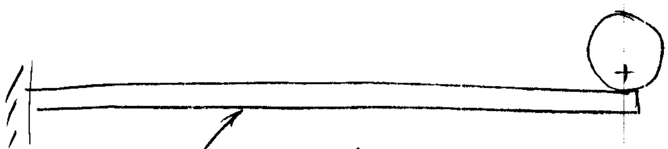
ME 356 Fall 2001

Solutions to Homework #6

By: Nick Silverman

Questions, comments, gripes, and most importantly praise should be directed towards your loving TA during office hours MTF.

Text Example 7-13.



32" x 2" x 1/4" leaf spring.

For the data on p. 405,

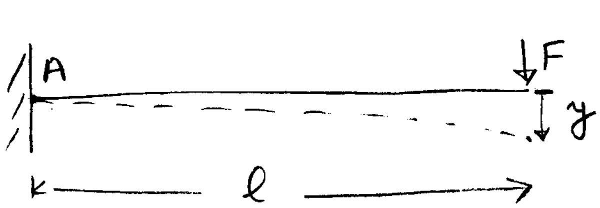
(a) Plot the Goodman failure locus and the load line.

(b) find the factors of safety guarding against fatigue failure (n_f), and guarding against yielding in the first cycle (n_y) corresponding to 2 in. and 5 in preload.

(c) what are the factors of safety based on preload deflection.

Preliminary Calculations. (FROM Text p. 406-407)

$I = 0.0026 \text{ in}^4$



$$y = \frac{F l^3}{3EI}$$

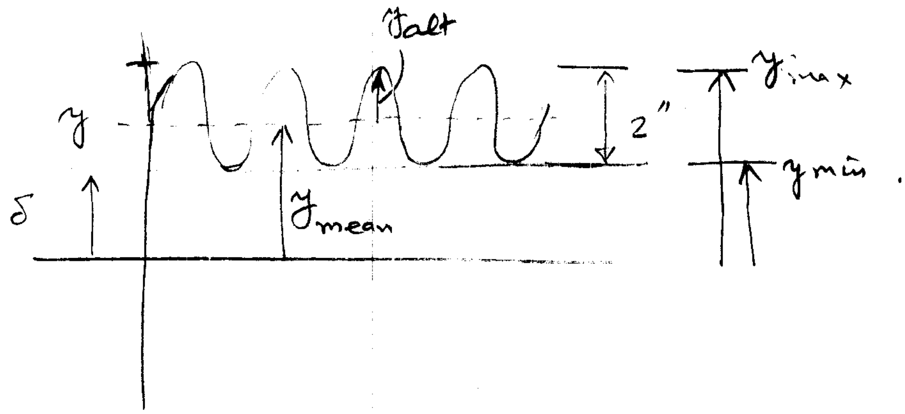
$$F = \frac{3EI}{l^3} \cdot y \quad , \quad M = Fl \quad , \quad \sigma = \frac{Mc}{I}$$

$$\sigma = \frac{Mc}{I} = Ky \quad \text{where } K = 11 \text{ kips/in.}$$

note, stress is being evaluated at the base, where M is maximum.

Let us plot a displacement diagram. Note, displacement, Force, and Stress are proportional to each other.

Let δ be the preload deflection.



$y_{min} = \delta$

$y_{max} = \delta + 2$

$\sigma_{max} = K\, y_{max} = K(2+\delta)$

$\sigma_{min} = K\, y_{min} = K\,\delta$

Now, $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{K(2+\delta) - K\delta}{2} = K$

$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{2K + K\delta + K\delta}{2} = \dfrac{2K + 2K\delta}{2} = K + K\delta$

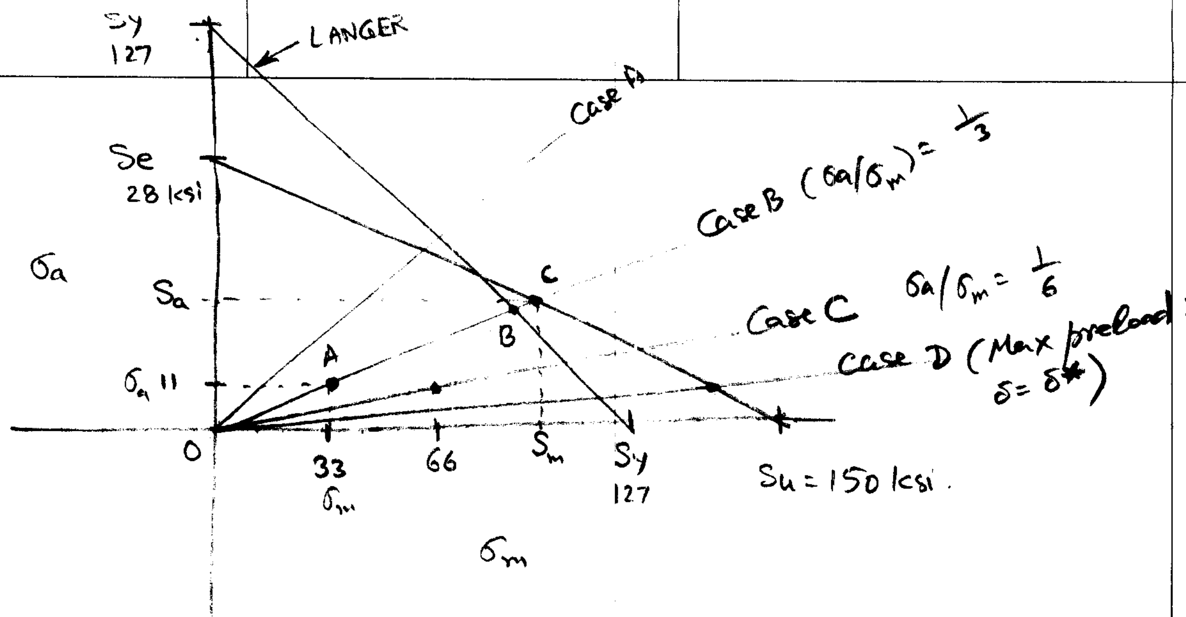
Now consider three cases:

Case A $\delta = 0$ (no preload) $\sigma_a = K$; $\sigma_m = K$ $\sigma_a/\sigma_m = 1.0$

Case B $\delta = 2$ in. $\sigma_a = K$; $\sigma_m = K + 2K = 3K$ $\sigma_a/\sigma_m = 1/3$

Case C $\delta = 5$ in $\sigma_a = K$; $\sigma_m = K + 5K = 6K$ $\sigma_a/\sigma_m = 1/6$

(note, here K = 11 kip. Here it has been multiplied by deflection in inches).



(Not to Scale)

Analytically: (Goodman)

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_f}$$

$$n_f = \frac{S_e\, S_{ut}}{\sigma_a S_{ut} + \sigma_m S_e} = \frac{(28)(150)}{(11)(150) + (33)(28)} =$$

for yielding: (Case B)

$\sigma_{max} = 4K = 44$ ksi. (note, Von Mises $\sigma' = \sigma_{max}$ in this case, since σ is the only stress)

$$\therefore \quad n_y = \frac{S_y}{\sigma_{max}} =$$

Case D: $\delta = \delta^* =$ Max preload based on Goodman:

$\sigma_a = K$

$\sigma_m = K + K\delta^* = K(1+\delta^*)$

Now, with $n_f = 1$: $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = 1$, or $\frac{K}{S_e} + \frac{K(1+\delta^*)}{S_u} = 1$

Solve for δ^*, then $n_f = \frac{\delta^*}{\delta}$

Example 7-14

TABLE E-20 GIVES: $S_{UT} = 440$ MPa and $S_{yt} = 370$ MPa

$S_e' = .506(440) = 223$ MPa

$k_a = 4.45(440)^{-.265} = .886$

$k_b = \left(\frac{d}{7.62}\right)^{-.107} = .833$

$\therefore \quad S_e = (.886)(.833)223 = 165$ MPa

(a) USE TABLE E-16 $\quad a/D = \frac{6}{42} = .143 \quad \frac{d}{D} = \frac{34}{42} = .810$

$\therefore$ USING LINEAR INTEPOLATION WE GET:

$A = .798$ and $k_t = 2.366$ FOR BENDING and

$A = .89$ and $k_{ts} = 1.75$ FOR TORSION

So $\quad Z_{net} = \frac{\pi A}{32D}(D^4 - d^4) = \frac{\pi(.798)}{32(4.2)}\left[(4.2)^4 - (3.4)^4\right] = 3.31 \text{ cm}^3$

$J_{NET} = \frac{\pi A}{32}(D^4 - d^4) = \frac{\pi(.89)}{32}\left((4.2)^4 - (3.4)^4\right] = 15.5 \text{ cm}^4$

NOTCH SENSITIVITIES From 5-16 and 5-17:

$q_{Bending} = .78 \qquad q_{Torsion} = .96$

$k_f = 1 + q(k_t - 1) = 1 + .78(2.366 - 1) = 2.07$

$k_{fs} = 1 + .96(1.75 - 1) = 1.72$

BENDING STRESS:

$\sigma_x = k_f \frac{M}{Z_{NET}} = 2.07 \frac{150}{3.31} = 93.8$ MPa

TORSIONAL STRESS:

$\tau_{xy} = k_{fs} \frac{TD}{2 J_{Net}} = 1.72 \frac{120(4.2)}{2(15.5)} = 28.0$ MPa

FIND VON MISES STRESSES:

$$\sigma'_m = 0$$

$$\sigma'_a = (\sigma_{xa}^2 + 3\tau_{xya}^2)^{1/2} = (93.8^2 + 3(28.0)^2)^{1/2} = 105.6$$

$$\therefore \quad \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n_f} \quad \text{so} \quad n_f = \frac{S_e}{\sigma'_a}$$

$$= \frac{165}{105.6} = \boxed{1.56}$$

$$n_y = \frac{S_y}{\sigma'_a} = \frac{370}{105.6} = \boxed{3.50}$$

(b) $T_a = \dfrac{160 - 20}{2} = 70 \text{ N}\cdot\text{m}$

$$T_m = \frac{160 + 20}{2} = 90 \text{ N-m}$$

$$\tau_{xya} = k_{fs} \frac{T_a D}{2 J_{NET}} = 16.3 \text{ MPa}$$

$$\tau_{xym} = k_{fs} \frac{T_m D}{2 J_{net}} = 21.0 \text{ MPa}$$

THE STEADY BENDING STRESS:

$$\sigma_{xm} = k_f \frac{M_m}{Z_{NET}} = 2.07 \frac{150}{3.31} = 93.8 \text{ MPa}$$

VON MISES COMPONENTS:

$$\sigma'_a = \left[3(16.3)^2\right]^{1/2} = 28.2 \text{ MPa}$$

$$\sigma'_m = \left[93.8^2 + 3(21)^2\right]^{1/2} = 100.6 \text{ MPa}$$

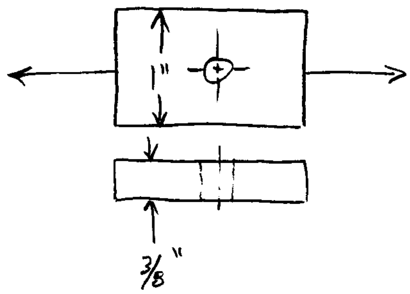
FIND n_f:

$$\frac{1}{n_f} = \frac{\sigma'_a S_{ut} + \sigma'_m S_e}{S_e S_{ut}} = \frac{(28.2)(440) + (100.6)(165)}{(440)(165)} = .39954$$

$\therefore \boxed{n_f = 2.50}$

and $n_y = \dfrac{S_y}{\sigma_a' + \sigma_m'} = \dfrac{370}{28.2 + 100.6} = \boxed{2.87}$

7-16) CD AISI 1018 STEEL



$F_{max} = 3000$ lbf
$F_{min} = 800$ lbf

ESTIMATE FACTORS OF SAFETY USING MODIFIED GOODMAN CRITERIA.

TABLE E-20 $S_{UT} = 64$ kpsi $S_y = 54$ kpsi

$A = .375(1 - .25) = .281 \text{ in}^2$

$$\sigma_{max} = \frac{F_{max}}{A} = \frac{3000}{.281}(10^{-3}) = 10.67 \text{ kpsi}$$

$\therefore \; n_y = \dfrac{S_y}{\sigma_{max}}$ SINCE THERE IS ONLY ONE STRESS.

$$= \frac{54}{10.67} = \boxed{5.06}$$

NOW WE NEED TO FIND CORRECTED ENDURANCE STRENGTH:

$S_e' = .506(64) = 32.4$ kpsi

$k_a = 2.67(64)^{-.265} = .887$

$k_b = 1$ Since it is Axial

$k_c = 1.23(64)^{-.078} = .889$

$\therefore \; S_e = (.887)(1)(.889)(32.4) = 25.5$ kpsi

FROM TABLE E-15.1 $w = 1$ in $d = 1/4"$ $d/w = .25 \; \therefore \; k_t \approx 2.45$

$$k_f = \frac{k_t}{1 + \frac{2}{\sqrt{r}}\left(\frac{k_t - 1}{k_t}\right)\sqrt{a}} = \frac{2.45}{1 + \left(\frac{2}{\sqrt{.125}}\right)\left(\frac{2.45-1}{2.45}\right)\left(\frac{5}{64}\right)} = 1.942$$

$$\sigma_a = k_f \left| \frac{F_{max} - F_{min}}{2A} \right| = 1.942 \left(\frac{3.000 - .800}{2(.281)} \right) = 7.60 \text{ kpsi}$$

$$\sigma_m = k_f \left| \frac{F_{max} + F_{min}}{2A} \right| = 1.942 \left(\frac{3.000 + .800}{2(.281)} \right) = 13.13 \text{ Kpsi}$$

USING GOODMAN CRITERIA :

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{UT}} = \frac{1}{n_f} \quad \therefore \quad n_f = \frac{S_e S_{UT}}{\sigma_a S_{UT} + \sigma_m S_e}$$

$$= \frac{(25.5)(64)}{(7.60)(64) + (13.13)(25.5)}$$

$$\boxed{n_f = 1.987}$$

7-17) USE INFORMATION FROM 7-16.

$$\sigma_a = 1.942 \left| \frac{3.000 - (-.800)}{2(.281)} \right| = 13.13 \text{ kpsi}$$

$$\sigma_m = 1.942 \left| \frac{3.000 + (-.800)}{2(.281)} \right| = 7.60 \text{ kpsi}$$

$$n_f = \frac{S_e S_{UT}}{\sigma_a S_{UT} + \sigma_m S_e} = \frac{64(25.5)}{(13.13)(64) + (7.60)(25.5)}$$

$$\boxed{n_f = 1.578}$$