctoral candidate at his examination, not only the candidate but also my colleagues on the committee thought this guy from Physics is really strange. So I re-phrased the question as "Is there more to Music than Sound?" and we had a lively discussion.

with some local winemakers and apple growers. I pulled out all the stops, and talked about Bach, Goedels Undecidability Theorem, Quantum mechanics – this whole book in one hour

At the reception after my talk, an elderly lady came up and said:

"Teacher,"

Wow! I have been called Professor, Dr. Chaloupka[9] ", "Vladimir", or "Dr. Vladimir" (mostly by my Japanese students), but nobody ever called me "Teacher" before. It sounded almost biblical.

But then the lady continued:

"I have not understood a word of what you were talking about".

Needless to say, the biblical feeling instantly evaporated. But then she completed the sentence:

"I have not understood a word of what you were talking about, but I felt that it was about something great."

That, I must say, was one of the high points in my public speaking / teaching career. That lady may not have understood the subtleties of quantum-mechanical non-locality, or of the intricacies of the unequal temperament, yet she took away more than a scientist who understood all the technical details, but who left the lecture puzzled what it was all about.

All teaching involves various proportions of instruction and education, and the emphasis depends on the subject, and on the audience. When I teach Quantum Mechanics, my main goal is to introduce students to the marvelous microworld and its strange rules which we - somewhat inexplicably - seem to be able to use, if not fully understand. So the process is mostly instruction, with me trying to make students able to calculate what I can calculate. And I believe it is about this process that Gibbon said: *"The power of instruction is seldom of much efficacy, except in those happy dispositions where it is almost superfluous."* Instructors should always try to teach as well as possible, but even the best come to grief when they forget about Gibbons observation.

However, when talking about interpretation of science, and about the worldview which science represents for me, the balance changes in favor of education. When I teach a course about Science and Music, or Science and Society, I fancy myself, rightly or wrongly, as transmitting all the wisdom which I may have accumulated throughout my life, all the knowledge but also all the doubts, interconnection and complexities. I have devoted a whole chapter of this book to "Teaching PHIS216": a course on Science and Society for a highly diverse group of science- and non-science students meeting in the same classroom. For now,

[9] Once as I was boarding a plane on a business trip, the stewardess noticed the "Dr." my secretary puts on the ticket, and asked me: "Listen, are you a real doctor, or just an MD?" I laughed and said "I am just a PhD - thats how most people see it". And she replied: "Oh no, I used to date a PhD, and he explained the proper significance of these titles to me."

Later in this book, I will have an opportunity to describe a couple of occasions when I felt like a "real doctor".

may I just quote - without implying any efficacy of the quixotic process of “teaching wisdom” - a couple of quotes from the students response papers:

“Even if I hadnt enjoyed the rest of class, having my fear of death allayed to some degree would have been worth taking it.”

“I found this class and the grand finale to be absolutely beautiful and [it] gave me inner peace.”

It is perhaps not a common result of a physics class to give a student inner peace, or to allay another students fear of death. In a sense, this book is a PHIS216 course expanded from one college term to several hundred pages. What reaction it will produce and what fears it will allay (or cause) is a question that can only be answered by completing and publishing the manuscript.

1.10 The goal of the book

Coming back to the unifying thread of the book: I will argue that Bachs Art of Fugue is in fact a triple art:

art of writing a fugue

art of playing a fugue

and an art of listening to a fugue.

It is my goal and my hope that some of my readers will be inspired to cultivate, at least, the art of listening to a fugue, and share the exuberance and humility that science, music and human affairs evoke in me.

Chapter 3

Physics is Different

The physicist who coined the term black hole – John Archibald Wheeler - wrote some time ago:

Recent decades have taught us that physics is a magic window. It shows us the illusion that lies behind reality - and the reality that lies behind illusion. Its scope is immensely greater than we once realized. We are no longer satisfied with insights only into particles, or fields of force, or geometry, or even space and time. Today we demand of physics some understanding of existence itself.

If this quote seems too modest, here is another, from Charles Misner:

The organist chemist, in answer to the question,"Why are there ninety-two elements, and when were they produced?" may say "The man in the next office knows that". But the physicist, being asked, "Why is the universe built to follow certain physical laws and not others?" may well reply "God knows that".

And what about this from a well-known physicist who shall remain anonymous:

"Fundamental physicists are titillated by the thought that perhaps only one more step separates them from the Ultimate Design. ... .. we are beginning to feel that we are on the threshold of really knowing His thoughts."

In this chapter, we will take a brief tour of some rather elementary physics, as a beginning of our effort to find out what is the worldview that science represents and nurtures. Our goal will be to find out which - if any - of the above quotes reflect the true nature of modern physics.

We will begin by describing, in some detail, four cases where everyone - independent of their physics background - will be able to proceed from the simplest beginnings, through simplest possible steps, to some of the most profound views of Nature modern physics provides. In fact, I humbly recommend even to physicists not to skip this part – they may find it interesting how one of their colleagues teaches Einstein, Feynman and Heisenberg to a very diverse enrollments of students (there is a whole chapter later in the book on this topic.)

3.1 The Central Mystery of Quantum Mechanics

We met Richard Feynman already in the Prelude, and we will come back to him quite often in this book. Many Nobel Prize winners are quite narrow, highly specialized, and downright ordinary or even boring and silly outside their discipline. Feynman was a true polymath, and more: in addition to his work on theoretical particle physics (which brought him the prize) he was - with a single article - at the origin of the field of Nanotechnology. With another article, he foresaw the field of Quantum Computing long before others came to realize that the mysteries of Quantum Physics could be exploited for computations. He was also a talented artist, musician and story teller. He was a show-off, but he was also, in his own way, a rather wise and profound philosopher (due to his rather strong accent, some called him "philosopher from Brooklyn".)

Feynmans Lectures on Physics became an instant classic soon after they were published. They were originally written as a transcript of his freshman physics lectures at Caltech in 1963. Soon after I started teaching at the University of Washington, I assigned them as an optional reading, to accompany the standard College textbook we were using. After a few days, a student came to see me, and complained bitterly: "Those Caltech kids are lucky to have such a fascinating book, while we have to suffer through our pedestrian textbook". And I had to tell him that in fact the Lectures are not really used at Caltech for the intended purpose - the level is too high even for the smart students there. But it is not the mathematical level being too advanced. The books represent a window into the mind of a genius. At CERN, I knew a retired physicist, rather well known, who would come every day to his office to read Feynmans Lectures on Physics. He would say: "*now* I can start to really understand Physics".

Well, when Feynman comes to Quantum Mechanics he writes:

Things on a very small scale behave like nothing that you have any direct experience about. They don't behave like waves, they don't behave like particles, ... or like anything you have ever seen. ... Because atomic behavior is so unlike ordinary experience, it is very difficult to get used to and it appears peculiar and mysterious to everyone, both to the novice and to the experienced physicist. Even the experts do not understand it the way they would like to ... In this chapter we shall tackle immediately the basec element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery..

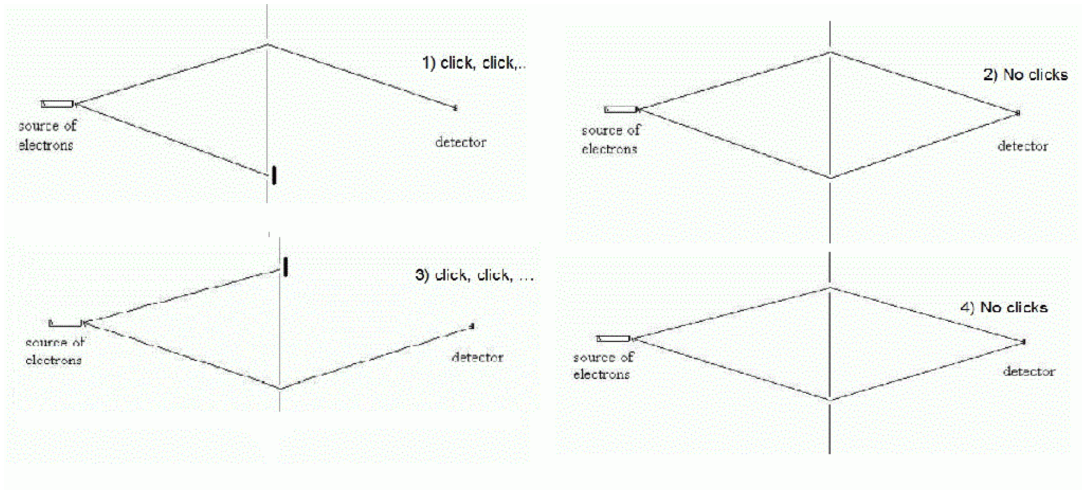


Figure 3.1: 1) there is only one path, and you hear a click for each arriving electron 2) you open additional path and the clicks cease!!! 3) you close the first path and the clicks resume 4) closing it again stops the electrons from coming

So, let us follow Feynman, and imagine an electron gun spraying a wall with two holes (two "slits" in Physics jargon); one hole is presently closed (see Figure). Some distance after the wall there is an electron detector - imagine we have an old-fashioned counter which makes an audible click when it is hit. It is important to place the detector at the right place to experience the effect we will describe. It is not difficult to determine the proper location - more about this later.

The gun is not very good (it is a quantum gun) - the electrons fly, randomly, in every which direction. Obviously, many electrons hit the wall, and that is the end of that electron. But some electrons do make it through the open hole, ricochet up or down, and some will hit the detector: click .. click, .. click,click, .click.

Anticipating the mind-boggling and headache-inducing development to come in a minute, you spend quite some time verifying that everything makes sense. You make sure that each click signals an arrival of a whole electron - that the particle does not split into two. An electron either arrives, whole, at the detector, or it does not. And you measure the timing, and indeed - every click arrives at the right time delay after being emitted by the gun, convincing you that it was indeed emitted by the gun, and went in a straight line to the hole, and from there in a straight line to the detector (see Figure). Also, you make the electron flux so low that there is never more than one electron in the apparatus: an electron is emitted, goes through, gets absorbed at the screen or goes through and is registered by the detector, then you wait for a long time, then you shoot another electron ...

When you are done with all the checks and cross-checks you can think of, you open the second hole, *and the clicks cease*. Silence – no clicks. The electrons stopped coming. Previously, they were coming through the top hole, and you did not do anything to that possibility: you just opened up an *additional* possibility for an electron to go from the gun to the detector, and as a result the electrons stopped arriving.

You cannot believe what happened. You cover the bottom hole - click, clikclick, click. Open it again: silence. Cover the top hole: click click click. Open it: silence.

That is the Central Mystery of Quantum Mechanics. In fact, it is more than that. As the Intelligent Design people like to say about evolution, Quantum mechanics is just a theory - more about this later. But what we have described is not result of any theory - it is an *experimental fact* (and a simple one to describe, if not to understand). So perhaps it should be called Central Mystery of Nature

I always start a course in Quantum Physics with an exposition like this, and students are always fascinated. "Professor", they say, "this is so interesting. We should not proceed with all those equations we saw ahead in the textbook. Instead we should discuss this until we really understand it." But I know better, and I reply: "Yes, yes, this is very interesting indeed, and if there is time at the end of the course, we will come back to this."

Two things invariably happen. First, there is never time at the end of the course. More importantly, by the end of the course students have learned many useful things. They now know how to calculate all kinds of quantum magic, and most of them rarely if ever recall that the original mystery was never resolved.

Now, readers with some previous physics knowledge will point out that in fact there *are* things we do understand about all this. I mentioned the importance of placing the detector in the right position. What happens if we move it a little up or down? It turns out that as soon as we move it, clicks start to happen even when both holes are open. As we keep moving the detector, the frequency of the clicks keeps increasing, and at some position the number of clicks per second for both holes open is twice the number when only one is open. Thats what you would have expected to start with. But if you keep moving the detector, more and more clicks keep coming, until they come *four-times* faster than when a single hole is open. That is the maximum. Additional detector shift results in a gradual decrease in click frequency, until you reach a new position where clicks cease when both holes are open.

So it seems we are dealing here with some kind of a wave phenomenon, with an interference of waves coming through the individual holes. And indeed: in the rest of the course students learn about the psi (ψ) wave-function, about the Schroedinger equation which governs it and how the solution leads to all kinds of extremely precise and useful calculations.

But the quantum wave is something very strange: we know how to use it, but we don't know what it really means. Since the celebrated Einstein-Bohr discussions in the 1920s, hundreds of scholarly (as well as not-so-scholarly) articles and dozens of books have been written about its interpretation. We know that the value of the wave function at some position can be used to determine the probability of finding the particle at that particular position. But as soon as we find it there, the probability of finding the particle anywhere else must instantly become zero - this is the infamous collapse of the wave function which is the first of the many non-local aspects of the Quantum Theory.

Many solutions and "interpretations of Quantum Mechanics" were proposed; some decidedly worse that the disease they were trying to cure. And recently, something exciting happened: the decades old, interminable debates (starting with Einstein vs. Bohr in the 1930s) were replaced by a pragmatic attitude: the paradoxes and mysteries were transformed into a technological resource, and the brand new fields of Quantum Computing and Quantum Information were born. We will have many opportunities to discuss the significance and implications of this development.

3.2 Why is there Something rather than Nothing?

There will be very little of real science in this book, but there *will* be some, and we may as well begin with a spectacular example. We will start with some very simple considerations, and we will see how quickly we get to some rather esoteric heights - I promise that even readers with not even high school physics will progress, by few pages, from what looks like a dreadful, boring physics lecture to something quite remarkable.

Perhaps the most famous equation in whole of Physics is Einsteins $E = mc^2$. The mass of a body is a measure of its inertia, i.e. of the extent to which the body resists attempts to change its state of motion. It also happens to be the measure of the force which the body experiences in gravitational field (incidentally: the equivalence of the inertial and gravitational mass led Einstein to his General Theory of Relativity). And Einsteins equation $E = mc^2$ tells us that the mass is also a measure of the energy contents: mass m, when at rest, has energy $E = mc^2$, and equivalently, rest energy E corresponds to mass $m = E/c^2$.

In various physical, chemical or biological processes, energy is always conserved:

$$total\ energy\ of\ the\ initial\ state = total\ energy\ of\ the\ final\ state$$

A specific example: consider a simple reaction: one atom of carbon joins with a molecule consisting of two atoms of oxygen, and the result is a molecule of carbon dioxide, and energy. In this case, the energy is produced in the form of heat, and the process is the ordinary burning of coal.

$$C + O_2 \Rightarrow CO_2 + energy$$

Numerically, the conservation of energy is, as always:

energy of the initial carbon and oxygen is equal the energy of the resulting product plus the energy released

The law of conservation of energy is very fundamental: in the past, whenever we thought it might be violated, it always turns out we forgot to include a particular form of energy (chemical, nuclear, energy in a spring,).

So far so good, I hope. Now consider a system consisting of two components bound by a mutual attraction: take for example the Moon orbiting around Earth. The question is how does the total mass of the (Earth-Moon) system relate to the sum of the mass of Earth plus the mass of the Moon. The first reaction is likely to be that they are equal: after all, if you load a 1 ton pickup with a half-ton lawn mower, the whole thing will have 1.5 tons, right?

And then you recall I just discussed energy: clearly, the Moon is attracted to Earth (and Earth is attracted to the Moon), there is gravitational field between them, this field has some energy, and this energy, according to $E = mc^2$ corresponds to some mass. So now tell me: is the mass of the (Earth-Moon) system equal, smaller than or greater then the sum of the masses of Earth plus Moon:

$$m_{EM} = m_E + m_M \ ?$$

$$m_{EM} < m_E + m_M \ ?$$

$$m_{EM} > m_E + m_M \ ?$$

The understanding of this question is so important that I suggest you spend at least five minutes pondering it before turning the page.

Well, I tested this on a bunch of Physics majors and graduate students. About three out of four majors and one out of four graduate students quickly reply that the mass of the system is larger than the sum of masses - *"because the binding energy corresponds to some mass, too."* They get it right when they think about it some more, but it shows you that even a simple question like this is far from being trivial.

To see what is the correct response, consider what you have to do if you wish to split the system into its constituents - for example you want to remove the Moon from its orbit and place it somewhere far from Earth (so that it is no longer an Earth-Moon system but an independent Earth here, and an independent Moon somewhere else). Clearly you have to deliver energy to the system to accomplish this - the Moon is bound to Earth by gravitational force, and you have to "convince" the Moon to fly away - say by hitting it with a giant golf club. So for this process, the law of conservation of energy says

$$E_{EM} + \textit{ extra (binding) energy needed to break the system} = E_E + E_M$$

where E_E is "rest energy of the Earth, E_M is the "rest energy of the Moon, and E_{EM} is the "rest energy of the Earth-Moon system. But since the rest energy is $E = mc^2$ in general, this means

$$m_{EM}.c^2 + \textit{binding energy} = m_E.c^2 + m_M.c^2$$

which is the same as

$$m_{EM}.c^2 = m_E.c^2 + m_M.c^2 - \textit{ binding energy}$$

which is the same as

$$m_{EM} = m_E + m_M - \textit{binding energy}/c^2$$

In plain English, the mass of the system (m_{EM}) is *smaller* that the sum of the masses of the constituents ($m_E + m_M$)).

But the speed of light is a very large number, and - for the Earth-Moon system - the binding energy is small, because gravity is so weak. Therefore the effect is very small.

However, you can see that if the binding energy becomes larger, the resulting mass gets smaller. This is certainly counter-intuitive and perhaps interesting, but you are probably not impressed by all this - not yet. But keep thinking along the same direction. We see that the more strongly are two particles bound to each other, the *smaller* is the resulting mass. What if you had binding so strong that the term

$$\textit{binding energy}/c^2$$

became equal to the term

$$m_E + m_M$$

Wow: the two terms cancel, and the result is zero! I hope you agree that this starts to get interesting: you take two particles, each with non-zero mass and - if you bind them strongly enough - you get the composite particle with zero mass!

But you say: this is just two academic particles - who cares about them. So you think some more, and say: wait a minute: the Universe contains many “particles” (some of them quite large, such as planets, stars,) and they are all bound to each other by gravity. As we mentioned, gravity is very weak, but could it be that the overall binding of everyone to everyone is just sufficient to cancel all the masses of all the stars and planet, to produce net mass equal zero and net energy equal zero?

And the answer to this is: detailed calculations show that yes, it is possible. There are many additional factors to consider, but yes, it is possible that the total mass and the total energy of the Universe are zero. But if they are zero, then it is possible to create all the Universe, with all its galaxies, stars and planets, out of nothing - it does not violate the law of conservation of energy.

Every Physics Department – faculty, postdocs, graduate students and ambitious undergraduates – meet once per week for a secular Mass called “Physics Colloquium”. The speakers are instructed to aim at a level where Professors are not bored and students are not lost (or is it the other way around?).

A few years back we had a Colloquium on the subject:

Why is there Something rather than Nothing?

ANd yes, it was a Physics Colloquium. The speaker (our own UW Physics Professor David Kaplan) went through one hour of calculations and reasoning, quite advanced[1] but along the same lines as our simple account, and at the end the answer was:

Maybe there is Nothing, cleverly disguised as Something.

So there you have it. Centuries-old philosophical question gets a new meaning, and every single student in my Science and Society course - English majors and all - is able to follow at least the outline of the reasoning, and share the wonder.

[1] As you may suspect, there are many difficulties: what does it really mean for a Universe to have zero mass? And could something have a negative mass? or an imaginary mass? etc.

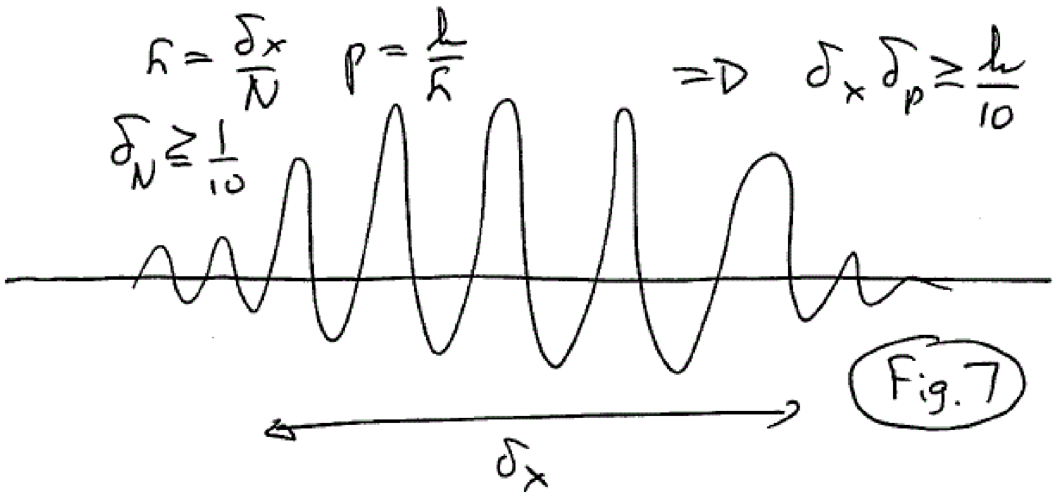


Figure 3.2: Caption for Heisenberg1

3.3 The Heisenberg Uncertainty, and the nature of Vacuum

In this mini-investigation we will start with one of my favorite subfields of physics - acoustics - and we will try to estimate the frequency of a signal depicted on Figure 2. It is a simple tone, starting from silence, and after some finite duration ending in silence again. The most precise way to determine the frequency is to choose some "reasonable" duration, count the number of periods during that time interval, and then to determine the length of one period by a simple division. For several reasons, physicists are very fond of Greek alphabet, and of indexing symbols. I know that this is disconcerting to many, but it is how all Uncertainty Relations are always printed, so you may as well get a glimpse of this notation. Here it is:

the duration of the signal (the "uncertainty in time") will be denoted by δ_t
the uncertainty in the frequency determination will be denoted by δ_f
the uncertainty in the number of periods will be denoted by δ_N

Apart from this "Greek difficulty", everything else is a succession of several easy steps. However, the outcome will be quite non-trivial.

First: if the number of "wiggles" (i.e. number of periods) occurring during δ_t is estimated to be equal N, then the length of one period is simply

$$T = \frac{\delta_t}{N}$$

For the signal on the Figure, I have chosen $\delta_t = 13.7$ milliseconds, and I estimated there to be $N = 6.3$ periods in that duration of time. Therefore my measurement of the period is $T = \frac{\delta_t}{N} = \frac{13.7}{6.3} = 2.2$ milliseconds $= 0.0022$ seconds.

Now, "frequency" expresses how many times the period occurred per second, so if the duration of one period is T, then the frequency is

$$f = \frac{1}{T} = \frac{N}{\delta_t}$$

In our example, the estimate for the frequency is $f = \frac{1}{T} = \frac{1}{0.0022} = 455$ Hz.

What is the *uncertainty* on the estimate of frequency that we have obtained? Well, the value of δ_t was my choice, and I can choose it as precisely as I wish (i.e. I can declare it to be $\delta_t = 13.700000000...$ milliseconds in our case.) However, the estimate of the number of periods N is subject to an unavoidable uncertainty ("margin of error") δ_N, i.e. the answer is $N \pm \delta_N$. The value of δ_N cannot possibly be as large as $\delta_N = \pm 1$ – that would mean missing a whole wiggle, or adding one. On the other hand, it is surely impossible to do as well as $\delta_N = \pm 0.01$. So let us settle on a "reasonable guess" of the "best possible determination" with $\delta_N = \pm 0.1$. In our example, that means that the number of wiggles is known to be equal to 6.3 ± 0.1.

The implication of all this is that the final estimate of frequency has an uncertainty δ_f:

$$f = \frac{N}{\delta_t} \qquad \Rightarrow \qquad \delta_f = \frac{\delta_N}{\delta_t} = \frac{0.1}{\delta_t} = \frac{1}{10\delta_t}$$

In our example, we get $\delta_f = \frac{1}{10\delta_t} = \frac{1}{10 \times 0.0137} = 7.3$ Hz, so that the frequency estimate is $f = 455 \pm 7$ Hz.

Multiplying both sides of the above general equation by δ_t yields the celebrated Heisenberg Uncertainty Relation:

$$\delta_f \delta_t = \frac{1}{10}$$

Had we applied the considerably more powerful apparatus of calculus, we would have obtained the relation in its correct form

$$\delta_f \delta_t = \frac{1}{4\pi}$$

Since $4\pi \sim 12 \sim 10$, our simple considerations produced an essentially correct result!

So far, all this is "only" acoustics, but it has all the aspects of the "real thing": the product of the two uncertainties cannot be arbitrarily small, i.e. if e.g. δ_t is small, then δ_f must be large, and vice versa. When you think about it, it explains why you can trill with a flute but not with a tuba - and there are other, more significant applications, too.

We get to the "real thing" by adding another, simple but ground-breaking equation to the mix. About 100 years ago, Max Planck (reluctantly) came to the conclusion that the electromagnetic radiation in a cavity is quantized in integer multiples of a "quantum of energy" which needs to be $E = hf$, where $h = 6.6 \times 10^{-34}$ (in metric units) is an exceedingly small constant - now obviously called "the Planck constant". Einstein subsequently obtained his Nobel Prize for applying the same equation on the photoelectric effect, and finally a young French doctoral student Louis de Broglie proposed that the same equation applies to everything, not just light: there is a "wave function" ψ associated with everything, and the relationship between the frequency of that wave function and the energy of that "anything" is

$$E = hf$$

Unlike Einstein, Dr.deBroglie never achieved anything much after getting his degree, but this alone was sufficient for him to get to meet the king of Sweden. And his Nobel Prize was well deserved - the wave function was found to obey a particular equation (Dr. Schrodinger $\rightarrow$ trip to Stockholm) and - a few years later - its meaning (at least for the purposes of calculations) was understood (Dr. Born $\rightarrow$ trip to Stockholm).

When you now recall the "acoustical" Heisenberg relation we derived above:

$$\delta_f \delta_t = \frac{1}{4\pi}$$

you should feel an urge to multiply both sides of the equation by the Planck constant h. Using $hf = E$ and therefore $h\delta_f = \delta_E$, you get instantly the real "quantum mechanical Heisenberg"[2]:

$$\delta_E \delta_t = \frac{h}{4\pi}$$

What is more, by a completely similar reasoning you get, for a particle of mass m (restricting ourselves, for simplicity, to particles moving with velocity v much smaller than the speed of light) a relation

$$\delta_v \delta_x = \frac{h}{4\pi m}$$

with an interpretation that it is impossible to assign to a particle both a value of its velocity as well as a value of its position - if the velocity is known, the position is not, and vice versa. This means that the concept of a "trajectory" loses its meaning, to emerge only as an illusion for particles (such as bowling balls, or people, ...) with mass m so large in comparison with the value of the Planck constant h that the right hand side of the last equation is, for all practical purposes, negligible. However, in the atomic and subatomic world, with tiny particles, the trajectories are replaced by a "quantum fuzziness" with which we have no direct experience, as explained by Richard Feynman.

But there is more, much more. The time-energy relation can be interpreted not only as "prohibitive" ("there shall not be a product less that $h/4\pi$") but also as a "permissive": violation of energy by an amount δ_E may be possible if the duration δ_t is short enough so that $\delta_E \delta_t = h/4\pi$. This leads to the current view of vacuum as constantly "boiling" with emission and reabsorption of particle-antiparticle pairs - of all kinds: electron/positron,

[2] Note that all Heisenberg-like uncertainty relations are really inequalities, as the product of the two uncertainties cannot ever be smaller, but it can be larger (the right-hand side represents "the best we can do".)

proton/antiproton/ ... - with the duration of each occurance subject to the Heisenberg time-energy relation. Thus in modern physics, the vacuum, rather than being the simplest of all phenomena, is in fact the most complicated one.

The list of books for “general audience” on this fascinating subject is very large, and most of them contains no equations whatsoever. If you made the effort to understand the simple equations used here, you will be able to actually *understand* the books which would otherwise just grace your coffee table.

3.4 Scientific Notation and a Glimpse at Infinity

In the last (for now) mini-investigation of wonders of physics we will start with a topic that will seem quite boring. If you persevere, you will be rewarded (perhaps rewarded by another headache ...). Besides, we will need these “boring” skills in the rest of the book.

3.4.1 Scientific Notation

In any discussion of Nature, we need to be able to deal with an extremely large range of physical quantities. For some strange reason, most authors of popular books seem convinced that the general public is not capable of comprehending a simple system of powers of 10 known as the “scientific notation”. Even more bizarre is the apparent belief that expressing large numbers in the “ordinary way” such as “1,000,000,000,000,000,000” or “one hundred billion billion billion” provides any useful information at all.

In this book, we use the scientific notation frequently and consistently. Readers who never encountered this reasoning should be prepared to spend some time digesting and mastering this topic: please be assured that it is most definitely worth your while! And even physics majors - who may be bored by the apparent trivialities that begin this section - usually find the end of the section highly non-trivial and interesting.

The whole affair starts very simply indeed:

$$10^2 = 10.10 \qquad 10^3 = 10.10.10 \qquad 10^5 = 10.10.10.10.10$$

and so on. Inversely:

$$10^{-2} = \frac{1}{10.10} \qquad 10^{-3} = \frac{1}{10.10.10} \qquad 10^{-5} = \frac{1}{10.10.10.10.10}$$

It is easy to see that each additional power of 10 means adding a zero, so that the large number 1,000,000,000,000,000,000 in the first paragraph is nothing but simply 10^{18} (just count the zeros).

It is also easy to see that multiplying two powers of ten corresponds to simple addition of the exponents: for example

$$10^2.10^3 = (10.10).(10.10.10) = 10^{2+3} = 10^5$$

Therefore, the second large number in the first paragraph above (one hundred billion billion billion) is equal to $10^2.10^9.10^9.10^9 = 10^{2+9+9+9} = 10^{29}$.

For numbers smaller than 1.0, it is equally easy to see that it is the position of the first non-zero digit after the decimal point which gives the correct (negative) power of 10, so that for example:
$10^{-3} = 0.001 =$ one thousandth,
$10^{-6} = 0.000,001 =$ one millionth and so on.

And finally, any division by a power of 10 corresponds to subtraction of the powers, as in

$$10^6/10^2 = \frac{10.10.10.10.10.10}{10.10} = 10.10.10.10 = 10^4 = 10^{6-2}$$

We can summarize our results so far

$$10^2 10^3 = 10^{2+3} = 10^5 \qquad \frac{10^6}{10^2} = 10^{6-2} = 10^4$$

by two general equations: for an arbitrary choice of the values of m and n we have:

$$10^n 10^m = 10^{n+m} \qquad \frac{10^n}{10^m} = 10^{n-m}$$

Note also that just a moment of reflection will convince you that

$$(10^n)^m = 10^{n.m} \qquad e.g.\ (10^2)^3 = 10^2.10^2.10^2 = 10^{2+2+2=10^{2\times3}} = 10^6$$

As an exercise, you should be able to evaluate

$$\frac{2 \times 10^{12} \times 6 \times 10^{-9}}{3 \times 10^{-7}} =$$

and express in plain English the numbers:

$$\frac{10^4}{2} = \qquad and \qquad \frac{10^{-2}}{2} =$$

Also: most students know that $10^0 = 1$ but few can explain why is it so. Yet, you are now equipped to show this from the simple relations we just introduced.

The solutions of this (and other) “homework” exercises are in Appendix X at the end of the book. However, I advise all readers to spend at least a few minutes on each problem trying to solve it, before turning to the solutions.

3.4.2 Nicknames for some standard exponents

Some powers of 10 have nicknames (see below) but for really large or really small numbers there are no names - just the scientific notation.

Standard names and prefixes:

factor	name	prefix	example
10^{12}	trillion	Tera	TeV (tera-electronVolt)
10^{9}	billion	Giga	GW (gigawatt)
10^{6}	million	Mega	MΩ (megaOhm)
10^{3}	thousand	kilo	kg (kilogram)
10^{-1}	tenth	deci	dl (deci-liter)
10^{-2}	hundredth	centi	cm (centimeter)
10^{-3}	thousandth	milli	ms (millisecond)
10^{-6}	millionth	micro (μ)	μ g (microgram)
10^{-9}	billionth	nano	nm (nanometer)
10^{-10}			Angstrom (A)
10^{-12}	trillionth	pico	ps (picosecond)
10^{-15}		femto	fm (1 Fermi = femtometer)

3.4.3 Note on units

We will consistently use the metric system, with MKSA units of length (m), mass (kg)[sic], time(s) and current (A). Some derived metric units are:

work/energy: Joule (J), power: Joule/s = Watt, force: $kg\, m/s^2 = Newton\ (N)$, pressure: $N/m^2 = Pascal(Pa)$

Note that some dimensionless quantities have been given names, too (decibels, Radians, ...)

3.5 A glimpse at Infinity

The usefulness of the scientific notation follows from the ease with which we can represent the huge ranges of various quantities mentioned above. So for example, we will soon discuss the range of distances from 10^{-34} meters (the "Planck scale") all the way to 10^{26} meters (the distance to the edge of the observable Universe). So if you start from the Planck scale and increase it million times, your get $10^{-34}10^{6} = 10^{-28}$. If you take *this* distance and increase it billion times, you get $10^{-28}10^{9} = 10^{-19}$ - still a ways from 10^{+26}!

There is an anecdote that nicely illustrates the enormous ranges we deal with. An article on cosmology in a scientific journal was followed by an Errata in a subsequent issue, apologizing for an inadvertent error: the main result was off by a factor of *million*[sic]. However, the Errata says, *this does not change any conclusions of our paper.*

And now, to introduce a *really* large number, I should mention that a well known software company is named after a misspelled nickname for 10^{100} which is called a googol: it is a 1 followed by one hundred zeros. This is a number much, much larger than the number of seconds elapsed since the Big bang (which is only about 10^{17}). You see that a number billion times smaller or billion times larger than a googol is 10^{91} or 10^{109} – who cares that it is "not exactly 10^{100}".

Now, if you have a tendency for mischief, you might think: what about 10^{googol} ? That would be $10^{10^{100}}$: a 1 followed by a googol of zeros[3]. As you can read in Wikipedia, this number could not be printed even if all matter in the visible Universe was converted into paper and ink. Curiously, such a monster is important enough to have its own name: it is called a "googolplex", and even more curiously, it is (possibly) useful to think about numbers as large as this, as we will now see.

The part of Universe visible to us today is called a "Hubble volume", and it is some 95 billion light years (or about 10^{27} meters) across[4].

In a serious (it is claimed) scientific paper, a calculation is presented showing that if the Universe is infinite or just very, very large, then we can expect, at a distance of some $10^{10^{115}}$ meters from us, A Hubble volume identical in all details to our Hubble volume. This means that in "that Universe" there is a copy of you reading this book and thinking what you are thinking[5]

Curiously, it is quantum mechanics that allows this calculation to be made - Universe with continuous spectra of physical quantities would be much *more* complicated than our quantum Universe. However, in quantum physics many quantities can only occur at discrete, "quantized" values, and that makes the number of possible combinations finite, although still extremely large. Then an arbitrary combination that is possible, such as for example everything in our Hubble volume, *must* occur somewhere else.

Now, to get a glimpse at infinity, express that distance in terms of the size of "our Universe", i.e. how many of our Hubble volumes we would have to put next to each other

[3]Please note that $10^{10^{100}}$ is defined as $10^{(10^{100})}$ which is different from (much larger than) $(10^{10})^{100} = 10^{10.100} = 10^{1000}$ - the first number has a googol of zeros; the second has "just" 1000 zeros (it is still an obscenely large number)

[4]This is more than twice what light would cover since the Big Bang some 14 billion years ago, because the Universe has been expanding ever since then. Details are quite complicated, but - as you will see - even major corrections would be completely irrelevant.

[5]The article begins like this: *"Is there another copy of you reading this article, deciding to put it aside without finishing this sentence while you are reading on? A person living on a planet called Earth, with misty mountains, fertile fields and sprawling cities, in a solar system with eight other planets. The life of this person has been identical to yours in every respect ...*

You probably find this idea strange and implausible, and I must confess that this is my gut reaction too. Yet it looks like we will just have to live with it, since the simplest and most popular cosmological model today predicts that this person actually exists ..."

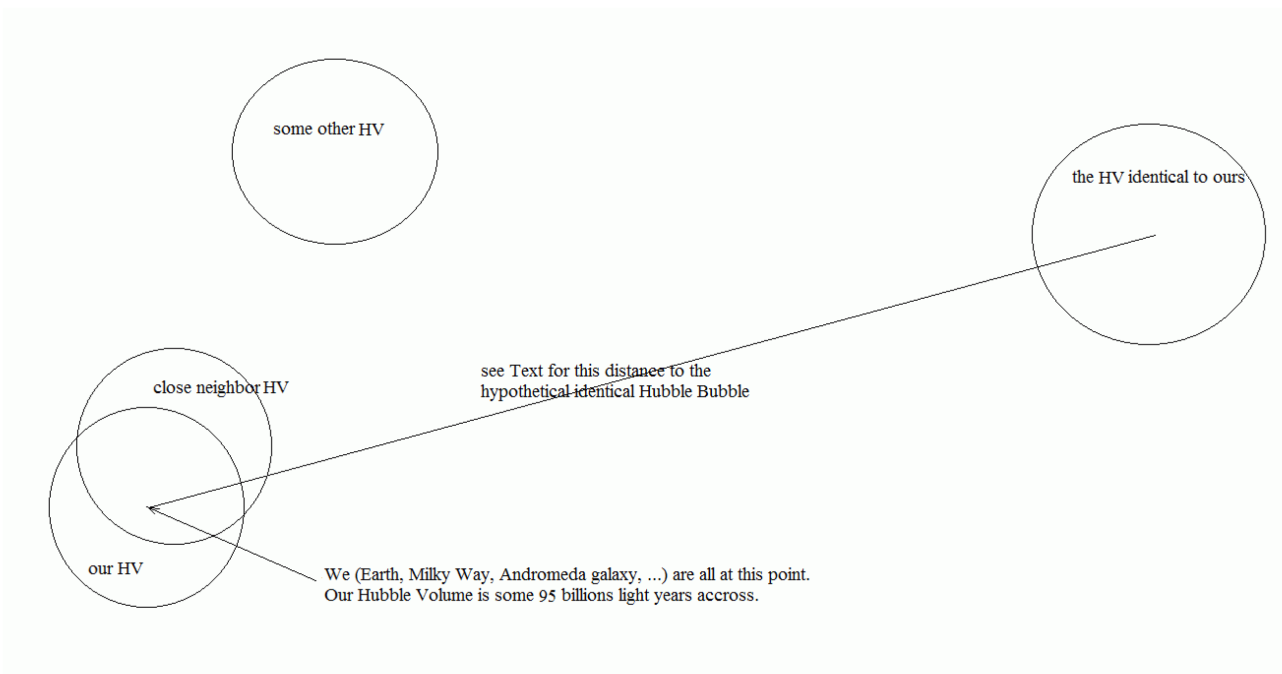


Figure 3.3: Part of a very large Universe. Seeveral Hubble Volumes (HV) corresponding to the present age of the Universe are indicated. Note that the Figure is, obviously, not to scale (see text). But note well that on any scale, Earth and the whole galaxy, as well as the Andromeda galaxy that is 2 million light years away from us, all this is indeed at a single point, relative to even just our own Hubble volume.

to get to that place. It is a straightforward consequence of our elementary introduction to the scientific notation that instead of

$$10^{(10^{115})}$$

the answer is

$$10^{(10^{115}-27)}$$

Now try to visualize the difference between 10^{115} and $10^{115}-27$! For "all practical purposes", they are the same.

This means that the distance to that "mirror" Universe is, for all practical purposes, $10^{10^{115}}$, *independent* of whether you measure it in inches, meters, light years or in Hubble volumes! Billion billion billion times smaller or billion billion billion times larger - does not make any difference.

You may have to spend a few minutes to fully comprehend how incomprehensible this result is. And if you do, you will get a glimpse at the nature of infinity. Galileo, who was not exactly the most modest of scientists, was asked once if he thought the Universe was finite or infinite. He replied, with uncharacteristic modesty, that *either* possibility was beyond his imagination.

3.5.1 A case study in modern cosmology

In 2002, the cosmologist Andrei Linde[6] (from the Department of Physics, Stanford University) published a paper on "Inflation, Quantum Cosmology and the Anthropic Principle" [7] . In it, he calculates that after an inflation lasting 10^{-35} seconds, starting from the Planck size $l_p \sim 10^{-33}$ cm, the Universe grew to the size $l \sim 10^{10^{12}}$ cm.

[6] in 2014, Dr. Linde has been a victim or a premature celebration when one of the leaders of the BICEP2 South Pole experiment visited him, a bottle of champagne in the backpack, to announce their discovery of the echoes of gravitational waves dating from the Inflation period. The visit was duly video recorded, right from Dr. Linde opening the door to answer the bell ring, and posted on Youtube. Sadly, another experiment soon spoiled the general joy (I was one of the victims, too) by finding another, more likely explanation that was much less exciting. Dr. Linde took it in stride - he already won the US\$3 million Fundamental Physics Prize in 2012, so the Nobel Prize would be mostly honorific. But we all hope that he will still get to go to Stockholm, after the next generation of experiments, with even better sensitivity.

[7] Inflation is a process similar to inflating a balloon, postulated to occur at the very beginning of the Big Bang. It 'explains' why today the apparently causally disconnected regions of the Universe seem homogeneous. Dr. Linde is the author of the "eternal chaotic inflation" scenario, in which the birth of new Universes and their subsequent inflation happens frequently, randomly, like bubbles in boiling water.

Quantum cosmology applies Quantum Mechanics to the Universe as a whole, bravely ignoring the role of an observer in quantum mechanics. The Many Worlds interpretation of the theory postulates that whenever there is a quantum choice, all possible choices actually happen: each time the Universe splits into "parallel Universes".

The Anthropic Principle 'explains' the apparent 'fine tuning' of constants and laws of Nature by pointing out that we - necessarily - live in a Universe which has the constants making intelligent life possible.

In 2006, Linde gave a review talk on “Inflationary Cosmology”. In it, he calculates that after an inflation lasting 10^{-30} seconds, starting from the Planck size $l_p \sim 10^{-33}$ cm, the Universe grew to the size $l \sim 10^{10^{10}}$ cm.

After this calculation, both papers continue with an identical text:

“This number is model-dependent, but in all realistic models the size of the Universe after inflation appears to be many orders of magnitude greater that the size of the part of the universe which we can see now, $l \sim 10^{28}$ *cm. This immediately solves most of the problems of the old cosmological theory.”*

HW: Contemplate these numbers, appreciating the duration of the inflation and the initial and final sizes. Then determine

a) what is the difference (rather: what is the ratio) between the two sizes of inflated Universe above (i.e. what is the error in one of the papers, if the other one is in fact correct).

b) what is the numerical result for the ”many orders of magnitude” in the text above.

3.6 Physics is indeed different

The reader must have noticed that so far I have been only dealing with rather old concepts - no mention has been yet made of the Big Bang, the Superstring theory in n+1 dimensions, or the possibility of creating a baby Universe in laboratory. We will have a chance to talk about some of these topics later on, but quite remarkably, the points I am trying to make in this chapter can be made using concepts known since the late 1930s.

What are we to make of all this? Physics indeed is different. With what Wigner called “the unreasonable effectiveness of mathematics”, we seem able to overcome what Weinberg calls an “equally puzzling phenomenon: the unreasonable ineffectiveness of philosophy”. In spite of our limitations (starting with our size, utterly negligible in comparison with the cosmic scale, and grotesquely large and awkward compared to subatomic scale) we seem able to “understand” more and more of Nature, from quarks to clusters of Galaxies, and from the Big Bang 15 billions years ago to the fate of our Sun 5 billions years from now.

We do it in a strange way, though: we don’t really know what we are doing. The late, great John Bell (about whom more later) and the late, great Arthur Koestler (about whom more later, too) both compared the procedure to sleepwalking. As mentioned above, Max Planck discovered the quantum in 1900. But he did not do it by any lofty and profound reasoning. By purely mathematical manipulation, he managed to get two different formulas to merge into a third one, which happened to agree with the experimental data so well, that he set out to investigate: ”What would I had to assume to obtain the third formula directly from my physics assumption?”. And he found the assumption - $E = hf$ - two months after he found the correct formula.

Similarly, Schrodinger did not know the meaning of the wave function ψ when he wrote down his famous equation for it. As mentioned above, again, it was another physicist, Max Born, who came up with the right interpretation.

And so it goes. As always, there are exceptions, most famous being Einstein's Special and General Relativity. But by and large, the process is not rational nor logical. It has been said that in physics, we often can formulate the question only when we almost know the answer, not before!

We shall postpone answering the question posed at the beginning - how justified are the claims of "reading His mind" - until later in the book. However, I hope it is already quite clear that Wheeler was right. Physics indeed is a magic window, our everyday reality is an illusion, and the quantum mechanical "illusion" is a reality. It is all marvelous and probably undeserved. As it often happens, the last words goes to Albert Einstein:

The most incomprehensible thing about the world is that it is comprehensible.

Chapter 15

Appendices (many more to come)

15.1 Appendix 1: The (ir)rational numbers

First recall from the main text: rational numbers are defined as numbers that can be expressed as a ratio of finite integers:

$$r = \frac{n}{m}$$

examples: $3/2 = 1.5 \quad 7/4 = 1.75 \quad 2/50 = 0.04$

or r = 2521/7000 = 0.360142857142857142857142857142857142857.....

That last example looks somewhat different from the previous ones: those have a finite number of decimal digits, while the decimal expansion of 2521/7000 never ends. But a brief inspection of the decimal expansion shows that after the initial 0.360 the expansion has six digits (142857) that repeat without end. And there is an infinite number of such cases (the simplest one being 1/3 = 0.33333333 or even 1/2 = .5000000 where the repeating group has just one digit - “3”and “0”, respectively. So all the “finite” expansions are in fact infinite, with an infinite series of zeroes repeating infinitely after the last non-zero digit.

As we discussed in the text, it was truly devastating for the Pythagorean worshippers of the rational numbers to discover that $\sqrt{2}$ is not rational – it cannot be expressed as m/n. Any expression with finite integers in the numerator and denominator is just an approximation:

$$\sqrt{2} = 1.4142135623730950488016887242097...... = \frac{14142135623730950488016887242097.......}{10000000000000000000000000000000.......}$$

It should be appreciated how difficult it is to prove that this decimal expansion never ends in an infinitely repeating group of digits. Attempts at actually calculating the digits can never prove irrationality, because there is no limit as to when the repeating block might start or how many digit it might have, while your attempt must end somewhere by you simply giving up at some point. This did not prevent some truly heroic calculations (up to 100 billion digits so far. Since we now have the proof, such calculations are being done for a different reason as we discuss below.).

15.1.1 The incredible[sic] proof that $\sqrt{2}$ is not a rational number.

So now please follow the reasoning (if you have never done something like this, you will have to be patient - the secret of most complicated math is a patient addition of several steps, each of which is very simple):

We start with the assumed property

$$\sqrt{2} = m/n$$

where both m and n are finite integers.

Then square both sides of the equation and you get $2 = m^2/n^2$
This means $m^2 = 2n^2$
This means m^2 is an even number.
This means that m itself is also an even number.
But the above equation $2 = m^2/n^2$ is equivalent to $n^2 = m^2/2$
Therefore, since m is even i.e. m is a multiple of 2, m^2 is a multiple of 4
Therefore n^2 is even
therefore n is even

So both m and n are even so you can divide both of them by 2 to get
$\sqrt{2} = m'/n'$ where $m' = m/2$ and $n' = n/2$
and you can repeat the whole argument to get
$\sqrt{2} = m''/n''$ where $m'' = m'/2$ and $n'' = n'/2$
and this never ends.

But it *must* end because both the initial m and n are finite.

So we obtained a logical contradiction, and it just cannot be done: $\sqrt{2}$ can not be expressed as a ratio of two finite integers. As illustrated above, by increasing the size of the integers we can get arbitrarily close to $\sqrt{2}$ but we will never get there.

In the heading of this section I called the proof "incredible". Well, it better be credible, since it is a proof. But it is really quite extraordinary. In a few lines, it accomplishes something that the most powerful computer imaginable could not accomplish if it ran since the Big Bang nonstop. In a sense, it shows that what we have in our heads is not just a computer. The infinite reach of this most extraordinary accomplishment is of the same kind as the reach of the Goedel Undecidability Theorem (very roughly, he proved that some truths cannot be proven) or of the Bell Inequality (restricting the properties of all imaginable future theories).

We will have more to say on this topic, but for now I just propose that the proof that $\sqrt{2}$ is not a rational number should be in everyone's intellectual toolbox.

15.1.2 More fun with irrational numbers

There are many more numbers that have been proven irrational. In fact, there is infinitely more irrational numbers than rational, although the number of the rational numbers is infinite to start with – you can say that some infinities are (much) more infinite that others :-)

Perhaps the most famous example of an irrational number is the value of π - the ratio of the circumference of a circle to its radius[1]. Proof is not easy here so you just have to take my word for it.

A sequence of digits or characters are called "normal" if any possible combination (of any length) occurs in that sequence. Obviously, all normal numbers are irrational, and instinctively you would think that all irrational number have to be normal, but that is not true. In fact it is extremely difficult to prove whether or not a given irrational number is indeed normal. For example, we do not know if π or even $\sqrt{2}$ are normal[2]. We just know that "almost all irrational numbers are normal" , or "we have evidence that π is likely to be normal".

The implication of the existence of normal numbers may seem staggering: the sequence of digits of any single normal number represents a realization of the proverbial army of monkeys, each typing madly at a typewriter (or a realization of the Library of Borges and other equivalent discussions). Everything that has been ever written, everything that will ever be written, the full sequence of my DNA and of any alien (if they have it or something similar), all of this and more is included in the sequence. It seems truly mind boggling, but it becomes somewhat less mind boggling if you realize that it is absolutely useless, to the point of being meaningless[3]. It is utterly useless because a quick back-of-the-envelope calculation shows that just a single sentence of just 100 characters, each of which can be one of 35 letters or punctuation signs, will have $35^{100} = 10^{154}$ possible different variations. For comparison, the number of atoms in the observable Universe is estimated at only about 10^{80}, and the number of seconds elapsed since the Big Bang is a pitiful 10^{18}. Some readers may want to re-read the section on scientific notation to appreciate this comparison. So even at 100 characters, it is clearly impossible to even contemplate a) producing such a library b) using it to find any *interesting* 100 character sequence. And the difficulty goes up by a factor 1,000 of with every two additional characters in the sentence - you want to do a book? or all books? or a genome? :-)

And yet: a little thought about such schemes does provide us with yet another glimpse at infinity, so it is not completely useless. And since similar considerations are often used in the discussions of the origin of Life, we will revisit these topics again ....

And last (for now – there is a quasi-infinite amount of fascinating result in the theory of numbers - see Wikipedia for a good starting point) but not least: in Musical Acoustics there

[1]Note: I am typing this section on March 14, 2015; i.e. on 3.14.15! This is almost as scary as claims of Bach's cabalah :-)

[2]In cases like this, while waiting for a next Fermat to prove (or disprove) something, we do what might be called "experimental mathematics" – we use large computers to determine the values of π with 100 billions decimal digits, ad investigate the statistical properties of the sequence.

[3]This did not prevent a notorious producer of mind-boggling books about all kinds of science disciplines to re-discover this mind-boggling factoid, and then revel in the excitement it provoked in innocent readers of his blog whose minds were thereby boggled.

is an interesting analogy of a “normal” number containing (unbeknownst to us) all imaginable sequences. In acoustics, we find out that a single tone (such as when you sing a simple “aah”) contains, again: unbeknownst to you, all the consonant intervals: an octave, Major third, minor third, even a whole Major chord! In contrast to the above phantasmagoric property of the “normal” numbers, this acoustical phenomenon is actually of extreme practical importance in the development of analysis of sound in the brain of the newborn baby (more about this in the Physics of Music chapter).

15.2 Mandelbrot Set

In our final warm-up we will consider the famous Mandelbrot set. Most people have seen the spectacular images, and many sleepless nights have been spent with one of the many free browsers exploring the limitless zooming into the set[4] . But very few really understand the math, and implications of all this. As I say to students: this will not necessarily improve your grades or job prospects, but it will make you a better-educated person.

The Mandelbrot set has been called the most beautiful set in all of Mathematics. It is remarkable by the utter simplicity of its definition, compared to the *infinite* complexity of the resulting pattern. We will use this set as a metaphor for the concept of emergent complexity, as an illustration of the nature of the laws of physics (including the possible(?) Theory of Everything), and we will compare its patterns to the subtle arrangements of musical notes in Bachs Art of Fugue. So it is worth your while to spend some time (patiently, if necessary) to really understand how it works. As a byproduct, you will learn a valuable lesson about "complex numbers".

If you already know all this, please skip down to the section on Mandelbrot, Dirac and Bach.

15.2.1 Complex Numbers and the Mandelbrot Set

As we said, the definition is simplicity itself:

Take any number (x_0) and calculate the next one (x_1) and then the next one (x_2), then x_3, x_4, and so on, according to the prescription:

$$x_{n+1} = (x_n)^2 + x_0$$

for n= 0, 1, 2, 3, ...

In words: start with an initial number x_0 - a "candidate for Mandelbrot membership". Then determine the next number as "the previous number squared plus the initial number", and keep repeating this.

This is known as a "recursive formula" where there is a prescription how to calculate the next member of the series if you know the preceding one. This, and a choice for the very first member (x_0) allows sequential calculation of all the members of the sequence. We then say: if the sequence of numbers x_n escapes to infinity, then we say that the starting number x_0 is not a member of the Mandelbrot set; otherwise x_0 is a member of the set.

[4]I highly recommend the QUICKMAN at http://sourceforge.net/projects/quickman/ . The comment by quantumvlad is by me, so you can be sure ai really like it!

The recursive formula

$$x_{n+1} = (x_n)^2 + x_0$$

may look simple enough, but perhaps somewhat dense to people without much math background, so here are two examples to illustrate the process:

a) start with $x_0 = 1$ and apply the recursive equation for n=0,1,2 ...:

$$x_1 = x_0^2 + x_0 = 1^2 + 1 = 1 + 1 = 2$$

$$x_2 = x_1^2 + x_0 = 2^2 + 1 = 4 + 1 = 5$$

$$x_3 = x_2^2 + x_0 = 5^2 + 1 = 25 + 1 = 26$$

and you dont need to be a Math major to see that this goes, inexorably, to infinity. Therefore, the number $x_0 = +1$ is not a member of the Mandelbrot set.

b) now take a different starting number: $x_0 = -1$ and calculate:

$$x_1 = x_0^2 + x_0 = (-1)^2 + (-1) = 1 - 1 = 0$$

$$x_2 = x_1^2 + x_0 = 0^2 + (-1) = 0 - 1 = -1$$

$$x_3 = x_2^2 + x_0 = 0$$

Clearly, $x_4 = -1$, $x_5 = 0$ and so on: the numbers alternate between 0 and -1, so the sequence never escapes to infinity, and the number $x_0 = -1$ is a member of the set.

So this procedure really is simple, but the resulting set is quite simple, too - in fact it is boring: with just a little more math one can show that the set is just a straight line starting at $x_0 = -2$ and ending at $x_0 = +0.25$.

15.2.2 Complex numbers

Things become interesting indeed when we keep the same procedure as above, but instead of "ordinary" numbers we allow the numbers to be "complex". The origin of the general need for complex numbers is the desire to be able to reverse the operation of squaring: we know that $2^2 = 4$, $3^2 = 9$ and so on. The reverse is called a "square root": the number whose square is 25 is called "square root of 25", and it is equal 5, because $5^2 = 25$. Similarly, the two statements

$$7^2 = 49$$

and

$$\sqrt{49} = 7$$

are equivalent.

The key to complex numbers is to ask yourself a question: what is the square root of -1, i.e. what is the number x such that $x^2 = -1$. Obviously, no ordinary number x has the property that $x^2 = -1$. But this does not stop our imagination: we imagine a new category of numbers, call them "imaginary numbers, and *define* one of them as $i = \sqrt{-1}$ i.e. $i^2 = -1$. It turns out that all other imaginary numbers are just plain multiples of i, for example:

$$\sqrt{-9} = \sqrt{9.(-1)} = \sqrt{9}\sqrt{-1} = 3i$$

and so on.

And finally, by adding an ordinary (or real) number x and an imaginary number iy, we get a *"complex* number z

$$z = x + iy = (x, y)$$

so that $d = 1 + 2i$, $e = 1.5 - .5i$, $f = 2.5$ and $g = -1.2i$ are examples of complex numbers; their real and imaginary parts are d = (1.,2.), e = (1.5,-0.5), f = (2.5,0) i.e. f is a pure real, and g = (0,-1.2) i.e. g is pure imaginary.

And finally, we plot complex numbers as points in a plane, with the real part in the x-direction (horizontal) and the imaginary part in the y-direction (vertical)- see Figure.

We are now ready to extend the Mandelbrot set to the complex plane. We simply repeat the basic procedure above, except now we are free to choose for the initial value an arbitrary complex number z_0, corresponding to a pixel in the display being prepared. We then evaluate[5] the sequence:

$$z_{n+1} = (z_n)^2 + z_0$$

for n= 0, 1, 2, 3, ... If the sequence of complex numbers z_n escapes to infinity, then we say that the starting complex number z_0 is not a member of the Mandelbrot set; otherwise z_0 is a member of the set. If z_0 indeed turns out to be a member, make he pixel black, otherwise we leave it blank. Then repeat this procedure for all pixels of our display, and you are done.

It should be obvious by now that the Mandelbrot set will be some kind of a blob in the x-y plane, located close to (x=0, y=0) - large initial numbers will get even larger when squared, and will quickly escape to infinity. And if asked for a prediction as to the shape of that blob, a reasonable guess might be: a circular or an elliptical disk or some other simple shape.

15.2.3 Mandelbrot, Dirac and Bach

However, the shape of the set is anything but simple. In spite of the simplicity of the algorithm, calculations to determine if an arbitrary complex number is or is not a member of the set are time consuming, and it took modern computers to even discover that the set looks interesting. Today, many software packages for exploration of the Mandelbrot set are available on www - but be warned: this is addictive! Many of these programs use color, to indicate how close a given point is to the set . For my purposes I find the use of color

[5]The Mandelbrot Appendix contains more explanations and technical details about the set and about complex numbers in general

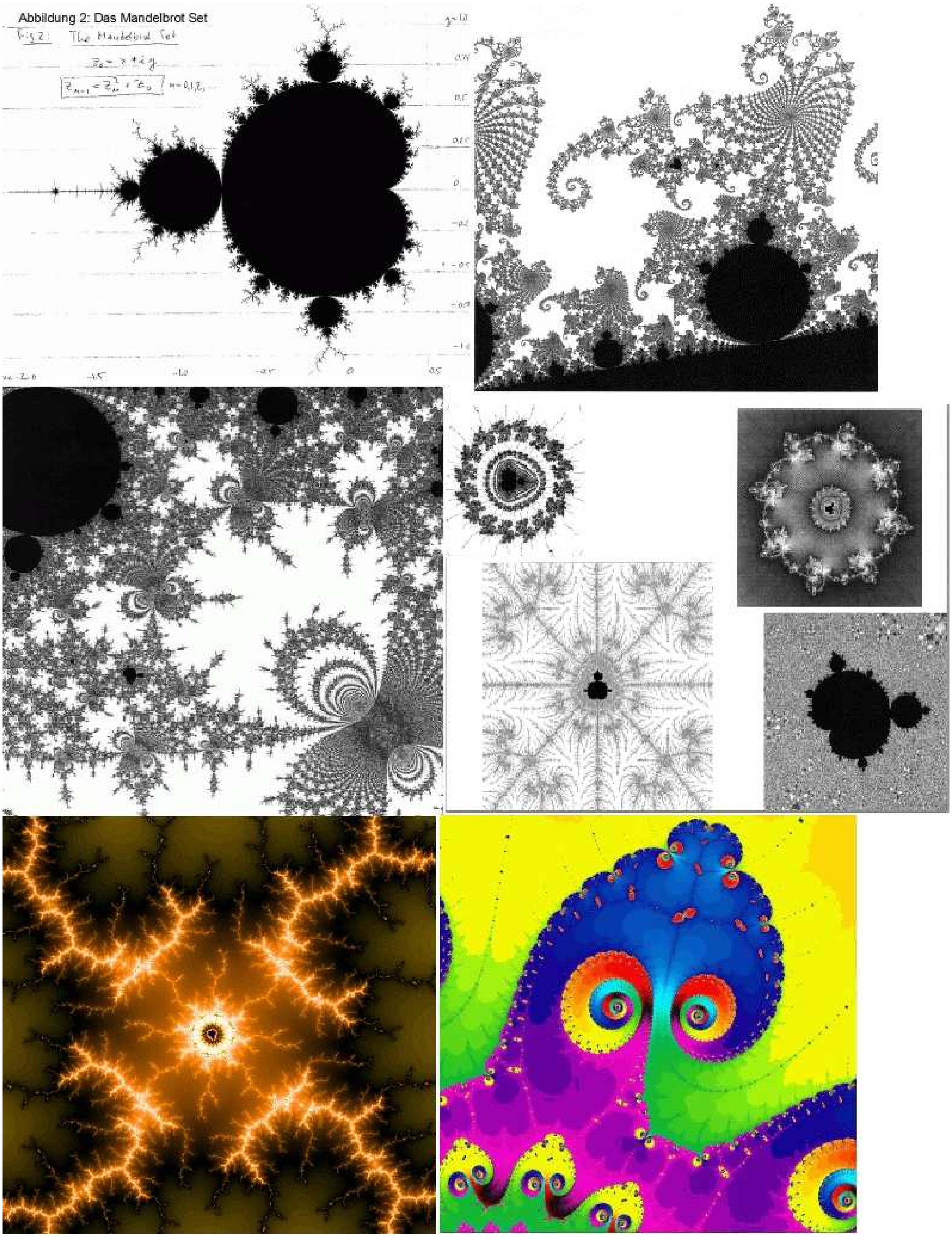


Figure 15.1: Caption for MandCollage

distracting - a point in the complex plane either is a member, or is not - and therefore I use black-and-white printing, with a procedure of my design to ensure that the thin filaments are not missed.

Figure shows the whole set - it is a beetle-like object, with cardioid-shaped main body, and many protuberances. It is upon zooming at various regions that you discover the true fractal nature of the set: the complexity of the set is inexhaustible. Delicate webs of filaments contain small, distorted versions of the original "beetle" - and by zooming on the periphery of that beetle, you find delicate webs of filaments containing small, distorted beetles ... and so on and so on. As you travel along the beetle surface, the webs are self-similar but not identical, and they smoothly morph from baroque ornamentations to jagged snowflakes and exuberant spirals (see Fig. 1.7). The extraordinary variety of the way in which the beetles are nestled in the filaments is illustrated on Figure. The highest zoom I generated (the neighborhood of the point

$$z_0 = -.7541521031327 + 0.050815525271\ i$$

magnified by a factor of over 100,000,000 - see Figure 4 top left) shows the distorted beetle surrounded by filaments which are so dense that the resolution of my printer - and of my eye as well - is not sufficient to see the details.

As mentined above, people often like to artificially add color to displays of the Mandelbrot set by calculating the isoclines on the "Mandelbrot mountain" and coloring the different height bands. This leads to sometime psychodelic, sometime beautiful (and sometimes both) images - see Figures for some spectacular examples. This is one of the lovely cases where science meets art in a happy cooperation.

Mandelbrot set is an infinitely complex Universe generated by an absurdly simple Law. It should remind you of those nerdish Physics students with Maxwell equations on their T-shirts. As the great Dirac said, the simple equations of electromagnetic interactions between the nuclei and electrons(in their quantized form) govern "most of Physics and all of Chemistry" - and he considered the "problem of life" as just another, just a little more difficult *Physics*[sic] problem ...

On the other side of the exuberance - humility spectrum, I am reminded of the Mandelbrot set by the structure of Bach's fugues, especially in the Art of Fugue. As I argue in the detailed description of many of the fugues, the main Theme and its variations can be seen as the Mandelbrot "beetles", all the way to the distortions upon "zooming". And the various motifs which accompany the Theme – and which make the fugues so interesting – cannot but remind me of the delicate filaments.

Finally note that the Mandelbrot set only became interesting when we switched from real numbers to complex numbers. This alone would have made the invention of complex numbers worthwhile. But in fact, complex numbers provide powerful and beautiful tools in innumerable Math, science and engineering applications, and we shall see that quantum mechanics (and therefore the very existence of the Universe) would be impossible without complex numbers. I believe that a brief introduction such as the one in this chapter should be part of the standard *high school* curriculum.

15.3 Mandelbrot Appendix

15.3.1 The Vladibrot Set

I propose, as a very useful exercise, to first consider a "Vladibrot Set"[6] defined by a procedure:

start with a complex number $z_0 = x_0 + iy_0$. Then keep evaluating

$$z_{n+1} = z_n \sqrt{x_n^2 + y_n^2}$$

for n= 0, 1, 2, 3, ... If the sequence of complex numbers z_n escapes to infinity, then we say that the starting complex number z_0 is not a member of the Vladibrot set; otherwise z_0 is a member of the set.

This procedure looks as complicated (if not more) as the Mandelbrot procedure; yet the shape of the resulting Vladibrot set is simple indeed - determine the shape as an exercise[7].

The solution of this exercise can be found at the book website. However, I urge the reader not to give up easily. You will see why the Vladibrot set never became famous ...

15.3.2 Back to the complex Mandelbrot

Recall the examples of complex numbers given in the main text:

$d = 1 + 2i$, $e = 1.5 - .5i$, $f = 2.5$ and $g = -1.2i$

The real and the imaginary parts of the above complex numbers are

d=(1.,2.) e=(1.5,-.5) f=(2.5,0.) and g=(0.,-1.2) respectively.

Note that the real numbers (e.g. the $f = 2.5$ above) and imaginary numbers (e.g. $g = -1.2i$ above) are special cases of complex numbers.

We need to be able to add and multiply complex numbers. Adding is easy: add the real parts to obtain the real part of the sum, add the imaginary parts to get the imaginary part. So for example:

$$d\ +\ e = (1 + 2i) + (1.5 - .5i) = (1 + 1.5) + i(2 - .5) = 2.5 + 1.5i$$

Multiplication is not much more difficult: just remember that $i^2 = -1$:

$$d \times e = (1 + 2i)(1.5 - .5i) = 1 \times 1.5 + 1 \times (-.5i) + 2i \times (1.5) + 2i \times (-.5i) =$$

$$= 1.5 - .5i + 3i - 1 \times i^2 = 1.5 + 1 + i(-.5 + 3) = 2.5 + 2.5i$$

[6]Last year, a student wondered how I could have the chutzpah to name a set after myself. Please see the last sentence of this section and construct the "Vladibrot" set to appreciate my self-deprecating joke.

[7]Hint: from Dr. Pythagoras we know that $\sqrt{x^2 + y^2}$ is simply the "magnitude" of the complex number $z = x + iy$ - see Figure 1. Therefore, for the Vladibrot procedure, all the numbers z_n are "in the same direction" as the original z_0 - they just have different magnitudes ("lengths").

Let us test[8] a purely imaginary starting point $z_0 = i$ for Mandelbrot membership

$$z_1 = z_0^2 + z_0 = i^2 + i = -1 + i$$

$$z_2 = z_1^2 + z_0 = (-1+i)^2 + i = ((-1)^2 + 2(-1)i + i^2) + i = (1 - 2i - 1) + i = -i$$

$$z_3 = z_2^2 + z_0 = (-i)^2 + i = -1 + i$$

In this case, we are lucky: z_3 turns out to be is the same as z_1 - therefore, the sequence will oscillate between $-i$ and $-1+i$, never escaping to infinity. Therefore, $z_0 = i$ is a member of the Mandelbrot set.

Together with our previous results from the real axis, we have determined two members of the Mandelbrot set: $z_0 = -1$ and $z_0 = i$ (see Figure 1). As an exercise, please verify that $z_0 = -2$ and $z_0 = -i$ are members of the set, and $z_0 = -2i$ is not.

15.3.3 The amazing and indispensable complex numbers

Originally invented as a purely mathematical exercise, the complex numbers became extremely useful - you could say that we now believe that "imaginary numbers really exist". Personally I consider the complex calculus (differentiation and integration of complex functions of complex arguments) as not only the most useful, but also one of the most beautiful parts of mathematics.

But it is in Quantum Mechanics that the coplex numbers are truly indispensable - if you want to sound funny or profound, you can say: "Quantum World is Complex". There are two main reasons for this[9]:

First: the Schroedinger wave function is complex, and obviously the Schroedinger wave function is complex, too. But you might say: that is not a big deal. If you have an equation of the form

$$f(x) = g(x)$$

where f(x) and g(x) are two complex functions, this is simply equivalent to two real equations:

$$Re(f(x)) = Re(g(x))$$
$$Im(f(x)) = Im(g(x))$$

But the Schroedinger equation has an extra "i" on the left-hand side:

$$i\ f(x) = g(x)$$

and therefore the functions f(x) and g(x) are coupled in a much less trivial way:

[8]note: to calculate $(-1+i)^2$ I used the fact that $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$ - and of course $i^2 = -1$

[9]More about this in the chapter on Quantum Mechanics

$$Re(f(x)) = -Im(g(x))$$
$$Im(f(x)) = +Re(g(x))$$

An even more physical reason is the need for a sensible interpretation of the wave function. If it is complex, it provides us with the remarkable, and much needed ability to have a constant magnitude, yet to exhibit an oscillatory behavious at the same time (more about this in the Quantum Mechanics chapter).